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Donald W. Mertz

Essays on Realist Instance Ontology and its Logic

Predication, Structure, and Identity

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To Polly

Sine Qua Non

Contents

Preface

The essays collected in this volume have as their subject matter the realist ontology of unit attributes, or 'instances', and aspects of the logic it implies. Here a relation (or property as the limiting one-subject case), insofar as it qualifies/characterizes jointly the entities of a particular *n*-tuple of subjects, e.g., Part-of$_i$(*a*,*b*), is held to be unique and individuated to that set of subjects. No other *n*-tuple of subjects can be characterized by numerically the same instance, e.g., if Part-of$_i$(*a*,*b*), Part-of$_j$(*c*,*d*), and $<a,b> \neq <c,d>$, then Part-of$_i \neq$ Part-of$_j$. The fact that there are multiple exactly resembling instances of a 'kind', e.g., Part-of$_i$ and Part-of$_j$, is, on this ontology, the result of numerically the same intension-content being a constituent of each such instance, i.e., exactly resembling instances share as their sole qualitative content one and the same universal, e.g., Part-of. Hence, the use of the term 'realist'. Though left mostly undeveloped, realist instance ontology has had a persistent history, it being found as far back as Plato and Aristotle, was dominant among the medieval scholastics, and is a thesis advocated by various modern and contemporary philosophers. A prominent contemporary version (e.g., as expanded upon by E. J. Lowe) is one developing the four-category ontology Aristotle briefly describes in the *Categories* (2a20-b9). In contrast and more widely discussed in current ontology is the nominalist alternative that denies the possibility of numerical sameness of the qualitative content of distinct unit attributes. It is the nominalist theory of predicate instances that is usually intended by the contemporary term 'trope theory', though the doctrine has been advocated variously back at least to Ockham.

In the contained essays I argue against both classic Aristotelian substance/attribute ontology and nominalistic trope theory. This is done in the context of advancing a positive theory of ontic predication with the following key theses:

1. *Structure* or *system* is a ubiquitous feature of reality and requires an ontological account.

2. A structure as a non-random, qualitatively determined/specified unification among yet diverse elements has among its essential compo-

nents polyadic relations, full and unreduced (monadic properties being the limiting case).

3. Polyadic relations are ontic predicates which as such are properly analyzed as *intension-determined agent-combinators* among *n*-tuples of subjects.

4. As an agent-combinator, an ontic predicate is unique as a unifying act/event among its particular *n*-tuple of subjects, and in this unrepeatability it is an *individual* or *particular*. I.e., ontic predicates exist as individuated instances.

5. The intension aspect when conceptually abstracted from a predicate instance, and what is numerically the same qualitative content of all exactly resembling instances, is non-combinatorial, i.e., an intension/universal is *not* an ontic predicate.

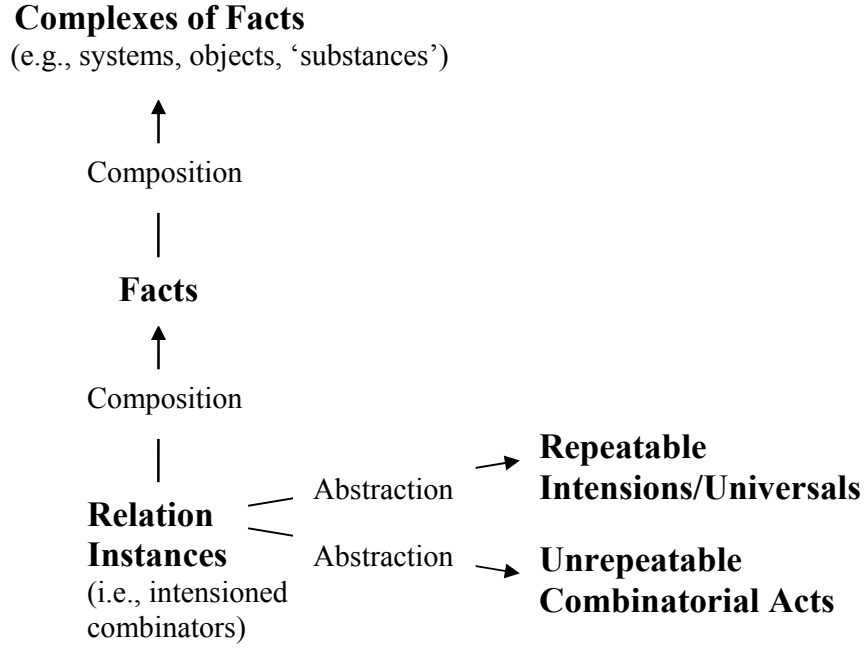
6. The agent-combinator ('predicable') aspect of a relation instance renders the instance an ontically dependent entity, requiring/presupposing as such other subject entities that it unifies into a fact. Hence, an ontic predicate never exists independently of a subsuming fact whose unity it effects.

7. In consisting of both a unifying agency and an intension content aspect, the latter controlling/specifying the former's application, an ontic-predicate/relation-instance has the nature of a 'composite simple', where the unity of these aspects is 'closer than relation'.

8. It is ontologically possible, and indeed necessary, for there to be an atomic ontic level consisting of only ontic predicates, and from these there can be built-up hierarchical complexes of facts that are the entities of our experience and physical theories.

9. As does every account of ontic predication, the one inherent in realist instance ontology carries with it a, albeit more refined and powerful, predicate logic, what is an intensional logical in the literal sense, and what corresponds to Gödel's programmatic 'Theory of Concepts'.

Principal aspects of this ontology can be given diagrammatically as:



Ontological dependence is in the reverse direction of the arrows. Intensions and combinatorial acts when separated in abstraction from any relation instance have a conceptual existence only, as do any constructed non-instantiated intensions, e.g., Unicorn, Phlogiston, and any wholes that supposedly derive from combinatorial agency separated from any controlling intensions—i.e., 'free association', e.g., sets or mereological sums.

It is in substantiating the above theses that a number of traditional and otherwise intractable ontological problems can be solved. A sampling of these problems and the theses that solve them are:

a. Theses 2-8 provide the needed details for an account of structure as a specifically organized, 'ordered', unity among a yet plurality of entities, an account whose necessity is asserted in Thesis 1.

b. Theses 3-7 refine our understanding of ontic predication, the absence of which has been the source of serious distortions, including the classic but specious containment/inherence model of predication.

c. Theses 3, 4, and 7 provide ontology with a *principium individuationis*, what for each individual is not simply a posited '*haecceitas*' or a necessitated but incoherent 'bare particular' residuum.

d. Theses 3 and 5 explain the confusion that leads to, and in this they disarm, the classic Bradley's Regress argument. They also nullify the critique by contemporary realists and nominalists that advocating both predicate instances and intension universals is redundant.

e. Theses 3, 6, and 8 eliminate the need for the traditional posit of non-dependent, non-predicable substrata, e.g., 'substances' as *entia per se*.

f. Theses 3, 5, and 6 correct the maxim 'There are no unexemplified universals' to 'There are no subject-less ontic predicates', allowing for a uniform and global account of intension universals, including those that are uninstantiated, e.g., Phlogiston.

g. Details demonstrating Thesis 8, which presuppose Theses 2-4 and 6, give content to the proposed proto-ontology for mirco-physics termed 'structural realism', an ontology for the apparent fact that physical reality is at its foundation 'pure relatedness'.

h. The details demonstrating Thesis 9, which include the formal implications of Theses 3-6, render more perspicuous what is the relationship between ontology and predicate logic, providing insight into what sense the latter is 'formal ontology'. The power of the resulting logic is exemplified by its ability to differentiate legitimate from paradoxical self-reference and to provide a precise means of defining identity and differentiating it from indiscernibility.

The details of how these and other problems are solved in realist instance ontology are provided in the following essays.

With the exception of Essay 7, original versions of the included essays have been previously published. I have herein taken the opportunity to correct errors, both formal and material, distributed across them, and to render them uniform in format. Because these essays were separately published, there is some redundancy to the arguments given, in particular, of the key argument for the individuation of attributes/ontic-predicates. However, the essays were written/re-written with the intent that the differing contexts and examples given would further clarify the repeated arguments.

I express my gratitude to the respective editors and publishers for their permission to republish the included essays; the citation for each is given with it. I would also express my gratitude to the University of Missouri – St. Louis for its support in the research that has led to these essays. I extend particular thanks to my colleague David Linzee for his considerable help in matters of form and grammar. Most of all, I acknowledge my wife, Polly, whose support and encouragement have been unwavering.

1

Combinatorial Predication and the Ontology of Unit Attributes*

> The topic of relations may prove a peculiarly rewarding one for the theory of universals as a whole. For it may be expected that important classifications and distinctions which can be made at the level of polyadic universals will vanish, or become merely notional, in the limiting monadic case.
>
> D. M. Armstrong
> A Theory of Universals

1. Introduction

A primary task of ontology is to provide a satisfactory theory of the ultimate categories of entities and the relationships between them. It is no exaggeration to say that the majority of the issues and distinctions proposed in this regard are tied, directly or indirectly, to what would be answers to a single, intuitive, and simply stated problem. This is the *Triple Aspect Problem*:

> What coherent account is there for the ubiquitous, fundamental epistemic given of apparently *unrepeatable* entities (individuals, particulars) *possessing* (instantiating, having as ontic ('material') predicates) apparently *repeatable* attributes (universals, intensions)?

Central here is the pre-critical *unrepeatability/possession/re-peatability* trichotomy everywhere exemplified in our internal and external experience. Apple *a*, for example, is apparently an unrepeatable individual possessing as predicates a number of repeatable properties, e.g., Red, Round, Has-mass. Likewise for relations, e.g., it is intuitive that the dyadic Square-root relation is numerically identical among distinct relata pairs: <2,4>, <3,9>, <4,16>, etc. Independent of whether natural numbers are universals or par-

ticulars,[1] pre-critically each relata pair jointly possesses as a shared ontic predicate the same Square-Root relation.

Of the three thus identified ontic aspects of *individuality, predicability, and universality*, predication is epistemically primary. For, there is no explication of either the unrepeatable aspect (the non-recurrent 'thisness' (*haecceitas*) of individuals) or the repeatable aspect (the recurrent 'suchness' (quiddity, *natura*) of universals) without an appeal to the 'possession' of predication. Aristotle described this pivotal role saying: "Some things are universal, others individual. By the term 'universal' I mean that which is of such a nature as to be predicated of many subjects, by 'individual' that which is not thus predicated." (*De Interpretatione* 17a38-40; cf. *Metaphysics* 1038b10) Accordingly, an entity that is not predicable, or at least not predicable of multiple subjects, is an unrepeatable individual.[2] Despite its fundamental and linking role in the order of explanation, the parallel role of predication in the order of reality has gone generally underappreciated, this due in large part to the traditional and misleading *containment (or inherence) model* of prediction. In the dialectic that is the history of ontology, ontic predication has been given a mostly secondary and derivative role, theories of predication following whatever were theorized as the more fundamental natures[3] of individuals and/or universals. Indicative is the fact that historically one finds the Triple Aspect Problem limited

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* Original version first published in *The Modern Schoolman* LXXIX (2002): 163-97.

[1] I have argued that natural numbers are best described as both particulars and universals in the following sense. As predicable of concepts, e.g., in the adjectival 'There are three apples on the table', numbers are individuated instances predicated of concepts, $(\exists 3_i)3_i$(Concept of apple-on-the-table), and in abstraction from its instances a number is a universal, e.g., 3 is the universal content common to its instances 3_i, 3_j, 3_k,…, and is referred to in substantival number sentences such as 'Three is a prime number.' The advantages of this view are given in D. W. Mertz, *Moderate Realism and Its Logic* (New Haven: Yale University Press, 1996), pp. 259ff.

[2] It is to be noted that Aristotle's criterion as stated here does not require that an individual be a non-predicable entity. And indeed, in the realist version of instance ontology advocated herein (and for which there is evidence in Aristotle in regard to properties—see *Moderate Realism*, pp. 98-111) it is individual relation (including property) instances R^n_i that are the only predicable entities.

[3] The term 'nature' is used herein as equivalent to essence or quiddity ('whatness') and as meaning that which is described in a definition.

to the Problem of Individuation,[4] and/or the Problem of Universals,[5] but there is no comparably explicit 'Problem of Predication'. The closest to such an identified problem is what is now called Bradley's Regress, an ancient and perennial argument from which have been drawn contradictory conclusions. It is a thesis herein that this ambiguity is mutual with the secondary role given predication, and that both are the consequence of a failure on the part of ontology to give an adequate analysis of ontic predication's *combinatorial*[6] *nature* and to draw what are its significant implications.

Using polyadic relations as a means of epistemic 'magnification', it will be argued herein that all ontic predicates insofar as they are *predicable* of their subjects are properly and succinctly characterized as *individuated states of intensioned connectivity*. Among the consequences of this analysis of ontic predication is, first, that it provides a solution to the Problem of Individuation that avoids the inadequacies of current theories. These theories are principally: a) the posit of what must be absolutely contentless or 'bare' individuators (Armstrong, Moreland, Bergmann)[7], b) the individua-

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[4] The *Problem of Individuation* can be stated as: What explanation can be given of the fact that, though absolutely every characteristic of an individual is a repeatable intension, that entity can nevertheless be unrepeatable *qua* individual?

[5] The *Problem of Universals* can be stated as: What explanation can be given of the apparent fact that numerically one and the same intension can be predicably 'in' diverse subjects—*unum in multis*?

[6] Cognizant of its mathematical usage, I have nevertheless chosen the term 'combinatorial' as appropriate in a literal sense since on the following analysis predicates are 'combinators', i.e., agents of plural unification.

[7] D. M. Armstrong, *A World of States of Affairs* (Cambridge: Cambridge University Press, 1997), pp. 68, 109; James P. Moreland, 'Theories of individuation: A Reconsideration of Bare Particulars', *Pacific Philosophical Quarterly* 79 (1998): 251-63, and *Universals, Qualities, and Quality-Instances: A Defense of Realism* (Lanham: University Press of America, 1985); Gustav Bergmann, *Logic and Reality* (Madison: University of Wisconsin Press, 1964), pp. 133-34, 277. In regard to Armstrong, he refers to bare particulars as 'thin particulars'. By this designation he seeks to emphasize his thesis that in extra-conceptual reality there are no particulars without some property attached as an ontic predicate, i.e., there are only states-of-affairs or facts. A thin (bare) particular is a conceptual abstraction from a state-of-affairs, as is the property or relation universal, the only other constituent of a state-of-affairs. Also see Armstrong's *Nominalism & Realism: Universals & Scientific Realism*, Vol. I (Cambridge: Cambridge University Press, 1978), pp. 113ff. The point I wish to emphasize is that Armstrong's thin particulars are for him fundamental and distinct ontological entities, inseparable as they may be from universals, *and* that they are themselves

tion-by-instantiation of especially empowered substance universals (Loux, Lowe)[8], or c) the assigning to individuation the status of an unexplainable primitive (a nominalist tack, e.g., Ockham, Campbell)[9]. Contrary to these theories that take individuation to be either primitive or the posited attribute of a posited primitive entity, on the following individuation will be seen to follow logically from what must be the nature of ontic predication. Secondly, the combinatorial assay of predication contributes to solving the Problem of Universals by explaining the sense in which intension universals are 'in' and yet, as necessitated on the following, not ontically predicable of the individuals they characterize. For the problem with universals is not simply that numerically the same entity is 'in' *as a constituent of* each of distinct complex wholes, this type of *unum in multis* being a common-place, e.g., *a* in {*a*,*b*} and {*a*,*c*}, or as a part that is shared by multiple machines. Assertions to the contrary may derive from what is a thesis of naturalism that for an entity to exist it must necessarily be a relatum of spatial relations, this on the relationist analysis of space asserted here,[10] and

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bare of any content or intension—"…The thin particular, the particular apart from its properties" (*States of Affairs*, p. 115).

[8] Michael Loux, *Substance and Attribute* (Dordrecht: Reidel, 1978), pp. 163ff. Also, Loux, *Metaphysics: A Contemporary Introduction* (New York: Routledge, 1998), pp. 117ff.; E. J. Lowe, *The Possibility of Metaphysics* (Oxford: Clarendon Press, 1998), pp. 180-83, 197.

[9] Indeed, some nominalists take the line that individuation is a non-problem. In the words of Ockham, "One does not have to look for a cause of individuation…. Rather one has to look for the cause why it is possible for something to be common and universal." Paul Spade, *Five Texts on the Mediaeval Problem of Universals* (Indianapolis: Hackett Publishing Co., 1994), p. 172. Keith Campbell takes this line in *Abstract Particulars* (Oxford: Basil Blackwell, 1990), p. 69.

[10] Adopted here is the relationist over the substantivist view of space. For a justification of this view see *Moderate Realism*, pp. 39-42. There it is argued that the substantivist view that space(-time) is composed of points(-instances) fails because the latter reduce upon analysis to what would be bare particulars reserved as relata for spatial(-temporal) relations. Apparently these bare point-instances would be the same discrete space-time minima essential to the geometrodynamics conception of real space-time Campbell has advocated, e.g., in 'Unit Properties, Relations, and Spatio-Temporal Naturalism', *The Modern Schoolman* LXXIX (2002): 151-62. Further, it is instructive to note that Campbell holds space to be real as demonstrated by its property of right- & left-handedness (enantiomorphism) and hence concludes that a Leibnizian relationist conception of space is false, since the latter includes a characteristic foundationism (a view advocated by Campbell himself) claiming the eliminative reduction of relations to properties of their relata. The point I would make is that relationism need not be Leibnizian. The counter-theses advocated here are that foundationism is false, that re-

the fact that the same entity can not be in two distinct places at once. Associated naturalist theses are that the existence of spatial(-temporal) relations is prerequisite to, or at least concomitant with, the existence of causal relations, and that 'There is no entity without causal efficacy', causality here being efficient causality.[11] Yet, the spatial criterion for existence is contradicted by irreducible categories of entities that do not support spatial (or causal) relations as relata, e.g., concepts, propositions, logical operators, sets, numbers; at least some instances of the laws of logic, mathematics, and science; and even instances of spatial relations themselves (e.g., try to cogently specify the spatial location of the Taller-than relation when person *a* is taller than person *b*). Any attempt to limit the real to the concrete, even with space conceived substantivally as a plenum of geometric points, will be defeated by the necessity of appealing to abstract relations, e.g., the relation Located-at. For, under this point-plenum conception of space, the reality of an entity *a* is a function in part of its being *located at* some set or other of spatial points, *s*, at a temporal instance. That is, for what are non-identical *a* and *s*, it is the reality of the further and necessary asymmetric Located-at relation existing between them that is essential to conveying an existential status from *s* to *a*—*a*'s existence ontically presupposes both that of *s* and its being Located-at *s*. But then to have the requisite reality by the same criterion, the Located-at relation between *a* and *s* itself ontically presupposes being located, presumably at *s*, and as located in space this relation between *a* and *s* would be individuated, i.e., be a unit attribute or instance. Yet the locating of the latter instance would itself be a further ontically presupposed instance of the Located-at relation existing between the first instance and *s*, and clearly this is the beginning of a vicious regress. The reality of every relation instance in this chain presupposes the reality of a prior one. Consequently, the instance of the Located-at relation existing between *a* and *s* cannot be located, i.e., it is not concrete, and so the exclusive spatial criterion for existence must be rejected.

Further and relatedly, under the spatial criterion the abstract entities and relations of, say, logic and mathematics would have to reduce to either illusion or to a mysterious 'supervenient' state somehow between being and non-being and inherited from, presumably, the spatial locatability of brain states. But then in regard to the latter alternative a regress analogous

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lations and their instances are fully real and unreducible, and that space(-time) is constituted by instances of space(-time) relations.

[11] Armstrong, *States of Affairs*, pp. 5-6, 41-43.

to the one above would be evoked by the necessitated reality of the Supervenience relation, and this in addition to the Located-at regress in regard to brain states. Moreover, here it would seem one cannot avoid concluding that, for example, natural numbers, if they have any reality at all, must inherit the status of entering into spatial relations, and this is counterfactual. It is not the case that 2 is spatially related to 3 anymore than spatial point *x* is numerically (or causally, legally, ethically, emotionally, etc.) related to spatial point *y*. Yet, none of this in itself implies anything about the reality or unreality of numbers, spatial points, or any of the relations mentioned. The ontically privileged role reserved for relations with spatial intensions is as much of an exaggeration (and limitation) as the Pythagorean thesis that what is real is marked ultimately by being relata for abstract relations with numerical intensions. The naturalist's only alternative for countering these objections is to argue that relations (and their instances), particularly asymmetric relations, can be eliminated from one's ontology by reducing them to property instances of their spatially located relata. But this is an impossibility as maintained below.

Setting the spatial bias aside, realists have pointed out correctly that there is nothing inherently incoherent in the notion of numerically the same entity being a constituent of diverse wholes, and this whether the latter enter into spatial relations or not. Rather, 'the problem' with universals is at bottom, I propose, a problem concerning ontic predication, one that derives from the classic *mis-identification of universals with ontic predicates*. This mis-identification results in the puzzle of how a monadic property whose intension specifies a predicable nexus with exactly one subject can, as a universal, be simultaneously predicable of multiple subjects. The puzzle is, of course, repeated for predicates of every *n*-adicity, though it is monadic properties that have been the traditional paradigm. And, it is precisely this latter 'tyranny of the monadic' that has served to mask what is a corrective more obvious in the case of polyadic relations. For, given the unreducibility of the polyadic to the monadic claimed herein, the intension of an *n*-adic relation (e.g., Taller-than) requires a simultaneous predicability among multiple, i.e., *n*, subjects. Hence, the puzzle over predication is not simply that one and the same predicate can have multiple subjects, but is rather how numerically the same *n*-adic predicate, that as such specifies exactly *n* subjects, can be simultaneously predicable of (repeatable over) an indefinite number (not necessarily *n*) of *n*-tuples. A now obvious answer to the puzzle is that *n*-adic predicates, including the monadic as the limiting case,

are in fact not repeatable, i.e., are individuated to specific *n*-tuples. That this is indeed the solution to the Problem of Universals will be seen to follow from the formulation and solution of the more basic Problem of Predication. Using relations as a perspicuous medium for analysis we shall see that ontic predicates themselves are not repeatable and they are not 'in', i.e., not internal constituents of, their subjects, though the predicates' characterizing intensions are 'in' them as constituents. That is, an intension can be a non-predicable constituent of each of multiple predicates, but the subsuming predicates are neither universal nor in their subjects. The result is a *moderate realism* absent the obscurities found in the versions of Aristotle and the scholastics.

Specifically, it will be argued herein that an ontic predicate is a simple entity with a dual nature—one aspect a combinatorial state to or among one or more subjects, the other aspect a content or intension ('sense') that delimits as to kind and, when the predicate is polyadic, the number and order of the unified subjects. The intension is also the source of a polyadic predicate's formal/logical properties (e.g., asymmetry, transitivity, reflexivity), attributes absent in the limiting case of monadic properties. More specifically, it will follow that 1) there is no 'real distinction', i.e., no pluralizing composition, in a predicate between its combinatorial and intension aspects (in scholastic terms the distinction between the aspects is a *distinctio formalis a parte rei*), and 2) the combinatorial state of a predicate among a set of subject relata is unrepeatable, i.e., individuated. Under 1) a predicate is a non-complex entity in the proper sense of a whole having no third constituent in addition to the combinatorial and intension aspects that unifies these aspects into the resultant predicate. Significantly and contrary to tradition, conclusion 2) means that predication provides metaphysics with a *principium individuationis*, an explanatory principle absent from popular trope theory[12] and from bundles-of-universals theories[13], and one that is sus-

[12] Campbell, *Abstract Particulars*; D. C. Williams, *Review of Metaphysics* 7 (1953): 3-18, 171-92; John Bacon, *Universals and Property Instances: The Alphabet of Being* (Oxford: Blackwell, 1995); Peter Simons, 'Particulars in Particular Clothing: Three Trope Theories of Substance', *Philosophy and Phenomenological Research* 54 (1994): 553-74, and 'Farewell to Substance: A Differentiated Leave-Taking', *Ratio* 11 (1998): 235-52.

[13] Bertrand Russell, *Inquiry into Meaning and Truth* (London: Allen and Unwin, 19-40), p. 98, and *Human Knowledge, Its Scope and Limits* (New York: Simon and Schuster, 1948), pp. 292-308. A. J. Ayer, 'The Identity of Indiscernibles' (1954), in Michael Loux, *Universals & Particulars*, 2d. ed. (Notre Dame: Notre Dame University Press, 1976), pp. 263-70. Brand Blanshard, *Reason and Analysis* (LaSalle: Open Court, 1962), pp. 399-401.

tainable, unlike the 'bare particulars' of substratum theories[14] or the individuation-by-instantiation of contemporary substance theories[15]. It follows that the basic ontic units are individuated relation (including property) instances R^n_i, each of which is a simple entity having the abstractable dual aspects of outwardly directed unrepeatable predicability correlative with a repeatable content or intension R^n. Intension R^n is numerically identical across exactly resembling but distinct instances R^n_i, R^n_j, R^n_k, …. Due to its nature as a *combinator*, a relation instance R^n_i exists only in the context of a fact or state of affairs, $:R^n_i(a_1,a_2,...,a_n)$, the latter being the unique complex resulting from the unrepeatable predicable state of R^n_i among the distinct entities within the relata set, $\{a_1,a_2,...,a_n\}$. (The colon locution will be used to distinguish a fact, if any, from a corresponding proposition.) Though the specific relata set of a relation instance is the 'occasion' or secondary cause of its individuation, it is its predicable union with these relata that is the primary cause of its unrepeatability.

The combinatorial or predicable agency of relation instances, together with intension universals, are the potent features of this unit attribute ontology and what distinguish it from its chief rival, nominalistic trope theory. Under trope theory individuated properties 'free float' in the sense that they are by definition not predicable—each is a self-sufficing 'little substance'. Any gathering of tropes into a thick individual, e.g., apple *a*, requires the posit of a specific relation to provide this unification, either 'compresence' (e.g., Campbell)[16] or some internal 'foundation relation' in the style of Husserl (e.g., Simons, Denkel)[17]. This predicably inert status is

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The problem for bundles-of-universals theories is simple: a bundle of universals is itself repeatable, i.e., universal, and so a bundle theory that admits only universals and bundles thereof cannot account for individuation. Cf. Campbell, *Abstract Particulars*, p. 19.

[14] For the bogus nature of 'bare particulars' see Campbell, *Abstract Particulars*, p. 7; Loux, *Metaphysics*, pp. 113-17; Mertz, 'Individuation and Instance Ontology', *Australasian Journal of Philosophy* 79 (2001): 45-61.

[15] These theories do not give an account of individuation, they simply posit a category of entities, substance universals, and assign to them the duty of individuation without explanation how this is possible. How is it that a property numerically identical across a plurality of individuals can, contrary to its repeatable nature, render these instantiating entities unrepeatable? See Alica Rothstein and Nathan Oaklander, 'Loux on Particulars: Bare and Concrete', *The Modern Schoolman*, 78 (2000), pp. 97-102.

[16] Campbell, *Abstract Particulars*, pp. 21-22.

[17] Simons, 'Particulars in Particular Clothing' and 'Farewell to Substance'. See Edmund Husserl, *Logical Investigations* (London: Routledge & Kegan Paul, 1970), p. 478. Arda Den-

plausible, though speciously so, for limiting monadic properties but not so for polyadic relations that have an obvious existential dependence on, i.e., a 'need to latch onto', the members of relata *n*-tuples insofar as the relations function definitionally as relating. Hence the telling need within trope theory of reducing relations to properties (Campbell) or treating relations as, in effect, a radically different kind of entity from properties (Simons' both non-predicable monadic as well as predicable polyadic tropes, and his 'foundation' relation which is not a trope at all). Both of these options are errors and it is the implications of their correction that yield in great measure the instance ontology defended here.

The ontology of relation instances outlined above not only provides solutions to the nexus of problems inherent in the Triple Aspect Problem, but also provides a solution to a second and equally fundamental problem for ontology. This is the *Problem of Complexity*:

> What coherent account is there for the ubiquitous, fundamental epistemic given of organized wholes or complexes (structures, systems, *Gestalten*) that are more than mere collections ('heaps', sets, mereological sums), i.e., plural wholes not necessitated by the mere existence of diverse entities (as are sets and mereological sums), but existing as a function of constituent inter-connections conditioned by the specific natures or essences of the entities so structured?

Extending common experience, science tells us that reality consists of a diversity of hierarchical complexes: sub-atomic, molecular, biological, cognitive, mathematical, etc. The common sense 'continuous solid' conception of classical substances (humans, trees, statues, etc.) has been replaced by one of wholes consisting of mostly empty space occupied by scattered entities, where the latters' specific structuring is equally essential to the natures of the wholes. These wholes have properties and enter into relations nonexistent at lower levels of complexity. It is intuitive and widely held that what is needed to account for structural connections are polyadic relations, but what is not appreciated is the explanatory power of this view refined to include relation instances as the real and individuated connectors. Specifically, relation (including property) instances are *necessary for* and *sufficient as* both ontology's 'primary substances' and as the cause *sine qua*

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kel, 'On the Compresence of Tropes', *Philosophy and Phenomenological Research* 57 (1997): 599-606.

non of all plural wholes whatsoever, and both notwithstanding instances' ontic dependence upon their subject relata. For, contrary to the traditional Aristotelian requirement on primary substances (*Metaphysics* 1028a20, 1028b34-1029a30), a view reiterated in contemporary ontology (e.g., in Campbell, Armstrong)[18], the existential dependence of instances on their relata results not from some defect of *being* (*ens*), but rather derives from their positive status as ontically productive and unifying principles. Indeed, it is an argument below that atomic ontic entities must be predicable. This and other results support the thesis that, as a single category, relation instances of a diversity of intensions sustain a complete ontology characterizable as *network instance realism*. Here from an atomic level of inter-predicated instances hierarchical networks of cross-connected complexes emerge that include at sufficiently structured levels ordinary 'thick particulars', (e.g., apple *a,* Socrates, this chair) and more abstract complexes (e.g., minds, social structures, logical and mathematical systems). As complexes emerge they themselves can be single relata for further emergent and *sui generis* relation instances, the resultant networks being next-level complexes. When the details are supplied for instance ontology, we would have, I contend, an explanatorily adequate version of the thesis advanced by Wittgenstein and recently argued by Armstrong that the world is a world of facts, not things.[19] Wittgenstein and Armstrong, however, hold that there are no or few facts about facts.[20] In complete contrast, the ontology for which the following lays the foundation is what I take to be a Whiteheadian one where the *res verae* are "individual facts of togetherness"[21], i.e., relation instances, and where the universe is a hierarchical system of systems composed ultimately of inter-connecting and dependent relation instances, atomic and emergent. Specifically, at the atomic level of this ontology there exist only relation instances having other instances as relata—'facts about facts'—and higher ontic levels exist because certain relation instances have complexes of facts as single relata—'facts about systems of facts'.

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[18] Campbell, *Abstract Particulars*, pp. 98-99; Armstrong, *States of Affairs*, pp. 38, 99, 267.

[19] Ludwig Wittgenstein, *Tractatus Logico-Philosophicus* (London: Routledge & Kegan Paul, 1961), p. 7, Prop. 1.1; p. 13, Prop. 2.05. Armstrong, *States of Affairs*.

[20] Wittgenstein, *Tractatus*, p. 13, Props. 2.061, 2.062; Armstrong, *States of Affairs*, pp. 139-147. For a critique of Armstrong's independence thesis see my review of his *States of Affairs* in *The Modern Schoolman* LXXV (1998): 227-31.

[21] Alfred North Whitehead, *Process and Reality* (New York: The Free Press, 1978), p. 20, also see pp. 27, 73.

2. The Problem of Ontic Predication and Locating the Predicable Nexus

Ontic, or 'material' predication (hereafter simply 'predication') at the level of fact is traditionally distinguished from linguistic/grammatical, or 'formal' predication at the descriptive level of language,[22] and refers to the nature and cause of the *nexus* or *union* attributes (extra-linguistic or linguistic) have with one or more entities they qualify. In contemporary terminology, facts formed from material predication are the *truthmakers* for true affirmative propositions formed from formal predication. Providing a coherent account of material predication is the fundamental *Problem of Predication*. Stated in its most general form the problem is:

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[22] It is instructive to note how nominalists who wish to eschew ontic predicates from their ontology cannot retreat to nominalism's 'home field' of language and there feel secure. At the 'formal' level of grammar, Plato long ago observed (*Sophist* 262a-e) that one could make no assertion with a string of only nouns (e.g., 'lion stag horse') or only verbs (e.g., 'walks runs sleeps'), but that one of each is required in order to 'interweave' (e.g., 'man understands'). The common correspondence view is that to assert something as a fact requires the joining of a linguistic predicate to a linguistic subject, and, when true, the assertion corresponds to the material predication of a property to a subject entity or of a relation to multiple subject entities. For example, 'Apple *a* is round', corresponds to a fact :Round(*a*) where property Round is predicably joined to subject entity *a*. A nominalist might counter by saying that, despite the fact that linguistic predication is pervasive and uneliminable in at least some languages, this is not in itself evidence that it marks a fundamental aspect of reality. This objection misses the mark. For, even at the level of grammar/syntax which is itself part of 'reality' there are full syntactical *relations*—functioning as polyadic predicates—between terms (tokens if not types) from various linguistic categories, e.g., non-symmetric relations of Linguistically-Predicable-of, Adjectivally-Modifiable-of, Adverbially-Modifiable-of. Each such relation has a definite content that together with that of would-be subjects determines whether or not they are predicably combined in a fact, as in :Linguistically-Predicable-of('walks','Theaetetus'), but there being no such fact corresponding to the proposition Linguistically-Predicable-of('Socrates','Theaetetus'), or in :Adverbially-Modifiable-of('beautifully','sings'), but there being no such fact corresponding to the proposition Adverbially-Modifiable-of('runs','sings'). Even if these and like relations are language specific, they are nonetheless relations, i.e., polyadic predicates. Relations are ubiquitous in the given of language (as they are in perception and cognition), and hence like everyone else, nominalists must make *use* of polyadic predication. If, as maintained here, it is true that ontic complexity is a reality (including multiple components unified in the functioning of language), that relations are essential to complexity, and that relations are not reducible to properties of their relata nor to relata themselves, then relations, i.e., polyadic predicates, must be admitted into one's ontology.

> How are we to understand the predicational unification between an *n*-place attribute with content or intension R^n and an *n*-tuple of subjects, a_1, a_2, ..., a_n, that so unified constitutes a fact or state of affairs $:R^n(a_1,a_2,...,a_n)$, a whole where R^n mutually characterizes a_1, a_2, ..., a_n and is thus distinct from and 'more than' the mere list R^n, a_1, a_2, ..., a_n, or set $\{R^n,a_1,a_2,...,a_n\}$?

What, for example, is the difference between the fact that John loves Mary, :Love(John,Mary), and the list John, Love, Mary, or the set {Love,John, Mary}? What is an obvious relationship between the Problem of Predication and the Problem of Complexity is based upon the consequential fact that facts or states of affairs are the simplest possible complexes. Understanding the unification constitutive of a fact is central to both problems. As a preliminary, among the obvious differences between the unity of a fact and that of a mere list or set are the following. 1) In a fact $:R^n(a_1,a_2, ...,a_n)$ there is a non-arbitrary asymmetric order to the predicable nexus between subjects *qua* subjects a_1, a_2, ..., a_n, and the predicate *qua* predicate (with intension) R^n, whereas the order in the list R^n, a_1, a_2, ..., a_n, is arbitrary and there is no order in the set $\{R^n,a_1,a_2,...,a_n\}$. This order is independent of whether or not there exists an ordering *among the subjects* as specified by the intension R^n. That there is a 'direction' to predication is evident in the monadic case $:P^1(a)$ where the subject is a 'thick' particular, e.g., apple *a*, for here a predicate, say Fruit, is predicable of *a*, i.e., it is the case that *a* is a fruit, but not vice versa, i.e., it is non-sense to assert 'Fruit is an *a*.' Importantly, the order in predication is evidence that the cause or principle of the predicable nexus cannot be equally shared by subject(s) and predicate, the nexus being 'from one type to the other type'. When predication is limited to the monadic it is not immediate where to attribute the cause of the nexus, an ambivalence found in the tradition. However, if polyadic predication is admitted full and unreduced, contrary to much of Western ontology, then it is apparent that because a single fact can have multiple subjects but not multiple predicates then predicates are the cause/source of the predicable nexus. Appealing to a version of a classic maxim [crucially, whose scope must be restricted—see Essays 2 and 4 in this volume], unity is from the one, or unit, and its being 'shared' by the thus unified. The single *n*-place predicate 'shared' among *n* relata is the source of the unification requisite of the resulting fact. If, from the side of the *n* subjects, each of the constituent subjects were singly the cause of the factual unity with the single predicate then there would be *n* facts, not one.

And alternatively, if the *n* subjects were a unifying cause only collectively then the unity of this collection would have to be accounted for by appealing to some further unifier distinct from each and all of the subjects as well as from the predicate, and this leads either to a Bradley-type vicious regress or to an incoherent 'bare linking'. Both of the latter issues will be expanded upon below. 2) For a contingent predicate, the predicable nexus, and hence resultant fact, can come into and/or go out of existence, whereas the corresponding set is atemporal 'eternal' in not being a relatum for temporal relations. The predicable nexus in :$R^n(a_1,a_2,...,a_n)$ is ontically distinct from any of R^n, a_1, a_2, …, a_n, taken singly, or together in the manner of a list or set. And 3), the existence of a fact whose predicate intension is R^n precludes the existence of other facts with the same subject *n*-tuple but whose predicate intensions are contraries of R^n, yet there is no such exclusion among the corresponding lists or sets. What this says is that for the predicable nexus the contents or intensions of subject(s) and predicate are essential, whereas it is only their existence that is relevant to the corresponding list or set. In regard to 3) a traditional assumption has been that in any fact it is the predicate as an intension universal which is the cause of the predicable nexus. This will be shown to be an error. Relevantly, it is to be noted that in the above formulation of the Problem of Predication the phrase '*n*-place attribute with content or intension R^n' has been deliberately chosen in lieu of what would be the more traditional '*n*-place universal R^n', i.e., where with the latter the nexus of attribution would be said to hold between the *n*-place *universal* R^n and an *n*-tuple of subjects. Methodologically, the adopted phrasing is motivated by its neutrality in regard to whether predicable entities are universals or particulars. Further, however, the now explicitly allowed predicate-as-particular option will emerge in the following as cutting reality at its joints.

As a preliminary to an analysis of the predicable nexus proper, it is necessary to examine proffered theories on where the cause of this unification is located. Indicative of its obscurity in the tradition, philosophers have variously located the principle of predicational unity: inside the fact, outside the fact, and under the former as either a function of the predicate or of the subject. Concerning the inside/outside debate, the dominant and intuitive view has been that the predicable nexus is 1) an internal constituent of the resultant fact. However, in reaction to apparent problems with this common sense view, recently philosophers have proposed that the predicable nexus be assigned to either 2) the subsuming fact itself (e.g.,

Armstrong, Olson)[23], or 3) to an agent external to the fact (recently by Vallicella)[24]. Views 2) and 3) are erroneous, I shall argue, though instructively so. In advocating 2) Armstrong is apparently motivated by his theses that a fact does not exist when only its constituents exist and that the postulation of a further unifying constituent leads to Bradley's regress. In regard to the first thesis, Armstrong holds that for a fact :P(*a*) its constituents are intension universal P and subject *a*, and, for contingent P, fact :P(*a*) might not exist though P and *a* both exist. Vallicella rightly criticizes Armstrong's theory by observing that it is self-contradictory to require a fact to be both a complex whole of parts and yet not a whole of parts insofar as it is more than its constituents. The error can be seen in this way: for Armstrong we have the three entities P, *a*, and :P(*a*), where fact :P(*a*) is to be the fact-producing principle of unification among P and *a*, not the resultant of some further unifier among them. Now, according to Armstrong, :P(*a*) = P + *a* + (fact-producing principle of unification among P and *a*). Then by substitution, :P(*a*) = P + *a* + :P(*a*). Hence, fact :P(*a*) (on the left side) is identical to something that is more than itself (the right side). Or alternately, fact :P(*a*) is a proper part of itself, or, fact :P(*a*) is ontically prior to itself. Unfortunately, however, Vallicella's critique of Armstrong applies equally to his own externalist thesis 3). To say that a requisite unifier is 'external' to its fact :P(*a*) can only mean that it is not a constituent of its fact, and thus :P(*a*) as the 'sum' of its parts has its being, whole and complete, independent of this unifier. But this is precisely what is denied in the required posit of the unifier. The unifier essential to a fact :P(*a*) can be external to each of the constituents construed here as intension P and subject *a*, but it cannot be external to the subsuming fact of which it is of its defining essence—fact :P(*a*) is definitionally P-and-*a*-factually-unified. In scholastic terms, though the actuality or existence of an entity and its constituents may be the effect of a sustaining external agent (e.g., God), what is of the *essence* of an entity must be a constituent of it.

The error of both the subsuming-whole and externalist theories of factual unity is even more obvious in the case of facts composed of non-symmetric relations. Let R be a non-symmetric relation, e.g., Love, and let it be the case that :R(*a*,*b*) and :R(*b*,*a*) both obtain. Each fact requires its own unifier since either fact could obtain without the other (a fact whose

[23] Armstrong, *States of Affairs*, p.118; Kenneth Olson, *An Essay on Facts* (Stanford: Center for the Study of Language and Information, 1987), pp. 60-1.
[24] William Vallicella, 'Three Conceptions of States of Affairs', *Nous* 34 (2000): 237-259.

expansion will be central below). But, if these unifiers are not constituents of their respective facts, then, because the facts have otherwise exactly the same constituents under Armstrong's and Vallicella's analysis, i.e., R, *a*, and *b*, the facts must be identical, :R(*a*,*b*) = :R(*b*,*a*), which is absurd. This conclusion is based upon assuming the *Uniqueness by Composition Principle*:

> Entities having exactly the same constituents are identical.[25]

Though this principle is intuitive and consequential (similar to the analogous Axiom of Extensionality for set theory), Armstrong explicitly rejects it,[26] while Vallicella explicitly maintains both the principle and the thesis that a fact is more than its constituents,[27] an inconsistency on the above. The difficulty here, and elsewhere as we are about to see, is the assumption that in the subject(s)/predicate structure of a fact the predicate is an intension universal, and this independent of whether we assign the predicable nexus to the subject(s) or the predicate. For, if this is assumed, then the Uniqueness by Composition Principle does imply that :R(*a*,*b*) = :R(*b*,*a*), for the same R, *a*, and *b*. Now it might be countered in the manner of Armstrong[28] that universal R provides order among its relata by possessing different positions or 'slots' such that *a* occupies the first position and *b* the second in :R(*a*,*b*), whereas *a* and *b* are reversed in positions occupied within :R(*b*,*a*). The needed distinctness of facts is retained though Uniqueness by Composition is violated. The trouble here is that if it is *literally and numerically the same* universal R in both facts then its first position would be occupied by *both a* (in fact :R(*a*,*b*)) and *b* (in fact :R(*b*,*a*)), and *mutatis mutandis* for the second position. Even if one can make sense of this, it implies that every such relation R, having *a* (and *b*) in both first and second positions, is reflexive in regard to *a* (and *b*), which is counterfactual

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[25] See Mertz, 'Individuation and Instance Ontology'.

[26] D. M. Armstrong, 'In Defense of Structural Universals', *Australasian Journal of Philosophy* 64 (1986): 85-91; *Universals: An Opinionated Introduction* (Boulder: Westview Press, 1989), pp. 90, 111; *States of Affairs*, p. 118-121. In the latter Armstrong allows for a 'weakened version' of the Composition Principle that holds two complexes are identical if and only if they have the same constituents *and* organization. The implication is that the organization is distinct from the 'constituents'. In contrast, the argument herein is that 'organization' is an aspect of a constituent of a fact, i.e., the predicate, and so the full Uniqueness by Composition Principle holds.

[27] Vallicella, 'Three Conceptions of States of Affairs', p. 248.

[28] Armstrong, *States of Affairs*, pp.121-22.

(e.g., let R = Admires). Avoiding the false dilemma of having to give up either the Uniqueness by Composition Principle (Armstrong) or the existence of facts (Lewis)[29], the case will be made below that individuated relation instances are the true predicates in facts and that intensions are shared aspects of the instances. It will be seen that the equivocation of non-predicable intensions with their subsuming predicable instances is what makes Bradley's regress plausible.

The internal placement of the cause of predicational unification must be retained, i.e., the principle of unification of a resultant fact must be a constituent of the fact. Doubts to the contrary have been a function of the problems that arise in attempts to assign predicability to constituents of a fact as traditionally conceived. For a monadic fact :P(a), which has been the traditional paradigm (a distorting limitation on the following), it has been one part of the tradition to assign predicability, directly or indirectly, to the intension P. The weakness here is not a function of a commitment to intension universals. Indeed, I propose that there are compelling arguments for universals,[30] and offer the following as indirect evidence for them. Rather, the deficiency of the traditional analysis of facts arises from its commitment to either a) that universals are themselves predicable entities (the traditional 'Aristotelian' view, the widely approved Fregean theory of 'concepts' being a modern explication)[31], or b) that universals are themselves not predicable but rather work mutually with either an exemplification (or instantiation) relation (e.g., Moreland)[32] or with 'non-relational ties', i.e., intensionless linkings (e.g., Bergmann, Strawson, Hochberg)[33], to

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[29] David Lewis, 'Comments on Armstrong and Forrest', *Australasian Journal of Philosophy* 64 (1986): 92-93. Also see his *Parts of Classes* (Oxford: Blackwell, 1991), pp. 56-57.

[30] E.g., as found in Armstrong, *Nominalism and Realism*, and Reinhardt Grossmann, *The Categorical Structure of the World* (Bloomington: Indiana University Press, 1983).

[31] Gottlob Frege, *Translations from the Philosophical Writings of Gottlob Frege*, ed. P. Geach and M. Black (Oxford: Blackwell, 1970), pp. 46-47, 54-55, 152. Also see Frege, 'On the Foundations of Geometry' in *Essays on Frege*, ed. E. Klemke (Chicago: University of Chicago Press, 1968), pp. 569-71.

[32] Moreland, *Universals, Qualities, and Quality-Instances*, pp. 170, 194. Also see his 'Issues and Options in Exemplification', *American Philosophical Quarterly* 33 (1996): 133-47.

[33] Gustav Bergmann, *Realism* (Madison: University of Wisconsin Press, 1967), pp. 9, 42ff. Herbert Hochberg, 'A Refutation of Moderate Nominalism', *Australasian Journal of Philosophy* 66 (1988): 188-207. P. F. Strawson, *Individuals* (London: Methuen, 1971), pp. 168ff.

form wholes predicable of *n*-tuples of subjects. There are severe problems specific to each of the alternatives, a) and b). First, against alternative b) is the fact that the hypothesizing of a non-relational tie is equivalent to positing a 'bare linking', analogous to and as bogus as a 'bare particular'.[34] This point will be developed below in the context of a further demonstration of the untenability of alternative a)—that universals themselves are the cause of their predicable union with their subjects. There it will be argued that predicates *qua* predicable are necessarily *unrepeatable*. A further and relevant critique against alternative b) is the observation that there is nothing unique to an especially appointed exemplification relation in its unification of its relata that is not possessed by any other relation R insofar as R is predicable of, i.e., 'actually relates', its relata. This is a lesson of Bradley's Regress, as reviewed below, and it applies equally to the limiting case of monadic predicates. In general the question becomes: Is it reasonable that, if upon a certain ontological analysis (e.g., the construal under Bradley's regress of predicates as non-combinatorial; or the Wiener-Kuratowski extensionalist reduction of relations) it is concluded that entities of a category (e.g., properties and relations) lack a specific characteristic (e.g., a linking state among their subjects; or an intensional content), to then posit an entity from the same category (e.g., an exemplification or instantiation relation; or the Element-of relation) as exempt from the analysis and as having the explanatorily needed characteristic? Clearly, the answer is no, and the fact that there is pressure to posit such an entity as exempt is evidence that the ontological analysis of the subsuming category is erroneous. Consequently here, either all ontic predicates, properties and relations, are combinatorial among their relata or none are, including exemplification. (The direct relevance of intensions will be seen below.) This critique applies equally to contemporary trope theory where, as a surrogate for monadic predication, non-trope unifying relations—Compresence or 'founding' relations—are posited to account for the 'bundling' into thick particulars of non-predicable tropes, and, as a surrogate for universals, the non-trope Resemblance relation is posited as what links tropes in being intensionally 'the same'. Yet, if these dyadic relations (or their instances) are predicable then why not monadic tropes, e.g., Red_i, Round_j, Has-mass_k? To avoid this problem, Campbell in advocating trope theory is consistent in having argued that polyadic relations are reducible to monadic tropes.[35] But then under such a reduction to non-predicable tropes the predicable/intension-

[34] See note 14.
[35] Campbell, *Abstract Particulars*, pp. 97ff.

controlled-linking aspects of the Compresence/founding and Resemblance relations are eliminated, and yet it is precisely these aspects that give these relations their efficacy for the theory and is, if implicit, the *raison d'être* for their posit. That the polyadic is not reducible to the monadic will be taken up latter.

There is a more consequential critique of the present assay of singular facts into subject individuals and predicate universals, one independent of whether the predicable nexus is assigned to the subject or the predicate, or whether the predication is immediate as in a) or mediate as in b). Succinctly, the assay requires that an intensioned or 'thick' individual *a* reduce upon analysis to an incoherent qualityless substrate—a bare particular. The argument is straightforward: If every characteristic, quality, or attribute whatsoever of an individual is a predicate of it and on the present assumption a *repeatable* universal, then an individual as *unrepeatable* must be distinct from *all* its characteristics—it must be a bare particular. This is apparently Aristotle's conclusion at *Metaphysics* 1029a10-30 where he reduces to absurdity his ultimate-subject criterion for primary substance given in the *Categories* (1b10, 2a34),[36] i.e., the absurdity of the thesis that primary substances are non-predicable individuals, and every other kind of entity is predicable of a primary substance. The conception of primary substance here is analogous to that of the naive 'vessel' model of subjects (cf. *Categories* 15b17ff; *Metaphysics* 1023a6ff) where as a 'container' the substance subject serves to unify by gathering within it all its attributable characteristics. The analogy is one of a sack as it holds together its contents. Another analogy satirized by Russell is thinking of a thick particular as something like an individuating and unifying peg on which is hung all the particular's characterizing universals. Yet, whether considered analogous to a container or to a focal unifier, an entity distinct from absolutely all its attributes reduces upon analysis to a bare particular, which is, euphemism aside, to evaporate into non-being. For the mis-analogy of the sack or the peg is that either would have attributes which on the present analysis must be reassigned from the sack to the content of the sack or from the peg to the universals hung on the peg—sack and peg reduce to *no-thing*.

Notwithstanding contemporary attempts to salvage the doctrine by positing an external 'tied-to' relation between bare particulars and their es-

[36] See Mary Gill, *Aristotle on Substance* (Princeton: Princeton University Press, 1989), pp. 19ff.

sential properties (e.g., Moreland)[37], bare particulars remain in themselves devoid of all characteristics whatsoever, and this is their incoherence. The theorized 'tied-to' relation itself indicates how for bare particulars even their 'essential' attributes (e.g., simplicity, unrepeatability) are radically external to them, displaying how bare particulars can have absolutely no content or intension. Hence, even their *raison d'être*—their unrepeatability—is outside their constituting essence, and hence, incoherently, something they need not be. This and other arguments against the coherence of such a concept are, I contend, compelling (e.g., those of Campbell, Loux, and Mertz)[38], and I refer the reader to them. Of these additional arguments I would here mention two. First, based upon the above Uniqueness by Composition Principle that entities having exactly the same constituents are identical, there could be at most one bare particular, just as analogously there can be but one null set. Entities having absolutely no constituents [proper or improper (in having no essences)] have by default the same constituents and are thus identical. It follows, absurdly, that there could be but one ordinary thick individual.[*] Secondly, the predication of properties of a subject is a function of the compatibility of the contents or intensions of each of the properties *and* that of *the subject*. Even so-called 'external' relations presuppose intensions characterizing their relata, e.g., the external spatial relation in the fact :Left-of(*a*,*b*) requires *a* and *b* have (or have content that support) the content Being-Extended, it not being possible for the relation Left-of to take as subjects any entities whatsoever, e.g., *a* and *b* could not be the numbers 3 and 4. Now, a would-be subject devoid of all intension or content could not condition or delimit properties as to their predicability or nonpredicability of it. Hence, the predicates of a bare particular would be every one arbitrary and contingent, contrary to its necessary properties such as simplicity and unrepeatability.

The important lessons to be learned from the above is that a slide into a *reductio* of bare particulars results from the assumptions that a predicate (immediate or mediate) is an intension universal, and that any content- or nature-bestowing intension possessed by an individual is predicable of it. As universals, every characteristic of an individual is other than it ren-

[*][Moreland and Timothy Pickavance have responded to this argument in 'Bare Particulars and Individuation: Reply to Mertz', *Australasian Journal of Philosophy* 81 (2003): 1-13. My counter-response is found in the same issue.]

[37] Moreland, 'Theories of Individuation: A Reconsideration of Bare Particulars'.

[38] See note 14.

dering it a bare particular. These assumptions require of an ontology that every individual that is not a bare particular be a *complex* entity, and force bare particulars to be constituents of whatever is the ontology's lowest level intensioned particulars (e.g., substance in the *Categories*; Armstrong's states of affairs; Moreland's unit attributes). Consequently, to avoid these difficulties an ontology is needed where what is predicable is particular, and where atomic individuals have as their qualitative contents repeatable intensions that are not predicable of them, but where the unity of the individuating and intension aspects requires no additional linking predicates constitutive of the atomic individuals. (To reiterate, this latter requirement is what is meant by a *simple* entity—not that distinct and partially characterizing aspects cannot be abstracted from the entity, but that these aspects do not exist in it as subjects of a further predicable unifier that is also part of the entity.) The meeting of each of these requirements will be seen to follow from the defining combinatorial character of predicates. It is to be noted that Aristotle in the *Metaphysics* went some distance in satisfying these conditions with his doctrine of unit substantial forms. Here a substantial form is individuated (*Metaphysics* 1042a29, 1049a35, 1071a25-30), it is predicable of multiple secondary matter and in this is the cause of a resultant structured whole—hence the ontic equivalent of a polyadic relation (*Metaphysics* 1016b12-17, 1041b1-30, 1045a6-25), and there is no 'ontic distance' between its combinatorial mode and its intension (implicit in Aristotle (cf. *Metaphysics* 1045b8-24), but explicit in the scholastics, e.g., as referenced in Suarez[39]). Unfortunately, the explanatory virtues of this analysis are negated in Aristotelian ontology by the individuating role assigned to matter, which in the limit of prime matter must become a bare particular, and by the official reduction of all polyadic to monadic predicates as reinforced by a false containment model of predication.

3. The Erroneous Containment Model of Predication

It has been argued that there cannot exist bare particulars, i.e., every unrepeatable individual must possess in a manner more intimate than external predication a content of one or more repeatable intensions. To make intension universals characterizing an individual externally predicable of and hence numerically distinct from it is to introduce a separation fatal to the

[39] Francisco Suarez, *On the Various Kinds of Distinctions* (*Disputationes Metaphysicae, Disputatio VII, de variis distinctionum generibus*), trans. Cyril Vollert, S.J. (Milwaukee: Marquette University Press, 1947), I, 17-29, pp. 28-38.

individual *qua* individual. The alternative is to eliminate the 'ontic distance' between an individual and its intensions by placing the latter somehow 'in' the individual. We shall see that this is indeed possible and required for unit attributes with their single intensions, as based upon results below that the combinatorial and intension aspects of a predicate are abstractions from a simple entity and that the combinatorial aspect individuates the whole. However, as we shall see below this requirement of collapsing the ontic distance between an individual and its intensions is incompatible with the assumption that the individual is characterized by multiple intensions. Seemingly in line with the leading thesis is the long-prevailing but in fact insidious *containment model* of predication where a predicate is held to be 'contained in', 'immanent in', 'inherent in', its subject. It is held, for example, that apple *a* has mass because the property Has-Mass is 'in' *a*, a plausible theory given that apple *a* is a spatially extended object. The theory is less plausible, however, for non-spatial and apparently non-complex subjects, e.g., that the property Prime is 'in' the number 3.[40] The containment model was explicit in the semantics of the medieval inherence theory of the copula. In a standard categorical proposition such as 'Apple *a* is round' the copula 'is' was considered to be a *signum inhaerentiae* of the universal connoted by the grammatical predicate 'round', it having the status of *inesse subjecto*, here the containing subject being apple *a*.[41] Historically, the containment model has remained mostly implicit with its further implications either unrecognized as such or unattended. Two notable exceptions are the implications Leibniz drew from his explicit *Praedicatum-inest-subjecto* doctrine[42], and those drawn by Bradley from his thesis that every judgment is a differentiation of the predicate out of a subject which is then 'restored' to the subject by predication, the subject of every judgment being ultimately all of reality or the Absolute[43].

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[40] The number 3 is at most a simple intension, or at least, under structuralist theory, an empty place holder in an ordered structure that is the natural numbers system, and in either case no amount of attempted conceptual dissection could find properties Prime, Odd, etc., a literal interpretation of Kantian 'analyticity' notwithstanding.

[41] See John Malcolm, 'A Reconsideration of the Identity and Inherence Theories of the Copula', *Journal of the History of Philosophy* 17 (1979): 383-400. For a particular example of the inherence theory in Aquinas see Keith Buersmeyer, 'Predication and Participation', *The New Scholasticism* 55 (1981): 35-51.

[42] Gottfried W. Leibniz, *Philosophical Papers and Lett*ers, 2d. ed., ed. L. Loemker (Dordrecht: Reidel, 1969), pp. 264, 334, 337.

[43] F. H Bradley, *Appearance and Reality*, 2d ed. (1897: reprt. ed., Oxford: Clarendon Press, 1966), pp. 144-45, 147-49.

Under the containment model a multiply characterized subject can be conceived as either 1) a plural heterogeneous bundle, e.g., a bundle of tropes (Campbell, Simons, Denkel, LaBossiere)[44], or as 2) a monistic homogeneous 'blend' of its properties, e.g., analogous to gray paint as the identity-losing blend of black and white paint. The notion of a constituent blend is obscure, though I take it to be what is ultimately the implied nature of Bradley's Absolute[45] and as required by any monism. I shall attempt briefly to characterize it. According to Bradley, "Since diversities exist… they must be united. …But this means that A and B are united, each from it own nature, in a whole which is the nature of both alike. And hence it follows that in the end there is nothing real but a whole of this kind."[46] In 'blending' each element is 'transformed' effecting a 'transcendent' whole, "A whole in which distinctions can be made [by abstraction], but in which divisions do not exist."[47](my insert) The entities that are blended each lose their identity by a kind of mutual suffusion into a resulting whole, and this in such a manner that when these entities are intensions they are no longer truly assertable of the whole; they are "characters of a Reality which they cannot express"[48], analogous to the fact that blended gray paint is literally neither black nor white. In effect, under the blending version of containment we have unity with no real plurality.

Now, inherent to the containment model, predication as a state of union between a subject and its characterizing attributes ceases to be ontically primary, but is rather now secondary and derivative of what must be a more fundamental unification existing among the attributes and whatever else constitutes the subject (e.g., an individuator if the other constituents are universals). Yet, here neither the blending or bundling forms of unification are sustainable. Specifically, in opposite ways blending and bundling each contradict the facts that both diversity and unity among the diverse is real (i.e., complexes are real), and that there are irreducible polyadic relations that obtain among the diverse. In regard first to Bradlarian blending we have seen that it implies the counterfactual thesis that diversity is an il-

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[44] See notes 16 and 17. Michael LaBossiere, 'Substances and Substrata', *Australasian Journal of Philosophy* 72, 3 (1994): 360-70.
[45] Bradley, *Appearance and Reality*, pp. 154, 404, 509-10.
[46] Ibid., p. 510.
[47] Ibid., p. 128. Also see pp. 161, 519, 521.
[48] Ibid., p. 511.

lusion. Relatedly, since asymmetric relations imply an order and hence a distinction between relata, they too must be declared illusion. Let it be the case that :R(*a*,*b*) exists for asymmetric relation R. As a 'predicate' of both *a* and *b*, R must blend into both and so carry with it the mutual blending of each relatum with the other. As Bradley puts it, "A relation must at both ends affect, and pass into, the being of its terms"[49] and so "A relation is unmeaning, unless both itself and the related are the adjectives of a whole."[50] Yet this forces a loss of distinction between relata *a* and *b* that the asymmetry of R presupposes. In Bradley's words, "The terms and the relation must 'enter' one into the other, and yet again are ruined if they do so."[51] Bradley interprets correctly the consequences of blending as implying the absurdity of both polyadic and monadic predicates, blending being for Bradley the only way to account for predication and its requirement of a real distinction between subject(s) and predicate. In total, "All predication, no matter what, is in the end untrue and in the end unreal…"[52] Hence, all plurality is an illusion, and no proposition is in the end true. In effect, Reality, the 'One', becomes the cohesive at the loss of the cohered. Here unity is from the one insofar as it obliterates all distinctions among the united. But then of what is this a *reductio*? Surely if something is to be rejected, it is to be the mysteries of blending and monism over the pervasive given of the diverse and related.

Under the alternative bundle theory where real diversity is a given, the standard and necessary account of the needed unity is the posit of a special relation (e.g., Compresence, 'founding') unifying the constituent properties F, G, H, … into resultant *a* (the constituents including necessarily at least one individual, e.g., a trope, if resultant *a* is an individual). But then, the unity here is supplied by a relation in its role as a polyadic *predicate*, and under the containment model of predication all multi-subject *polyadic* predicates must reduce necessarily to single-subject *monadic* properties of their relata. For, containment taken literally requires that the predicable nexus a property (or property instance) P has with an 'encompassing' subject *a* be independent of and not conditioned by P's simultaneous predicable nexus with any other subject *b*, $b \neq a$. That is, consistent

[49] Ibid., p. 322. Also see p. 347.
[50] Ibid., p. 394.
[51] F. H. Bradley, 'Relations', in *Collected Essays*, Vol. 2 (Westport: Greenwood Press, 1970), p. 638.
[52] Ibid., p. 672.

with the containment analogy, property P's being wholly 'in' *a* implies that no part of P-insofar-as-it-is-predicated-of-*a* is 'left outside of' *a* to be shared by, and in this be an ontic bridge to, some other subject *b*. Consequently, the property reduction of relations implied by a strict interpretation of the containment model of predication itself implies that the would-be bundling relation between, say, constituent properties F and G must reduce without remainder to foundational properties 'in' each, and hence the requisite cross-constituent unification essential to resultant *a* would disappear into the then mutually isolated F and G. No real interposing connectedness would result. This strict implication is counterfactual.

The latter implication is no doubt why historically a modified reduction of relations inspired by the containment model became the standard analysis. Here a real relation (held to be at most dyadic) was said to be reducible to monadic properties of each of its relata, the relation's *esse in*, but where the latter each have a *sui generis* character of 'being toward' the other paired relatum, the relation's *esse ad*.[53] The 'toward' or *esse ad* aspect was admitted as a minimal concession to our intuition of and theoretical need for inter-subjects connectedness. Though it was historically enduring, the *esse in*/*esse ad* analysis is untenable, as signaled by the counter-intuitive theories that have been drawn from it. For example, in his rigorous championing of the *praedicatum-inest-subjecto* tradition, Leibniz ridiculed the notion of an unreduced relation as an attribute having to have "one leg in one [relatum] and the other [leg] in the other [relatum]"[54](my inserts), and, pressing the doctrine's implications, concluded to an ontology of isolated monads, each internally 'mirroring' the rest of the world. Since every entity enters into at least some relations with every other, each of Leibniz's monads would contain the *esse ad*'s (the 'reflections') for every one of these relations without being actually linked to any other entity. Importantly and the reason for its failure, the 'being toward' aspects of this reduction would have to be themselves intensionless, for otherwise they would be further full relations calling for further *esse in*/*esse ad* reductions, and so on to vicious regress. A *reductio* of these 'bare linkings' will be given below.

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[53] Mark Henninger, *Relations: Medieval Theories, 1250-1325* (Oxford: Clarendon Press, 1989).

[54] Leibniz, *Philosophical Papers and Letters*, p. 704.

Within this tradition it was but a short step to relegating the distinctive *esse ad* aspects of the reduction, and consequently relations generally, to the status of products of the mind (*entia rationis*). The 'toward' aspects became for some philosophers (e.g., Hume)[55] free or blank associations resulting from 'comparisons' of the mind but revealing no 'real connection', i.e., no unity controlled by the compatibility of relata and relation intensions, the latter having been abstracted away into the *esse in*'s. Among the notable casualties of this tack is a loss of the nomic necessity of causation, it being replaced with the residual alternative of contingent constant conjunction. Here we have plurality with no real extra-conceptual modes of connectedness, and no connections conditioned on anything more than the existence of their terms and the unifying agency of the associating mind. In an attempt to eliminate the subjective/ideal character of unity by cognitive agency, an extensionalist/nominalist tack is to adopt the strengthened thesis that the existences of entities, independent of their natures, is sufficient *tout court* for the existence of resultant wholes of all possible combinations thereof. Here either set theory or mereology is awarded a primary ontological standing, and ontology proper reduces to models, i.e., substructures, from one of these formal theories. Yet, these formal models are themselves 'extrinsically determined' substructures constructively selected, i.e., differentiated out, *by the agency of minds*. What would be in fact intensionally determined relationships making up a given *a posteriori* complex, some contingent, some necessary, guide the unofficial but prerequisite creative act of selecting out the primitive sets or sums of the *a priori* model. The model itself consists of the structure containing only modally necessary relations following upon these atomic sets or sums and that are definable in the terms of either the Element-of relation for set theory or the Part-of relation for mereology. The essential role of mind is not eliminated, just officially ignored. Equally telling is the fact that here the role of intensions has not and, indeed, cannot be eliminated from the theories. For, in either theory the primitive relation, Element-of or Part-of, is essentially intensional since the axioms of the theory are not random, i.e., their negations are not alternatively assertable, but rather are necessitated by the very intension or content (the 'meaning') of the relation, and the relation is in principle unreducible to any kind of extensional surrogate within the theory. Consequently, the extensionalist strategy of identifying complexes with formal

[55] David Hume, *A Treatise on Human Nature*, I. ii. 4; ed. T. H. Green and T. H. Grose (1886; rpt. ed., Germany: Scientia Verlag Aalen, 1964), pp. 352, 559.

models avoids neither the necessity of an appeal to cognitive agency nor the necessity of connectedness defined and delimited by intensions.

In sum, neither the blending nor bundling alternatives of the containment model of predication can provide an account of how a thick individual can be characterized by multiple attributes. In opposite ways each alternative proves contrary to the fact of real unity among the truly diverse.

4. Relations and the Combinatorial Nature of Predication

Historically, the property reduction of relations was an error whose correction was slow in coming. In the formal sciences of logic and mathematics the explanatory value of relations as full multi-subject predicates was explicit from at least the middle of the nineteenth-century.[56] However, it was not until Russell's *Principles of Mathematics*, 1903, that we have what is now the *locus classicus* for arguments showing the ontological impossibility of the monadic reduction of polyadic predicates.[57] These arguments turn on the order of the predication that some relations (e.g., asymmetric and non-symmetric) maintain among their relata, in each case an order that cannot be eliminated extensionally (cf. Hochberg)[58] but is rather a function of the relation's intension.[59] Recently I have reinforced Russell's arguments against counter-claims found in Campbell's version of contemporary trope theory.[60] I shall not rehearse these arguments here. The crucial point for ontology is: Relations are unreducible to properties. Relations are not characterized as *esse in*/*esse ad*, but rather as *esse inter*—as fully real 'interconnectives' among their relata.

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[56] There were explicit attempts in the sixteenth and seventeenth centuries to demonstrate that proofs in Euclid's *Elements* are reducible to syllogistic form. See Neal Gilbert, *Renaissance Concepts of Method* (New York: Columbia University Press, 1960), pp. 89-90. Ian Mueller has shown that not even the first proposition of the *Elements* is reducible to syllogistic form. Ian Mueller, 'Greek Mathematics and Greek Logic', in *Ancient Logic and Its Modern Interpretation*, ed. John Corcoran (Dordrecht: Reidel, 1974), pp. 35-70.

[57] Bertrand Russell, *The Principles of Mathematics*, 2d ed. (1903: reprt. ed., New York: Norton, 1938), pp. 221ff.

[58] Herbert Hochberg, 'The Wiener-Kuratowski Procedure and the Analysis of Order', *Analysis* 41 (1981): 161-3.

[59] Bertrand Russell, *My Philosophical Development* (London: Allen & Unwin, 1959), p. 67.

[60] Mertz, *Moderate Realism*, pp. 163-71.

It is important to note that the non-reducibility, non-eliminability of ordering relations is strong evidence against the would-be deflationary thesis that necessary or 'internal' relations are supervenient upon and so represent no ontic addition over and above their relata (e.g., Campbell, Armstrong)[61]. For, given, say, the numbers 2 and 3 it is necessarily the case that the Less-than relation exists between them, i.e., necessarily :<(2,3), yet on the arguments cited the Less-than relation is not reducible to monadic properties of 2 and 3 or anything else. In particular, the ordering of the Less-than relation cannot be accounted for by any monadic property, or set or mereological sum thereof. The only seeming alternative to admitting an uneliminable entity into one's ontology is an argument showing that the very concept of such an entity is incoherent and is therefore illusion or 'appearance' in the relevant sense of Bradley's arguments against relations generally. Yet authors who advance the supervenient view do not want to grant such 'unreality' to internal relations.

Relative to traditional substance/attribute ontology, an analysis of the nature of relations motivated by their non-reduction produces a kind of 'Copernican Revolution' in both analytic power and ontological gain. In broad terms what is gained is the liberating perspective where, as it were, the monadic tail no longer wags the polyadic dog. Monadic properties are seen to be but the limiting case of polyadic relations, where the latter have ontically and logically significant characteristics obscured or absent in the former (e.g., ordering and formal properties such as asymmetry).[62] Specifi-

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[61] Campbell, *Abstract Particulars*, pp. 97ff. Also see Campbell's 'The Place of Relations in a Trope Philosophy' in *Proceedings of the Colloque International de Philosophie de Grenoble: La Structure du monde; Objets, Propriétés, Etats de choses*, in *Recherches sur la philosophie et le langage*, Universite Pierre Mendes France, Grenoble, 2003; Armstrong, *States of Affairs*, pp. 87-88, 92-93.

[62] Also globally, the non-eliminability of relations provides the ontic underpinning for the analytic methodology that holds that to discover the relations an entity *a* has with other interrelated entities, i.e., to find *a*'s 'position' in the context of a complex, is to have an explanation of *a*. As a slogan, 'To explicate is to interrelate.' This contrasts with the once common methodology that 'To explicate is to eliminate.' See Peter Strawson, *Subject and Predicate in Logic and Grammar* (London: Methuen, 1974), p. 37, 54-55. Also see Jerrold Katz, *The Metaphysics of Meaning* (Cambridge: MIT Press, 1990), p. 186. The containment model of predication, requiring as it does the property reduction of relations, not only abets the latter and, I claim, impoverished reductive methodology, but, insidiously, eliminates relations as the corrective to it. See *Moderate Realism*, p. 37.

cally, the non-reduction of relations to properties motivates, first and primarily, the recognition of what is the *sui generis* unifying nature of predicates among their subjects—predicates are *combinators*, i.e., agents of unification. This point will be established presently. The combinatorial nature of predication is what Frege attempted to capture with his metaphorical ascription to concepts of an 'unsaturated' character. As unsaturated, a concept is "capable of serving as a link", for in every complex whole "at least one [constituent] must be 'unsaturated' or predicative; otherwise they would not hold together."[63](my insert) Medieval philosophers noted the necessity of the combinatorial aspects of forms and properties in order for them to be "*per se* and directly joined without the medium of another mode", here referred to as 'modes' (Suarez)[64] and by others as 'dispositions' (Buridan)[65]. Recently, Armstrong has adopted the 'way' metaphor suggested by David Seargent where, in response to Bradley's Regress, it is said that properties and relations as ways "form so much closer a unity with the things involved that perhaps further relations are not required."[66]

The unifying nature of polyadic relations is ontically primitive, and necessarily because of this is accessible only by a kind of conceptual parallax focusing attention on and differentiating out this fundamental aspect of reality. The argument now known as Bradley's Regress is a classic and perennial heuristic device cited as effecting the insight that relations have a combinatorial nature.[67] Also instructive in making this same point, I propose, is this argument's interpretation, e.g., by Bradley and Ockham[68], as showing the self-contradictory nature of relations. Central to the analyses herein is understanding the grounds for these two interpretations. Bradley's Regress argument is as follows. In a fact :$P^1(a)$, e.g., Plato is bald, or formally :$Bald^1$(Plato), either property P^1 is predicationally attached to subject

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[63] Frege, *Translations*, pp. 54-55.
[64] Francisco Suarez, *On the Various Kinds of Distinctions*, I. 20, p. 31.
[65] John Buridan, *Questiones in metaphysicam aristoteles*, V, Q. 8, fols. 31, 33. Portions translated by Calvin Normore, 'Buridan's Ontology', in *How Things Are*, ed., J. Bogen and J. McGuire (Dordrecht: Reidel, 1985), p. 198.
[66] Armstrong, *States of Affairs*, pp. 30, 38, 98.
[67] Cited as displaying the unifying aspect of at least monadic properties, the regress argument is anticipated in Aristotle (in regard to substantial form), and found fully in philosophers such as Avicenna, Buridan, Suarez, Frege, Russell, and most modern ontologists. For references see my *Moderate Realism*.
[68] Bradley, *Collected Essays*, pp. 628-76. William of Ockham, *Ockham's Theory of Terms*, trans. Michael Loux (Notre Dame: University of Notre Dame Press, 1974), p. 170.

a or it is not. If not, then the unity between P^1 and *a* is not that of attribution but rather at best mere association, and in which case fact $:P^1(a)$ reduces to the juxtaposition of set $\{P^1,a\}$, e.g., $\{Bald^1,Plato\}$. Yet, clearly fact and set are not identical—in the example set $\{Bald^1,Plato\}$ no claim is made as to whether Plato is bald or not, yet with the complex $:Bald^1(Plato)$ we have the fact grounding the affirmative claim. Moreover and as previously noted, for contingent P^1 the fact $:P^1(a)$ can come into and go out of existence whereas the set $\{P^1,a\}$ exists necessarily and so is 'eternal'. Indeed, the set exists even when for some intensions (e.g., Unicorn, Phlogiston) no parallel fact ever obtains. In regard, then, to the initial fact $:P^1(a)$, an additional unifier, say relation R^2, is required to supply the *predicational unity* needed for the original fact, the latter now becoming $:R^2(P^1,a)$, e.g., $:Exp^2(Bald^1,Plato)$, where 'Exp' designates Exemplification. Now the same question can be asked of this new fact: Is R^2 predicationally linked with P^1 and *a*? If not, then fact $:R^2(P^1,a)$ becomes set $\{R^2,P^1,a\}$, which requires a further relation R^3 to supply the requisite predicational unity, i.e., the original fact has now become $:R^3(R^2,P^1,a)$, e.g., $:Exp^3(Exp^2,Bald^1,$ Plato). Clearly this denial of a predicational unification between predicates and subject(s) leads to vicious regress. And, to admit this predicational unity at any step implies its existence at the first step, $:P^1(a)$, the regress thus eliminated *ab initio*. Points to be observed here are that, first, at each step where predicational unity is required, it is intuitive that a polyadic relation should be called upon to do the job, and rightly so since the alternative of intensionless or bare linkings is incoherent as will be seen. Further and importantly, the initial plausibility of the ratcheting steps of the regress imply that we intuitively distinguish two 'versions' of an *n*-adic predicate: one as predicable in a fact, i.e., as a 'linking' among *n* relata under an intension, e.g., *a*-loving-*b*, and the other as the intension abstracted from its predicable aspect, e.g., Love. The regress plays on this possibility, and at each *n*+1-step it speciously equates the abstracted 'predicably inert' intension with what was in the *n*-step the full predicate as intension-plus-nexus-among-relata. That is, at each *n*+1-step the regress takes away the conjunctive or unifying aspect of the then relation, an aspect essential to the *n*-step, and leaves as a residue the relation intension as an unattached 'between'—as but one more element to be unified. Bradley is explicit in this: "An actual relation… must possess at once both the characters of a 'together' and a 'between', and, failing either of these, is a relation no longer…. On the other hand, if, to remain themselves, our terms retain their character as individuals, there is no legitimate way (we have seen) to their union in fact. We are without the 'together', which (like the 'between') is essential if any

relation is to be actually there."[69] Hence, the straightforward interpretation of what the regress demonstrates is that we cannot divorce the conjunctive or combinatorial aspect from polyadic relations treated as bare intensions, and analogously for properties as the limiting case. The relation of Exemplification (Instantiation or the like) is secondary and derivative, an artifact of analysis, analogous to the predicates True and False, having a conceptual existence only (the medievals' 'second intentions'). In sum, the unity between *n* entities does not always require an *n*+1 entity, the contrary being the road to regressive perdition. The latter error is abetted by the two easy misidentifications of a) a predicate with its abstracted non-predicable intension, and b) the vicious regress with the harmless progression of formal exemplification instances. Rather, there can be a unification of exactly *n* entities only if one of these entities has the categorical status of agent unifier, and this is what we intuit to be the nature of predication, especially as magnified in polyadic relations.

The above regress argument points to the proper understanding of polyadic predication without the distorting mis-analogies of the 'unsaturated' and 'way' metaphors, or a spatial interpretation of 'between'. The insight is: A polyadic predicate is an entity consisting of a qualitative content or intension controlling a *combinatorial or linking state* among a set of relata. Properties as monadic predicates are the limiting case of polyadic relations and thus have the same combinatorial character. Hence, all predicates, monadic or polyadic, are *ontoglial*—each is a rigid linking holding its relata both connected and yet separate, analogous to a connecting rod between connected nodes. As noted, one sees this 'relational' conception of predication in, for example, Aristotle's construal of substantial forms and their predicable structuring of plural matter in the *Metaphysics*. On the combinatorial conception of predication three points are to be emphasized. First, every predicate as predicable is a constituent of a complex—a fact—that emerges as a function of the predicate's unifying agency to or among its subjects. In this predicates are the cause *sine qua non* of all plural unity, i.e., of all complexity, organization, system, and structure, starting at the lowest ontic level of atomic facts. Every pluralist ontology must appeal ultimately to relations, polyadic or monadic, in order to account for the unity, however minimal, essential to every complex *qua* complex. This latter point

[69] Bradley, *Collected Essays*, p. 638.

is crucial to ontology yet is generally neglected.[70] Secondly, the intension and the combinatorial state of a 'relating relation', i.e., in its predicable role in a fact, are mutually necessitating. The intension is the source of (i.e., is the cause of) the number, order, and compatibility/admissibility of the linked relata, and, more globally, of the relation's formal/logical properties. This is the more obvious the higher the adicity. For example, in the fact :Between(*a*,*b*,*c*), which is the linear spatial arrangement of object *b* being between objects *a* and *c*, there is the unification of three objects under the intension Between. The intension specifies the second-order attributes of triadicity, the *a*—*b*—*c* order, and the logical properties such as the triadic version of symmetry, i.e., if :Between(*a*,*b*,*c*) obtains then so does :Between(*c*,*b*,*a*). Thirdly, the combinatorial agency of a predicate is 'outwardly directed', holding the predicate attached to but distinct from its subject(s). This is then equally true in the limiting monadic case—for fact :P(*a*), the predicate with P as its intension attaches itself to *a*, it is not a component of *a*. Significantly, then, all predicates are *external* to their subjects in the sense that what is predicable of a subject (or subjects) is not a constituent of the subject(s), contrary to the popular containment model of predication. Note that this does not deny that a single subject *a* can have predicates as constituents—the cross connecting relation instances making up *a* as a complex are such predicates—but only that none of these predicates can have emergent *a* itself as a relatum. Further, the externality of predicates does not require that at some base level atomic predicates must have bare particulars for subjects. Rather, the subjects of atomic predicates need only be other atomic predicates, all of which are particulars on the ar-

[70] Ontologies that attempt to avoid an appeal to relations as the agents of plural unity resort to classes or mereological wholes taken as primitives. Yet, some explanation of the source of unity of and correlative distinction among the elements of even these minimal, bloodless wholes is required of an ontology. One cannot simply say that a class or mereological sum exists when their elements exist, any more than one can say a wall exists when the bricks exist. For everything that there is exists, including all bricks across the world. Metaphorically, some entities and not others must be 'brought together' to form the whole. Separating out the spatial mis-analogy, what this means is that there must be a cause of the association of specific elements and in this differentiation from all other groupings. The simple criterion of existence in its non-differentiation could sustain at best the universal set. I propose that Russell was close to the truth in observing that "Although from the point of view of a formal calculus one can regard a relation as a set of ordered couples, it is the intension alone that gives unity to the set. The same thing applies, of course, also to classes. What gives unity to the class is solely the intension which is common and peculiar to its members." Bertrand Russell, *My Philosophical Development* (London: Allen & Unwin, 1959), p. 67.

gument to follow. Moreover, the above does not imply that predicates are radically external in the sense of their unifying agency being independent of the natures or intensions of their subjects. Reiterating a point made previously, even the 'external' spatial relation of Between places conditions an the possible categories of its relata, and is therefore not indifferent to what are the contents or natures of the respective relata, e.g., it is predicationally impossible for the triplet consisting of Plato, 3, and Circularity to be subjects in any order for this relation. This incompatibility of intensions is the source of what are termed 'category mistakes'. Succinctly, then, under the combinatorial analysis, the predication in fact :$R^n(a_1,a_2,...,a_n)$ is a rigid linking holding the non-identical relata among a_1, a_2, ..., a_n both united and distinct, and where the respective natures of a_1, a_2, ..., a_n are compatible with, and ordered (if any) according to, intension R^n.[71]

[71] It is worth noting here that reinforcing this insight into the linking nature of relations is its power in providing a viable 'absence theory' for solving the difficult problem of true negative propositions. The intractability of this problem under standard ontology is highlighted by George Molnar, 'Truthmakers for Negative Truths', *Australasian Journal of Philosophy* 78 (2000): 72-86. Atomic affirmative propositions, e.g., P(*a*), are held to be true because they correspond to a 'truth-maker'—fact :P(*a*). Some have held that, by simple analogy, there must be for true negative propositions, e.g., ¬P(*a*), corresponding 'negative facts'. But this tack, in addition to positing dubious entities, does not account for the fundamental syntactical fact that subjects cannot be negated, but only linguistic predicates, e.g., whereas one can say 'The ball is not red', it is nonsensical to say 'The not ball is red'. On the theory of predication defended herein for a negative proposition what is asserted is the *absence among* the referent(s) of the grammatical subject term(s) (e.g., the ball) of a combinatorial/predicable state controlled by the intension (e.g., Red) of the referent of the grammatical predicate. Here a predicable nexus is absent, i.e., non-existent, whereas the controlling intension is 'not absent' in the sense that it exists as an abstraction or construction independent of any possible instantiation within a predicable instance. The fact that it is ontic predicates that have this linking aspect explains why only the corresponding linguistic predicates can be negated. Recently Hochberg has attempted to eliminate the need to posit negative facts by proposing that the truth-maker for a negative proposition ¬P(*a*) is the set of all atomic facts corresponding to the true propositions of the form 'P(*a*)'. In addition to other problems, this construction commits an epistemic fallacy of presumed omniscience, a flaw that often infects the substitution of formal models for ontic entities. The set of atomic positive facts is to be the ground for the truth of the negative proposition ¬P(a). Hence, to know that the negative proposition is true we would have to survey the entire infinite set of facts in order to determine that the would-be fact :P(*a*) is not in it. Since this is an impossible task for finite minds, we would never be able to determine the truth of negative propositions. See Herbert Hochberg, 'Facts and Classes as Complexes and as Truth Makers', *The Monist* 77 (1994): 170-91.

5. The Formal Distinction between a Predicate's Combinatorial and Intension Aspects

It is important to now observe the negative consequence of positing a real pre-abstraction distinction between the combinatorial and intension aspects of predicates and thus requiring a predicate to be a complex entity. Specifically, the unifier aspect of a predicate separated from any intension is a 'bare linking', i.e., a contentless or intensionless unifier, a notion as untenable as its cousin the 'bare particular'.[72] Yet, this is apparently what some philosophers intend by the posit 'non-relational ties' as an answer to Bradley's Regress.

The trouble with bare unifiers is that, in having no intension to control or discriminate among possible subjects, such an entity would be nothing but a blank association or arbitrary conjunction. A bare linking would be the minimal and unconstrained unity of a list or set. Hence, if a bare linking were the unifying agent within a fact, then there would be, for example, no difference in the unification among R^2, *a*, and *b* in the fact $:R^2(a, b)$ and in the set $\{R^2,a,b\}$. Thus having precisely the same components, fact and set would have to be identical, which, for a contingent R^2, would require a set that exists necessarily to be identical with a contingent fact. Moreover, a blank or 'free' association, in having no intension, specifies no ordering among the entities linked, and hence despite, say, R^2 being an asymmetric or non-symmetric relation, there would be no difference between facts $:R^2(a,b)$ and $:R^2(b,a)$. For asymmetric or non-symmetric relation R^2, it is R^2-as-it-occurs-in-the-fact-$:R^2(a,b)$ i.e., as having the dual and mutually influencing aspects of a delimiting intension and a combinatorial state among *a* and *b*, that orders the latter. If the combinatorial nature is reassigned to a fourth constituent, the bare linking, then intension R^2 becomes just one more relatum for an orderless triadic linking among R^2, *a*, and *b*, with no more influence over ordering the latter than it has in the set $\{R^2,a,b\}$. Finally and for the same reason, the modal character of every fact becomes the same, i.e., all facts are necessary or all facts are contingent. For, if the unity essential to every fact is the same bare unifier with no intension R^n to control the modal attributes of these unions, then all such complexes have the same modal quality, which is counterfactual. In short, there are no 'bare unifiers' any more that there are 'bare particulars'. To anticipate, it is no accident that the arguments against bare unifiers and

[72] See note 14.

bare particulars are analogous, for as will be seen presently the combinatorial aspect of a predicate as it exists among its subjects is unrepeatable, i.e., a particular.

Importantly then in regard to the Problem of Predication, what the above demonstrates is that the combinatorial aspect of a predicate can exist only with a correlative intension, and equally significant, it is inherent in the nature of a predicate that there be no 'ontic distance' between these two aspects, i.e., the predicate must be *simple*. For as seen, to make a predicate internally complex by holding the unifier and intension aspects ontically distinct components is to force the linking aspect itself to be devoid of content, and this is untenable. What this means is that the distinction between these two aspects is 'outside the thing', i.e., they are abstractions representing partial realities not distinct in the predicate. This type of distinction is what Duns Scotus identified as a *distinctio formalis a parte rei* (a formal distinction on the side of the thing) and what other medievals referred to as a *distinctio rationis ratiocinatae* (a distinction of the reasoned reason). Under the designation 'modal distinction' the scholastic metaphysician Francisco Suarez draws this distinction in regard to the predicable 'mode' of a property to its subject. In the face of Bradley's regress, Suarez concludes that "inherence does not need a further union or inherence by which it may be united or may inhere, the reason is that the inherence does not add a real entity which inheres and is united, but is merely a certain mode that of itself is the reason for union and inherence…. Yet this mode is not properly distinct from the subject it modifies, as thing from thing."[73] This is a distinction of the reasoned reason where "Although the same object is apprehended in each concept, the whole reality contained in the object is not adequately represented, nor is its entire essence and objective notion exhausted, by either of them…. Things said to be thus distinct are real entities, or rather a single real entity conceived according to various aspects."[74] Recently, Armstrong has concurred with Scotus, saying, "It is concluded, therefore, that although particularity and universality are inseparable aspects of all existence, they are neither reducible to each other nor are they related. Though distinct, their union is closer than relation."[75] If the unifier and particularizer of the atomic ontic unit are the same, as will be argued

[73] Suarez, *On the Various Kinds of Distinctions*, I. 18-19, pp. 29-31.
[74] Ibid., I. 5, p. 19.
[75] D. M. Armstrong, *A Theory of Universals: Universals & Scientific Realism*, Vol. II (Cambridge: Cambridge University Press, 1978), p. 3.

presently, then Campbell's advocacy of the formal distinction is here apropos: "Although the idea here is that all particulars are particulars, and each of them has a nature, this does not involve conceding that a trope is after all complex (a union of particularity with a nature-providing property). The distinction is perhaps a 'formal' one, as Scotus used the term."[76] Against the formal distinction, Moreland has recently reiterated a position maintained by Ockham that if what is distinguished in thought are aspects of a single entity then they must be distinct within the entity.[77] This is an assertion, not an argument, and, to the contrary, there is no logical necessity preventing a single subject entity from being rich enough to support separate and partial yet veridical and complementary conceptions abstracted from it. This type of distinction is logically possible and necessitated on the above analysis.[78]

6. The Individuation of Predicates

Now, with this 'non-complex' character of predicates as a premise, it is possible to demonstrate that predicates are individuated as instances, R^n_i. Specifically, it is a simple matter to show that the combinatorial aspect of a predicate is unrepeatable, and this together with the subsuming predicate's internally simple nature implies the latter's inherited unrepeatability, i.e., individuation. In this we have the *principium individuationis* that eludes all bundle, substratum, and individuation-by-instantiation theories, and provides an explanation for what nominalists take as a primitive datum. The argument can be synopsized as follows.[79] Let it be the case that two facts $:R^n(a,b)$ and $:R^n(b,a)$ obtain, where R^n is a contingent and, for present purposes, a non-symmetric relation intension, e.g., Love. As non-symmetric, R^n imposes an order on its mutual combinatorial aspect accounting for the *directional unity* in each fact. But this directional unity in not identical across the two facts—in $:R^n(a,b)$ the predicational linking is from-*a*-to-*b*

[76] Campbell, *Abstract Particulars*, p. 56.
[77] Moreland, *Universals, Qualities, and Quality-Instances*, p. 68. Also his 'Keith Campbell and the Trope View of Predication', *Australasian Journal of Philosophy* 67 (1989): 386-7; William of Ockham, *Ordinatio*, d. 2, q. 6, trans. by Paul Spade, *Five Texts on the Mediaeval Problem of Universals* (Indianapolis: Hackett Publishing Co., 1994), pp. 153-90.
[78] See Mertz, 'Individuation and Instance Ontology', for an analogy to show the possibility of a simple entity with dual abstractable aspects.
[79] An expanded argument is found in my 'Individuation and Instance Ontology' [and in other essays herein].

whereas in $:R^n(b,a)$ the predication is from-*b*-to-*a*. If the predicable union were numerically identical in both facts, then having exactly the same constituents—intension R^n, relata *a* and *b*, and identical union—then by the Uniqueness by Composition Principle the facts would be identical, $:R^n(a,b) = :R^n(b,a)$. This would follow for all relata pairs of R^n, and hence R^n would be symmetric rather that non-symmetric. Further, no matter what the formal properties of intension R^n are (e.g., whether symmetric, asymmetric, or non-symmetric), on the assumption that R^n is contingent, if it were *numerically the same* fact-forming-union in fact $:R^n(a,b)$ and any other fact $:R^n(c,d)$, then if one fact ceased to obtain, i.e., its predicational union ceased to exist, then similarly all other like-intensioned facts would cease to exist. That is, all such facts with contingent R^n would come into and go out of existence together. All of this is a *reductio* of the present assumption that it is numerically the same predicable nexus in facts having the same intension. It follows that the combinatorial union of a predicate with its subject *n*-tuple is unique and unrepeatable relative to that *n*-tuple. Because this state of unification is individuated, and because it and its correlative intension R^n are distinct aspects in abstraction from but not distinct within the subsuming simple predicate, this predicate is itself an individuated instance R^n_i. In short, the predicable nexus is not repeatable, and hence a predicate *qua* predicable is not a universal.

It is the case, then, that predication is the principle or cause of individuation. Hence, contrary to a sometimes proposed thesis that it is the individuation of subjects that externally individuates their attributes, on the above an *n*-tuple of subject relata, $<a_1,a_2,...,a_n>$, is the secondary cause of the individuation of a predicate R^n_i, but it is the internal combinatorial aspect of R^n_i among these relata, what is essential to and inseparable from fact $:R^n_i(a_1,a_2,...,a_n)$, that is the primary cause of the predicate's individuation. Unlike for trope theory, where for exactly resembling tropes it is a brute fact that they differ *solo numero*, we have here the explanation that relation instances differ by their individuated linking among distinct relata sets. Further, because the individuator of a predicate R^n_i is its combinatorial state, which is not identical with its correlative intension R^n and not itself a further intension, we do not have the absurd situation where the same entity, instance R^n_i, has contradictory attributes, i.e., unrepeatability and repeatability. Nor do we have as with tropes the mysterious compression of individuation and intension into a simple entity that somehow removes the repeatability of the latter.

7. Conclusion: Network Instance Realism

The above results provide a successful, non-reductive means for solving ontology's fundamental Triple Aspect Problem and the Problem of Complexity, and those problems that, through the pivotal role of predication, are attendant to them.[80] With the single category of relation instances we have simple entities that are of their essence *predicable*, i.e., combinators, that by that very fact are *individuated*, and where each has a correlative non-predicable *repeatable intension*. Predicable particulars, some necessary, some contingent, along with their abstractable intensions, are sufficient to account for all of observable reality and conform to our best scientific explanations thereof. The resulting ontology would outline as follows.

First, as outwardly combinatorial, atomic instances are dependent for their existence upon other entities, but this does not make them less real ('supervenient') or incapable of sustaining a complete ontology. The status of ontic dependence of a relation instance on its relata, whether further characterized as necessary (an 'internal' relation) or as contingent (an 'external' relation), does not militate against the distinct reality of the instance anymore than does, say, the existential dependence of a cat upon its food make the cat 'less real than' or 'supervene upon' its sustenance. Indeed, the forgoing displays how from among the traditional characterizations of substance the condition of being independent ('self-sustaining') conflicts with that of being a cause of unity. Guided by the criterion for being real suggested by the Stranger in Plato's *Sophist* (247e), i.e., what is real has the power (i.e., active and passive *dynamis*) to affect and be affected by something else, this paraphrases on the above into the more general form: An entity is real that has the active power to go beyond itself and 'make a difference' in, i.e., qualify, something else, or the passive power to be so qualified. The former power is that of predicates, especially in the broad

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[80] The problem solving power of this analysis of individuated predicates with universal intensions is further magnified when the logic inherent in it is formalized. The result is a 'formal ontology' in the sense of Nino Cocchiarella, 'Ontology II: Formal Ontology' in *Handbook of Metaphysics and Ontology*, eds. H. Burkhardt & B. Smith (Munich: Philosophia Verlag, 1991), pp. 640-47. The logic inherent in realist instance ontology, what I have called 'PPL', has a number of powerful results, e.g., distinguishing legitimate from illegitimate self-referential predication, distinguishing identity from indiscernibility, and providing an ontology for arithmetic. PPL is developed in *Moderate Realism*, with an improved version in 'The Logic of Instance Ontology', *Journal of Philosophical Logic* 28 (1999): 81-111[Essay 6 herein].

polyadic sense and as conditioned by intensions, and the latter power is that of subjects, again as conditioned by their contents. This and the unreducible/uneliminable status of relations places them among the fundamentally real, and for all controlling intensions, not just those artificially restricted to spatial/temporal, efficient causal, or (*contra* Armstrong) contingent types. All of this with the further fact of their individuation renders predicates of any *n*-adicity *the* primary ontic category. Starting at the atomic ontic level, relation (including property) instances would have relata from among themselves, forming lattice-like complexes. [For details and models see Essays 2 and 4 in this volume.] One can model this possibility physically. In this we have the answer to philosophers (e.g., Campbell and Armstrong) who contend that if there were no non-relations (i.e., non-predicable entities—'objects', 'substances', etc.) there would be no relations, but not vice versa. We see that exactly the opposite is the case. What atomic relations there are (concrete or abstract (including logical/mathematical)) is for science in the broad sense to determine. These atomic lattices as unit wholes function themselves as single relata for further relation instances, resulting in complexes of complexes, and so on up through hierarchies of increasingly intricate wholes, static and dynamic (i.e., as 'event structures').

It is to be emphasized that a complex is not definable simply as any set or 'conjunction' of relation instances, but rather as a set of instances that are themselves 'connected' by means of strategically shared relata. That is, in its full generality a complex is a network of relation instances where, as such, each instance is joined to one or more other instances via shared relata nodes, all in such a way as to yield continuous connectivity, analogous to that facilitated by the sharing of vertices in a lattice. Simons has made this point precise using the topological analog of path-connectedness.[81] Loosely, two relata *a* and *b* are path-connected if and only if through a chain of relations and shared relata one can trace a path linking *a* and *b*. A complex is then a set of relations and relata where any two of the latter are path-connected through chains of the former. Alternately and with a combined precision and simplicity made possible with the refinement of instances, we can define a set of instances to be a complex if and only if any two disjoint and exhaustive subsets of it have at least one instance each that share the same relatum. The accuracy of this definition is verified by means of simple connected 'road and node' diagrams.

[81] Peter Simons, *Parts: A Study in Ontology* (Oxford: Clarendon Press, 1987), p. 327.

Complexes at any level are non-predicable entities ('objects' in the sense of Frege) and relatively independent (in the manner of traditional substances), but both their constituent instances and the complexes as wholes can be possible relata for further relation instances. In this ontology we have the unity-among-the-diverse requisite of a plural and multiply structured universe given in experience and theorized in science. Consonant with modern science (e.g., neural science and quantum mechanics)[82], at different levels *n*-adic relations emerge each with its own novel intension, and each non-existent at, and hence not reducible to, lower levels of complexity. At certain levels of neural complexity cognitive faculties arise sufficient for the then emergence of instances of logical, mathematical, ethical, emotional, etc., relations. At these levels of complexity there also arise both the facility to abstract the content intensions from relation instances and the facility to freely construct from them further intensions that may or may not be instantiated (this the basis for fantasy, theory, and the modal character of possibility). This possibility of intensions without predicable instances runs contrary to a popular principle that holds that every *n*-adic universal must have at least one *n*-tuple of entities that instantiate it. Uninstantiated intensions are the basis for an actualism that allows 'possible worlds' as conceptual constructs only—the possible is delimited by intensions, the actual by intensions that have instances. Over all, some intensions emerge only at the conceptual level and of these some are instantiated (e.g., True/False, the logical connectives, Exemplification, mereology's Part-of) whereas others are not (e.g., Unicorn, Phlogiston), while other intensions exist extra-conceptually as the content of instantiated instances (e.g., instances of space-time relations, or of causal relations). It is not, as some philosophers claim, just spatial and/or causal relations that form the 'cement of the universe', but rather instances of relations of every kind of intension. In general, because of the inter-relatedness under various intensions of all entities, whether transcendent or natural (i.e., a relatum in the space-time matrix), reality is to be characterized as the Total-Complex, this in contrast to the distinction-obliterating homogeneous One of monism.

Contra historically popular substance ontology, no natural entities exist *per se*; rather they exist as, or as abstracted or constructed intension

[82] On the need for emergent properties in science see Michael Silberstein and John McGeever, 'The Search for Ontological Emergence', *The Philosophical Quarterly* 49 (1999): 182-200.

nodes in, substructures in the all-encompassing physical/cognitive complex that is the natural world and that is sustained at every level *inter se*. The traditional 'substance' is here a sub-complex that as a unit endures through the 'accidental' changes of becoming or ceasing to be a relatum for relation (including property) instances not constitutive of ('outside') the complex, but that suffers 'substantial change' when one or more of the instances internal ('essential') to the complex cease to obtain and are not replaced by instances of the same kind, thus causing this complex as a whole to cease to exist. This is in contrast to such devices as Simons' theory of substratum 'kernels' of 'mutually founding' tropes that form the essence of the substance and around which there is a lesser bundle of accidental tropes.[83] In network instance realism 'substance' complexes are controlled and delimited by the internal real intensions of the constituent relation instances where a few of all possible *n*-tuples are related, this being opposed to the mis-identification of 'substances' with certain mereological sums[84] chosen from among a formal pre-existing set of sums, one for every *n*-tuple, the latter posited in a misguided attempt at avoiding ontological commitment.[85]

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[83] Simons, 'Particulars in Particular Clothing'.

[84] David Lewis has in his *Parts of Classes* advocated the view that mereology provides a general theory of composition. Against this view Simons has pointed out the limitations of mereology in explicating integrated wholes. See Simons, *Parts: A Study in Ontology*, pp. 324ff.

[85] Mereological sums are to be the composites of any disjoint objects whatsoever, though ontically no more than the member objects. Yet the theory prescinds from any account of the unification of the elements, a defect of omission seen even in the etymology of 'compose', i.e., 'to place together', and in this masks what is the ontic necessity of both an 'agent of unification' and a resultant whole distinct from all the elements. In addition, the actual universe is a selectively ordered and in part dynamically structured whole narrowly delimited as such from among all possible structurings by the required mutual compatibility of natures (i.e., the intensions) of the linking instances and their relata. Mereology, by contrast, posits for every fixed universe of disjoint entities whatsoever what would be a complete and static monolithic formal structure consisting of all possible sums and subsequent substructures under the Part-of relation. Modeling in mereology is selecting out from, i.e., 'subjectively creating', one formal substructure from what is in itself the total and undivided universe structured under the Part-of relation. There is nothing in this total mereological universe itself to distinguish one substructure from another. A substructure is an artifact of human practice (a 'fiat object') selected because of its fit with a prior structure of the experienced and theorized real. Mereology is a useful formal fiction but it is not ontology, and it is for this reason and not for what some philosophers (e.g., Armstrong) hold—because they 'supervene' upon their parts—that mereological sums have no ontological standing over and above

The equally important category of events, rather than being misidentified with artificial constructions, e.g., set-theoretical *n*-tuples of *n* subjects, a relation, and a time, are here explicable as relation instances. Processes are then seen as instance complexes having substructures consisting of sequentially related temporal or causal relation instances.[86]

In sum, on the above analysis the ultimate categorical structure of reality is the simple dichotomy of predicable and non-predicable entities.[87] Predicable entities are primary and are relation (including property) instances. Non-predicable entities are secondary and are either complexes of relation instances or intension universals abstracted from these instances. This accounts for the world as a 'totality of facts, not things', and in its observed and theorized form of rich interdependence and layered emergence. A world of hierarchical complexes requires a relational/structural ontology, and it is the correlative combinatorial and individuating aspects of predicates, writ-large in polyadic predicates, that make this ontology possible and intuitive.

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their parts. The same critique applies to set theory, *mutatis mutandis*. See note 70 above and *Moderate Realism*, pp. 51-58.

[86] For a description of the advantages of instance ontology in the analysis of events and the logic of causation see *Moderate Realism*, pp. 78-80.

[87] Characterizing the ontology advocated here as 'two-category' corrects its less accurate characterization in *Moderate Realism* as a 'one-category' ontology. Though the one category of predicable relation instances is primary, derivative entities in the category of non-predicable entities, complexes and universals, are equally real, though sufficiently distinct as to form two sub-categories.

2

An Instance Ontology for Structures: Their Definition, Identity, and Indiscernibility*

1. Introduction

In the modern history of ideas it has been a persistent thesis that *structure*, *complexity* or *system* is ontologically and epistemologically ubiquitous and fundamental. Indeed, our common experience as well as scientific theories are of cognitive and physical domains that are each a plenum of hierarchical structures. These structures can be *static* (e.g., force vectors in equilibrium, shapes of statues) or *dynamic* (i.e., event structures, e.g., the executions of a computer program, a tennis game); *abstract* (e.g., the Real Number System, topological spaces) or *concrete* (e.g., chairs, legal proceedings); *artificial* (e.g., machines, circuit diagrams) or *natural* (e.g., the metabolic cycles of living bodies, quantum phenomena). It is characteristic of structures or complexes that they are wholes which are 'more than the sum of their parts', i.e., they have attributes beyond the collection of those of their constituents taken singly. That is, structures themselves have (are single relata for) further *emergent* properties and relations with definite qualitative contents or intensions that delineate what are the *sui generis* 'natures' of their subject wholes. Moreover, these emergent relations serve to interconnect their relata structures into further subsuming structures of structures, iterated up through entire hierarchies. When these hierarchies are dynamic they exhibit increasingly complex behaviors in proportion to their internal complexity. A living body, for example, is not just a 'heap' of tissue and organs, but a hierarchy of these inter-related spatio-temporally and causal/functionally in various ways, and with emergent properties and relations at each level ('ontological emergence'), e.g., metabolic functions, or consciousness at a certain level of neural complexity.[1] Likewise but in the

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abstract, a proposition is a complex cognitive entity with emergent properties and relations, e.g., the properties of either True or False, or logical relations with other propositions, none of which are properties or relations of sub-propositional constituents. Even simple abstract additive who-les, i.e., sets, mereological sums, or random 'heaps', have emergent formal properties and relations non-existent at least at the level of 'urelements' ('mereological emergence'), e.g., Element-of, Part-of, Subset-of, In-1-to-1-Correspondence-with.

The explanatory power of complexity continues to be the motivation for its systematic study across disciplinary fields under the rubric of General Systems Theory.[2] More narrowly, it is a characteristic of modern science that its explicit methodology is one of generating *similarities of structure*—isomorphisms or homomorphisms—between hypothesized theoretical structures that are heavily formal and systems of observed phenomena as extended by experimental apparatus. Narrower still, there is literature in the philosophy of science arguing that the best ontological account of foundational quantum physics is structural realism, and this to the extent that at some atomic ontic level the objects-inter-related conception of structure is to be replaced by the purely relational conception of only relations-inter-related as the basic elements of physical reality.[3] Here the ultimate physical

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[1] For arguments for ontological emergence, especially from quantum mechanics, and its distinction from epistemological emergence see M. Silberstein and J. McGeever, 'The Search for Ontological Emergence', *The Philosophical Quarterly* 49 (1999): 182-200.

[2] E.g., classically, Ludwig von Bertalanffy, *General Systems Theory* (New York: George Braziller, 1969); Ervin Laszlo, *Introduction to Systems Philosophy* (New York: Gordon & Breach, 1972). There is an extensive literature on systems theory, and an active ongoing interest as evident from an internet search.

[3] See James Ladyman, 'What is Structural Realism?', *Studies in History and Philosophy of Science* 29 (1998): 409-24. Steven French and James Ladyman, 'Remodelling Structural Realism: Quantum Physics and the Metaphysics of Structure', *Synthese* 136 (2003): 31-56. Including an analysis of the historical structuralism of Cassirer and Eddington is Steven French's 'Symmetry, Structure, and the Constitution of Objects', in the *PhilSci Archives* (2001), Center for the Philosophy of Science, University of Pittsburgh at http://philsci-archive.pitt.edu/. For a more critical view of structural realism see in the same *PhilSci Archives* Anjan Chakravartty, 'The Structuralist Conception of Objects'. For a trope analysis of the 'relations-inter-related' conception of foundational physics see Andrew Wayne, 'A Trope Ontology for Classical and Quantum Field Theory', forthcoming in a volume edited by W. Myrvold in the *University of Western Ontario Series in Philosophy of Science* (Springer). Also advocating trope theory in this regard is Peter Simons, 'Particulars in Particular Clothing: Three Trope Theories of Sub-

particulars and fields assay without remainder into properties and relations, an analysis that may be extended to space-time itself. The pivotal problem here, one considered by some insurmountable and thus rendering (ontological) structural realism untenable, is how there can be relational structures without supporting non-structural relata nodes or 'substances' of some sort? How this is possible is a principal contribution of the following. The analysis below responds to the fact that, despite its explanatory potential, it has remained a declared unfulfilled desideratum of General Systems Theory broadly, and of contemporary philosophy of science in particular, that there exist an adequate ontology for structures.[4] Such an ontology would provide a definitional assay of structures that accounts for their natures as 'chains' or 'lattices' of interconnected entities (which may also be structures), each structure being a whole of interconnecting relationships in specific 'mutual arrangements', and where the composing relationships are delimited by their specific contents or intensions. In the following I shall detail with precision how a realist ontology of unrepeatable unit attributes or instances provides such a definition. In contrast to nominalistic trope theory, realist instance ontology recognizes individuated relation (including property) *n*-adic instances, R^n_i, R^n_j, R^n_k, …, together with sharable *n*-adic intensions (universals), R^n, the latter being constituent qualitative aspects numerically the same across their like instances and separable only in abstraction. (The superscripts indicate the number of subject places (of not necessarily distinct subjects) per predicable union, and the subscripts have a naming function that serves to distinguish instance tokens of the same intension type.) Elsewhere I have argued in detail how the combined elements of predicable and as such unrepeatable relation instances and their non-predicable but repeatable intensions make for an ontology and implied logic (analytic/inference engine) superior to standard substance/attribute and trope ontologies and a refinement on stan-

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stance', *Philosophy and Phenomenological Research* LIV (1994): 553-75. Building upon the analysis given herein, my claim is that, in regard to the current debate over 'ontological structuralism' in the philosophy of science, a realist ontology of unit attributes is superior in explanatory power to nominalistic trope theory.

[4] That we have not had an adequate concept of complexity or structure is a complaint of ontologically sensitive system theorists. The reason was identified as far back as by J. H. Marchal in 'On the Concept of a System', *Philosophy of Science* 42 (1975): 448-68, viz., "A general account of when a relation or set of relations *holds among* the members of a set is still needed." Such an account is provided herein, it being only possible within an (realist) instance predicate ontology.

dard predicate logics.[5] Herein I shall extend this display of power by showing precisely how the principles that yield and define instance ontology correct traditional theses concerning plural unity and predication, and provide the otherwise elusive definition of structure or complexity, the latter a serious omission in my previous analyses. The definition and the supporting principles will yield (by then) intuitive and insightful accounts of the identity and indiscernibility of structures or complexes. Indeed, I would propose that what is the standard troublesome notion of indiscernibility is clarified only in the context of complexes as assayed below, and as evidenced by the developed class of counter-examples to the Principle of the Identity of Indiscernibles. These accounts are invisible to standard ontologies where ontic predicates are repeatable universals. In particular, if predicates are universals, i.e., sharable *types*, it would be absurd to theorize at some atomic ontic level a network of only relations-inter-related since there could be but one case for each structural form or type, viz. the type itself, a base too poor upon which to build plural reality which exhibits multiple *tokens* of identically the same types, e.g., multiple methane molecules. These problems are solved with the availability of isomorphic structures composed of corresponding instances of the same type.

In its commonly recognized form, a structure or complex is a network or mesh of variously inter-related entities, and so a definition of complexity must make use of relations understood as constituent linkings or 'mediating combinators', the 'rods', between shared object 'nodes' that together make up an inter-connected whole. Even medieval philosophers whose official doctrine was the reductive elimination of polyadic relations nevertheless recognized that it is of the nature of a relation to be a sort of 'interval' (*intervallum*)[6]—relations bridge ontological space. The assay of relations presupposed here, and, as I shall rehearse, one that implies the individuation of relations into instances, is that each relation, insofar as it obtains among an

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[5] D. W. Mertz, *Moderate Realism and Its Logic* (New Haven: Yale University Press, 1996). The ontology is described succinctly in the more recent 'Combinatorial Predication and the Ontology of Unit Attributes', *The Modern Schoolman* LXXIX (2002) [Essay 1 herein]: 163-97, and 'Individuation and Instance Ontology', *Australasian Journal of Philosophy* 79 (2001): 45-61. The particularized predicate logic (PPL) inherent in the instance ontology and given initially in *Moderate Realism* is perfected in 'The Logic of Instance Ontology', *Journal of Philosophical Logic* 28 (1999): 81-111 [Essay 6 herein].

[6] See Jeffrey Brower, 'Medieval Theories of Relations', *The Stanford Encyclopedia of Philosophy* (Summer 2001 Edition), Edward N. Zalta (ed.), URL = <http://plato.stanford.edu/archives/sum2001 /entries/relations-medieval/>

n-tuple of relata (i.e., is an *ontic ('material') predicate*), is a cause of a unity of itself with and among each of its *n* subjects, and where this unity is conditioned or delimited by a specific *n*-adic content or intension, R^n (e.g., Taller-than, or Prime-Proper-Divisor-of) and its compatibility with the nature of each of the *n* subjects.[7] Exactly similar but distinct instances, R^n_i and R^n_j (e.g., $Circular_1$ and $Circular_2$), are tokens of the same type R^n because intension R^n is numerically the same constituent of each of its tokens, i.e., a shared universal (*unum in multis*), a thesis that follows from the standard arguments for universals which I take to be demonstrative and shall not rehearse.[8] The point here is that relations-as-predicable, i.e., relation instances (including property instances as the limiting monadic case) are *agent ontic unifiers* that form with their relata *individuated states of plural unity*, what are the simplest and basic complexes—resultant individual *facts* or *states of affairs* (e.g., the fact that *a* is taller than *b*, or that 3 is a prime proper divisor of 12). In the following a colon locution will be used to distinguish a fact, i.e., $:R^n_i(a_1,a_2,...,a_n)$, from a corresponding true proposition, i.e., $R^n_i(a_1,a_2,...,a_n)$. The unity in a fact is a *plural* one—the relata are both connected via an instance of R^n and yet by the same agency held in an identity-saving distinctness from each other and the relation. In the paradigm distinct-relata case an *n*-adic relation instance predicable of its *n* relata is, indeed, analogous to a rigid connecting rod holding its subject relata via itself both linked and distinct.[9] Even in limiting cases of facts whose dyadic intension R^2 is reflexive and the subject relata are identical, e.g., the fact $a = a$, there is a plural unity of the relation instance with its distinct subject. Here the rod analogy applies by representing the two 'attaching ends' (the dyadic nature) of a reflexive instance as bent back upon and unifying itself to the same relatum.[10] This insight into the nature of (instances of) *all* relations, as each an 'intensioned linking' or 'intension delimited agent

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[7] See my 'Combinatorial Predication' and 'Individuation and Instance Ontology'.

[8] E.g., as found in D. M. Armstrong, *Nominalism & Realism: Universals & Scientific Realism*, Vol. 1 (Cambridge: Cambridge University Press, 1978), and Reinhardt Grossmann, *The Categorical Structure of the World* (Bloomington: Indiana University Press, 1983).

[9] Even F. H. Bradley, one of the best-known modern antagonists of the reality of relations, noted that it is of the nature of a relation (at least if it is not reflexive) to be both a 'between' and a 'together', by which he meant that a predicable relation has a mediating role of holding its relata both apart and distinct as well as unified or joined, analogous to a rigid connecting rod between thus linked but separate subject nodes. See Bradley's 'Relations' in *Collected Essays*, Vol. 2 (Westport: Greenwood Press, 1970), pp. 634ff.

[10] Diagrams to this effect are found, for example, in Rudolf Carnap's *Introduction to Symbolic Logic* (New York: Dover, 1958), p. 118.

combinator', is the antidote to the sterile and/or misleading errors of founding all unity on either formal or psychological 'intensionless' concatenation (e.g., as with sets and mereo-logical sums), or shared containment in a subject (e.g., the classic theory that attributes 'inhere in' or are 'immanent in' their substance subjects), or an identity-losing mutual suffusion or 'blending' into a resultant homoeomerous One (e.g., Bradley's monism). The analysis also corrects the classic and influential notion of predicable 'forms' as will be outlined below.

The theses whose implications are developed herein are that all plural unity is relational and exists in its atomic form as facts, and that these facts, in turn, along with further relation instances, are the building-blocks of all other hierarchies of structures that go to make up all of reality, concrete and abstract. Closer to contemporary common experience as scientifically tutored, a traditional Aristotelian 'substance' (e.g., a man) or an artifact (e.g., a house) is now assayed as a hierarchy of structures where the 'secondary matter' consists of sub-structures and the unifying and organizing 'form' is actually multiple simultaneous relation instances existing among these structures as wholes, and where the bottom-most level of 'prime matter' is not that of incoherent bare particulars but of intensionally 'clothed' relation instances predicable among themselves. Consistent with Aristotle's conclusion in *Metaphysics* VII and VIII, it is 'forms' characterized as predicable unifers/organizers, what are in fact relation instances, that are most truly 'substance', and where, in keeping with Aristotle's other characterizations of substance, instances are also each a 'this' (particular), compose the ultimate ontic substratum, and at this atomic level are the ultimate subjects of all predication—being relata for each other.

That reality is a 'totality of facts' is a common thesis advanced by a number of philosophers, e.g., famously by Wittgenstein, Russell, and recently by D. M. Armstrong in his *A World of States of Affairs*.[11] Yet, what has

[11] Ludwig Wittgenstein, *Tractatus Logico-Philosophicus* (London: Routledge & Kegan Paul, 1961), p. 7, Prop. 1.1; p. 13, Prop. 2.05. Bertrand Russell's views on facts are distributed throughout his works, but is succinctly put in his summary *My Philosophical Development* (London: Allen & Unwin, 1959), pp. 112-13. D. M. Armstrong, *A World of States of Affairs* (Cambridge: Cambridge University Press, 1997). For an alternate analysis of concrete reality as purely structural, one built up from a single dyadic symmetric relation, see Randall Dipert, 'The Mathematical Structure of the World: The World as Graph', *The Journal of Philosophy* XCIV (1997): 329-58. This sort of reduction of reality to a single 'kind' of relation smacks of an *a priori* formal model-

remained deficient in these theories is not only a proper assay of facts but, following on this, the absence or vacuousness of proffered modes of composition among constituent facts in the formation of more complex structures, e.g., Armstrong's constructing the world from mostly mereo-logical sums of atomic states of affairs. Central below is the demonstration of how the ontology of individuated relation instances provides the means for remediating the latter deficiency. Specifically, the argument is that facts are the simplest complexes, and all other complexes are formed recursively by further instances either sharing relata with constituent instances of given complexes, or by taking given complexes themselves as relata. By this there is an emergent transitivity of connectedness of every instance's relata with the relata of other instances so chained together. The emergent unity belongs to the entire whole but is not, or not an effect of, any single proper constituent of the whole. Here the unity of a single complex is the combined effect of a 'team' of constituent unifiers, not a single shared constituent unifier, e.g., a form.

2. Specious Traditional Theses Regarding Unification

In most of Western philosophy the recognition of the *prima facie* interlinking or 'ontoglial' nature of relations among their own relata, as well as the possibility of an account based upon this of their mutual articulation into networks of complex wholes, were countered by the interplay of three deeply imbedded and pervasive theses. Two of the theses were erroneous in being overly restrictive, one in allowable predicate intensions and the other in the allowable number of constituent unifiers per whole. In regard to the former, the cross-subject combinatorial nature of *n*-adic relations was contradicted by the classic and false **Monadic Intension Thesis, M**:

> (**M**) *All ontic predicates are* ***monadic****, i.e., have intensions or contents that characterize their subject entities singly, e.g., Man, Horse, Circular, White.*

Relatedly and blocking a crucial insight to be exploited herein was the specious **Unity-by-the-Unit Thesis, U**:

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ing, whereas the analysis herein allows reality to be composed of various properties and relations of whatever kinds and adicities it may and apparently does have.

> (**U**) ***All*** *elements making up a plural whole must share a* ***single unifier*** *as the constituent cause of their collective unity and hence of the existence of the resultant whole.*

The logical and ontological link between theses **M** and **U** was the further, and what I shall herein clarify as a potent truth, **Ontic Predication Thesis, O**:

> (**O**) *All plural unifications into wholes (that are more than the sum of their constituents) have as their immediate constituent causes the agency of intensioned combinators—ontic predicates—whereby each unifies itself with other constituents delimited by its intension, and ordered, if any, according to that intension*.

As stated, thesis **O** allows for one or more predicate unifiers per whole, it being thesis **U** of the tradition that limited these to one. That there can be multiple predicable combinators contributing cooperatively to the unity of a single whole is a principal result below. Thesis **O** was observed in Aristotelian and scholastic ontology under the concept of predicable 'forms', the latter serving to effect all wholes of any ontological consequence, i.e., substances (e.g., a human, a horse) and artifacts (e.g., a statue, a house). The only exceptions were the loosest wholes of 'heaps' or 'groups' considered to be simply their constituents without any agent unifier (See Aquinas, *Comm. Meta*., VII, L.17, 1673)[12]. As intimated above and argued elsewhere[13], I propose all plural wholes are 'more than the sum of their parts' in the sense of having emergent properties and relations, and because of this presupposing predicable ontic unifiers. For this reason I shall drop the phrase in parentheses in the successively more precise versions of **O** offered below. I further propose that the failure to recognize that even 'loose wholes'—heaps, sets, mereological sums, etc.—require predicable unifiers is a theoretical artifact of the distortions abetted by theses **M** and **U**.

Thesis **U** is an extensional principle requiring that a cause of unification among multiple entities be one entity connecting itself to each and so linking all and only these entities, analogous in its simplest form to, say, a thread holding together a sequence of beads, or a jar enforcing unity upon

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[12] Thomas Aquinas, *Commentary on the Metaphysics of Aristotle*, 2 Vols., trans. John Rowan (Chicago: Henry Regnery Co., 1961).
[13] *Moderate Realism*, pp. 51-58.

its contents (*Meta*. 1023a11-16)[14]. Paralleling **U**, thesis **M** is an intensional condition on the nexus of predication requiring that an attribute's intension specify—be appropriate for—one subject per predicable union. That is, a monadic intension specifies a predicable nexus with exactly one subject as part of its very meaning, as made explicit in grammatical predicates where the copula is added, for example, 'is a man', 'is a house', 'is circular'. The latter examples are intensions of 'pure' ontic predicates, whereas, e.g., 'is right of *a*' and 'is right of something' are intensions of 'impure' ontic predicates, where 'impure' refers to ontic predicates that are further analyzable into polyadic predicates, i.e., having more than one subject place, e.g., 'is right of'. Impure ontic predicates carry with them one or more subject places 'filled with' specific subjects or that are quantified over. Pure ontic predicates with monadic intensions could exist in a universe with a single subject entity. In contrast, a dyadic intension of a pure ontic predicate specifies a predicable nexus with exactly two subject places, as with Cause-of, Above, Square-Root-of (though for reflexive relations these subject places may have the same entity, e.g., $a = a$). Similarly for triadic intensions, e.g., Between, and, in general, *n*-adic intensions for all *n*. In the following all reference to ontic predicates will be to pure ontic predicates. Theses **M** and **U** were perhaps considered mutually reinforcing on aesthetic grounds of symmetry or equality of proportion—that what is extensionally one is correlative with the intensionally one, and a predicable act that effects a single whole corresponds to a controlling intension specifying a single subject.

Historically, theses **M**, **U**, and **O** where utilized together most explicitly and influentially in the theory of predicable 'forms' of classic Aristotelian/scholastic hylomorphism. First and specifically in regard to **U**, Aristotle on the understanding that unity of wholeness is "in fact a sort of oneness" (*Meta*. 1023b35) further asserts that "Now most things are called one [have a unity] because they either do or have or suffer or are related to something else that is one [has a unity], but the things that are primarily called one [have the most unity] are those whose substance is one."(*Meta*. 1016b6-10; my inserts) And, "All that is is said to 'be' [and so be a unity] in virtue of something single and common"(*Meta*. 1061b13; my insert), it being also an often repeated principle in the tradition that being and unity are convertible (*Meta*. 1003b24-35, 1054a14, 1061a15). The view was reiterated later by Aquinas: "Things that are diverse do not come together in

[14] Aristotle's *Metaphysics*, translated by W. D. Ross and found in Richard McKeon, *The Basic Works of Aristotle* (New York: Random House, 1941).

the same order [i.e., in a structured whole] unless they are ordered thereto by some one being. For many are reduced to one order by one better than many: because one is the *per se* cause of one and many are only the accidental cause of one, inasmuch as they are in some way one."(*Sum. Theo*., I, q.11, a.3; See q.103, a.3; my insert)[15] Elsewhere and assuming **U** Aquinas asserts explicitly the denial of a fundamental thesis argued herein, saying "Nor can this unity [a unity among multiple things] come from diverse ordering causes [i.e., be the collective result of multiple unifiers], because they could not possibly intend one order in so far as among themselves they are diverse."(*Sum. Contra*., I, ch.42, par.7; my inserts)[16] Later in Scotus one finds thesis **U** in the form: "Just as unity in common follows *per se* on entity in common, so too does any unity follow *per se* on some entity or other."[17] What may have motivated, or at least reinforced **U** was the causal principle that: There can be nothing in an effect that is not in the cause(s) (See Aristotle, *Meta*., 993b22-25). So, a single unification into a whole among multiple elements cannot exist as an effect of multiple causes unless there is a single unification into a whole among these causes. But, then, the latter is just one (albeit complex) cause as is the effect. Hence, all unity among the diverse is by a single unifier. *Contra* the causal principle, the argument herein is that a complex can have emergent unity not caused by any single constituent unifier, but rather be the resultant of several.

In classic hylomorphic ontology the primary mode of unification of any ontically significant plural whole was by a *form*, substantial or accidental, ontically predicated of—being in 'act' as a unifier applied to—a subject or subjects, whether prime matter or existing substances. In *Metaphysics* VII Aristotle asserts that a predicable form answers the question "why one thing attaches to another", and it does so in the manner of a 'cause' and a 'principle', i.e., as an agent and a source from which the unity obtains, and not just as another element to be unified (*Meta*. 1041a6-41b30). Later, Aquinas is more explicit: "Each individual thing is actually a being through a form, whether in the case of actual substantial being or in the case of actual accidental being. And hence every form is an act, and as a consequence it is the

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[15] Thomas Aquinas, *Summa Theologica*, 3 Vols., trans. Fathers of the English Dominican Province (New York: Benziger Brothers, Inc., 1947).
[16] Thomas Aquinas, *On the Truth of the Catholic Faith (Summa Contra Gentiles), Book One: God*, trans. by Anton Pegis (Garden City: Doubleday & Co., Inc., 1955).
[17] John Duns Scotus, *Ordinatio*, II. d.3, part 1, qq. 1-6, in Paul Spade, *Five Texts on the Mediaeval Problem of Universals* (Indianapolis: Hackett Publishing Co., 1994), pp.101.

reason for the unity whereby a given thing is one."(*De Spirit. Creat.*, Art. 3)[18] Here thesis **O** is explicit and thesis **U** is understood. Perhaps most explicit in assaying subject/form-predicate unification—thesis **O**—was Francisco Suarez, considered by some to be the last great scholastic philosopher and synthesizer. Suarez distinguished between a form and its union of inherence in a subject, the latter being a 'mode' of the form. The union as mode was particular and unrepeatable, yet itself neither a substance nor a quality or form of something, but rather a modification of the predicated intension. The distinction between an intension and its union of inherence was held to be post-abstraction, though with a real basis within what is an internally simple predicate (what was classified as a *distinctio rationis ratiocinatae*).[19] (Suarez's work anticipates the assay of relation instances I am proposing.) In sum then, predicable forms were considered the immediate cause of the organizing unity of themselves with their subject or subjects. For example, in a substance such as a human, the substantial form, i.e., the soul (intensionally: Humanity), was held to be, in conformity with **U** and **O**, the one and immediate *agent organizer* into a structured whole of the underlying matter. Similarly for accidental wholes such as a house or statue. Yet, in conformity with **M** and **O**, predicable forms among plural matter and their causal role in its 'orderings' *were in every case limited to those with monadic intensions or contents*, each a species-intension causing their resulting substances to be one of that *kind*. The result was a 'split personality' for forms of composite wholes: specifically, the incoherence of ontic predicates each providing a structured unity among two or more subjects ('secondary matter'), yet also having an identifying intension or 'meaning' that is monadic and so specifying exactly one entity as the object of its agency. The problem is amplified when substantial form is taken to be predicated directly of prime matter (as soul was for Aquinas and Suarez). If prime matter is construed as such as an undifferentiated and amorphous single simple stratum of pure potentiality (yet as an individuator somehow numerically distinct for each distinct substance—what would have to be in the last analysis bogus bare particulars[20]), then a substantial form predicable

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[18] Thomas Aquinas, *On Spiritual Creatures (De Spiritualibus Creaturis)*, trans. M. Fitzpatrick and J. Wellmuth (Milwaukee: Marquette University Press, 1949).

[19] Francisco Suarez, *On the Various Kinds of Distinctions (Disputationes Metaphysicae, Disputation VII, de variis distinctionum generibus)*, trans. Cyril Vollert, S.J. (Milwaukee: Marquette University Press, 1947).

[20] For arguments against bare particulars see my 'Individuation and Instance Ontology'. For an argument that even Aristotle, from whom the tradition of individuating prime matter originated, would have rejected the notion of an absolutely qualityless

of it must be the proximate internal cause of *both* the differentiation of multiple parts *out of* it (e.g., Socrates' organs, tissues, bones, etc.), as well as the cause of the 'ordering' of the latter parts into the structured whole (e.g., into living Socrates as a complex of functioning systems). The substantial form, which is an intension or has a unique intension as a controlling aspect, is then required to be both monadic (as predicable of the bottom prime matter) and polyadic (as predicable organizer of the intermediate parts). Further and also inconsistent, the predicable act of the form as polyadic organizer here presupposes itself as *numerically the same* but prior differentiating monadic act on the internally simple prime matter. I note in this context once again the error to be corrected below that, in addition to a structuring function among differentiated parts (a role contrary to thesis **M**), a creative and nature-bestowing function effecting these parts (which is consonant with thesis **M**) is necessitated of substantial forms because they are required to be, at some foundational level at least, predicable of absolutely formless/qualityless prime matter. And, this is so because of the fallacious reasoning that in order to avoid an infinite vicious regress of analysis, what is predicable, i.e., form, requires at some base level something non-predicable, and hence formless, to be predicable of. The related and contemporary version of this fallacy is that relations (including properties) at some level require non-relational and non-predicable relata. To the contrary and answering both, we shall see that a base level of relation instances can among themselves provide both predicable unifiers and intensioned subjects.

In sum, the source of the above monadic/polyadic incoherence is the joint enforcement of two errors: the error under thesis **U** that a single complex whole of variously inter-related parts (i.e., having constituent relations of differing intensions, e.g., a human, a machine, a compound proposition) have one extensive unifier which by **O** is an ontic predicate, along with the error under thesis **M** that every agent unifier, i.e., an ontic predicate by **O**, have a monadic content that specifies one subject per emergent fact. The means for correcting **U** will be our major effort below. Here I will indicate briefly what have been historically the insidious consequences of **M**, and what was its definitive correction by Bertrand Russell in the last century.

A principal implication of thesis **M** is the doctrine of the monadic reduction of relations, which via a number of 'emendations' has had and con-

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substrate see Theodore Scaltsas, *Substances and Universals in Aristotle's Metaphysics* (Ithaca: Cornell University Press, 1994), pp. 222-28.

tinues to have distorting effects, e.g., the relegation of relations to the 'supervenient' (on property reducta),[21] or the reduction of relations to associations formed by the mind (*entia rationis*) as in the Humean non-nomic analysis of causality. Reinforced is Aristotle's assessment that relations are "least of all things a kind or entity" (Aristotle, *Meta*. 1088a23). The property-reduction of relations, traceable back to Plato and Aristotle and customized variously by medieval philosophers, eliminates polyadic (historically restricted to dyadic) relations in favor of monadic properties of one or more of their relata (an *esse in* aspect) but with each of the latter having a characteristic 'being-toward' the other relata (an *esse ad* aspect).[22] Of course, to be a successful elimination of the polyadic the being-toward aspect cannot be a further albeit more subtle relation, but rather must be a kind of intensionless 'pointing'. It is but a short step to making the toward-aspect a blank association independent of the natures of the relata and freely created by the mind, a position found in the subsequent 'modern' philosophies of Spinoza, Leibniz, Hobbes, Locke, and Hume.

Abetted by these distortions, the linking 'predicable' nature of relations disappears completely in the more recent nominalistic and formal Wiener-Kuratowski strategy for the reduction of relations to certain sets of sets, where the further assumption is that sets (and similarly for mereological sums) are wholes that do not need constituent unifiers among the elements. Here relations as intension universals are given an alleged extensional reduction in terms of their relata, and the unification of elements into a set is ignored as a non-problem. The latter is related to other instances of declared non-problems found in nominalism, e.g., Ockham's assertion that "One does not have to look for a cause of individuation…. Rather one has to look for the cause why it is possible for something to be common and universal."[23] In response to the Wiener-Kuratowski reduction strategy, Herbert Hochberg has shown that it is in fact unsuccessful in that it must surreptitiously appeal to ordering relations that are intensional and un-

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[21] This was the view of Keith Campbell in *Abstract Particulars* (Oxford: Basil Blackwell, 1990). Campbell has subsequently modified his views to allow for certain relations that resist foundational reduction. See his 'Unit Properties, Relations, and Spatio-Temporal Naturalism', *The Modern Schoolman* LXXIX (2002): 151-62.

[22] See Mark Henninger, *Relations: Medieval Theories, 1250-1325* (Oxford: Clarendon Press, 1989), and Jeffrey Brower, 'Medieval Theories of Relations', *Stanford Encyclopedia of Philosophy*.

[23] William of Ockham, *Ordinatio*, d.2, qq. 4-8, in Paul Spade, *Five Texts on the Mediaeval Problem of Universals*, p. 172.

reduced.[24] And, I have argued elsewhere that ignoring the necessity of internal unifiers is 'ostrich ontology' where set theory, which is a tool for formal modeling, is mis-identified with the reality modeled, and consequently, as with the Cheshire Cat in *Alice in Wonderland*, becomes analogous to a theory of grins with the ontically supporting cats (the constituent relations) abstracted away and ignored, though necessary and presupposed.[25] Plural wholes require internal causes of unity among their constituents, explicit or not.

The degeneration from intension-controlled unifiers down to blank associations or contrived formal models began with the erroneous restriction, under **M**, of the former to those with monadic intensions—the forms of classic hylomorphism. Starting only in the twentieth century has thesis **M** been widely recognized as false, and even then the ontological implications of alternative polyadic predication has received little attention. The latter accounts for the absence of overt rejections of thesis **U**. The *locus classicus* for demonstrating the error of the monadic reduction of relations, and hence of **M** that implies it, is Russell's analysis in *The Philosophy of Mathematics*.[26] The arguments turn on the non-eliminability of the ordering among relata by asymmetric and non-symmetric relations, a unique characteristic of polyadic relations, one not reducible to monadic properties singly or jointly. Elsewhere I have sought to reinforce Russell's arguments against contemporary defenders of the reductionist strategy (e.g., Keith Campbell).[27] I refer the reader there. Importantly, thesis **M** is rather to be replaced by the generalized **N-adic Intension Thesis**:

> (**N**) *An ontic predicate has an intension that specifies n subjects for a fixed n≥ 1.*

Thesis **N** now makes it possible to state more precisely the Ontic Predication Thesis, **O**, utilizing *n*-adic intensions and corresponding facts, viz.,

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[24] Herbert Hochberg, 'The Wiener-Kuratowski Procedure and the Analysis of Order', *Analysis* 41 (1981): 161-63.
[25] *Moderate Realism*, pp. 51-58.
[26] Bertrand Russell, *The Principles of Mathematics*, 2d. ed., (1903: reprt. ed., New York: Norton, 1938), pp. 221ff.
[27] *Moderate Realism*, pp. 163-73.

> (**O′**) *All plural unifications into wholes have as their immediate constituent causes the agency of intensioned combinators—ontic predicates—each having an n-adic intension R^n that delimits and orders (if any) other constituents into subject n-tuples, $<a_1,a_2,\ldots,a_n>$, the combinator unifying these subjects into single facts, $:R^n(a_1,a_2,\ldots,a_n)$, that are, or are unifying parts of, its resultant whole.*

Like **O**, left open with **O′** is the possibility of a single whole having multiple combinators and so multiple composing facts. In the next section I shall increase the warrant for **O′** and argue for a further refinement, viz., the individuation of ontic predicates. The latter will be prerequisite to correcting thesis **U**, i.e., to showing that a whole can have multiple partial unifiers whose effects 'add up' to the unity of the whole.

3. Bradley's Regress and Principles of Individuated Relations

Aristotle's argument for the unifying nature of forms (*Meta*. 1041b11-30) contains a condensed version of a historically reoccurring argument sometimes interpreted as showing the unreality of polyadic relations and now known as Bradley's Regress. Bradley himself intended correctly that the argument, if sound as he interpreted it, proves the absurd and illusory nature of all ontic predication whatsoever, monadic or polyadic.[28] The argument proceeds by observing that in the fact corresponding to the true proposition P(*a*), i.e., :P(*a*), if the ontic predicate is the intension *universal* P, e.g., if the intension Red is the ontic predicate in the fact :Red(*a*), then P (e.g., Red) and *a* are just two separate non-predicable subjects, each and in themselves making no reference to some other specific entity (neither having an 'esse ad' aspect indicative of something predicable). This is clear when fact :P(*a*) is contingent and so where P and *a* are identically the same P and *a*, respectively, that can exist when the predicable unity among them, and hence resultant fact :P(*a*), no longer exists. The unification prerequisite to contingent fact :P(*a*) requires something more than just, in themselves, intension/universal P and subject *a*. That is, the intension P as much as particular *a* are causally inert as themselves non-unifiers and, hence, there is required some further unifier to account for the unity of the original fact :P(*a*). The non-predicable nature of intension universals, e.g., Red, Triangle, Tall, Love, is the veridical base from which Plato could launch as plausible his further and false theory of separated Forms. An intension is

[28] Bradley, 'Relations', cited in note 9.

the same in its total being—what makes it to be what it is—in worlds with or without entities that exemplify it, and hence any principle of unification (e.g., 'participation') joining an intension and a subject entity is distinct from the intension. Now, the original fact :P(*a*) requires a unifier which, according to the regress, must now be a dyadic predicate, say the relation of Exemplification, E. The original fact then becomes the fact :E(P,*a*). But now, if it is Exemplification as an intension universal that is an element of the fact, then for the same reasons as with P, the three entities E, P, and *a* are distinct subjects none of which is connected to the other two inherently or by any nature of its own, and therefore the three are in need of a further unifier to account for the unity of the original fact, say, Exemplification′, E′. The original fact then becomes :E′(E,P,*a*). Clearly this is the beginning of a vicious infinite regress, where the predicate posited in the *n*-th step to account for the requisite unity is seen in the *n*+1-th step not to be capable of this role, calling for the posit of a further unifying predicate in the *n*+2-th step, and so on. Of course, the same regress results when the original fact has a polyadic predicate with *n*-adic intension R^n. Bradley concluded that "All predication, no matter what, is in the end untrue and in the end unreal…"[29] According to Bradley unity is not from ontic predication but rather from the all-encompassing One, the internally undifferentiated Absolute.

Pluralist philosophers who accept the validity of the regress argument (e.g., Ockham)[30] have no choice but to eliminate the unbridgeable (due to the regress) 'ontic distance' between each of diverse subjects and their attributes by placing the latter 'in' their subjects as container-unifiers—the classic inherence model of predication (*praedicatum inest subjecto*). The result is the forced adoption of thesis **M** and the property-reduction of relations—what is a *reduction* of the current assumptions. Pluralist philosophers who reject the regress argument do so by calling into question one of its premises. These underlying assumptions are, I propose, the following three. 1) In a relational fact :$R^n(a_1,a_2,\ldots,a_n)$ it is the relation-R^n-as-ontically-predicable-of-its-relata (what Russell termed the 'actually relating relation')[31] that is the cause of the unity of itself with its relata and hence of the existence of the emergent fact. 2) In a relational fact

[29] Ibid., p. 672.
[30] William of Ockham, *Ockham's Theory of Terms: Part I of the Summa Logicae*, trans. by Michael Loux (Notre Dame: University of Notre Dame Press, 1974), p. 170.
[31] Bertrand Russell, 'Some Explanations in Reply to Mr. Bradley', *Mind* 19 (1908): 373-8. Also see my 'Individuation and Instance Ontology'.

$:R^n(a_1,a_2,...,a_n)$ the relation-R^n-as-ontically-predicable-of-its-relata is identical to the intension universal R^n. And 3), No intension universal is in itself ontically predicable of any subject(s). Included here are monadic properties which are the limiting case of polyadic relations. The iterated appeal to these assumptions yields Bradley's Regress. Now, historically there have been two standard responses to the regress. One is to accept propositions 2) and 3) but reject 1), holding that the cause of the unity of a relational fact is a posited implicit 'non-relational tie' or 'nexus'.[32] The trouble with this maneuver is that if the tie has a specific content or intension then it is but a further relation with the effect that the regress is only put back one step, and if, alternately, the tie has no content or intension then it becomes a 'bare linking' analogous to the specious notion of a 'bare particular' and is open to equally serious challenges (e.g., the inability to account for ordering by and direction of an *n*-adic predicate among a relata *n*-tuple).[33] The second standard response to the regress has been to retain propositions 1) and 2) but reject 3), this thought by some to be in keeping with the dominant doctrine going back to Aristotle that universals are predicable entities (*Meta*. 999b35; 1038b15). There is, however, an argument[34] that I propose is demonstrative in showing that proposition 3) is true and that it is 2) that must be rejected. In particular, the argument establishes that in a relational fact $:R^n(a_1,a_2,...,a_n)$, where with 1) *relation-R^n-as-ontically-predicable-of-its-relata* is the cause of its unity, it is the case that, *contra* 2) but implying 3), *relation-R^n-as-ontically-predicable-of-its-relata* is an unrepeatable individual and hence is not identical to the repeatable intension universal R^n. Presupposed is the non-eliminability of polyadic relations (thesis **N**), in particular contingent non-symmetric relations, and this accounts for the argument's near-invisibility to a tradition focused on monadic properties. The argument can be put succinctly as follows: Let R^2 be a contingent non-symmetric relation, e.g., Irritates, Alters, Emits-Radiation-to, Moves-Toward, such that both facts $:R^2(a,b)$ and $:R^2(b,a)$ obtain, and $a \neq b$. The cause of the unification of fact $:R^2(a,b)$, i.e., the combinatorial act sustaining its existence as a complex whole, cannot be numerically identical to the cause in the same sense of the unity and hence existence of fact $:R^2(b,a)$. This is evident in that either fact can cease to exist while the other persists, and if it

[32] Gustav Bergmann, *Realism* (Madison: University of Wisconsin Press, 1967), pp. 9, 42ff; Herbert Hochberg, 'A Refutation of Moderate Nominalism', *Australasian Journal of Philosophy* 66 (1988): 188-207; P. F. Strawson, *Individuals* (London: Methuen, 1971), pp. 168ff.

[33] See my 'Combinatorial Predication' and 'Individuation and Instance Ontology'.

[34] Ibid.

were *one and numerically the same* cause—combinatorial act—sustaining the existence of both facts, then they would have to come into and go out of existence simultaneously, which is counterfactual. Hence, the combinatorial act sustaining fact :$R^2(a,b)$ must be unique to it, i.e., must be unrepeatable, and so particular and individual. Further and importantly, the agent cause of the unification in fact :$R^2(a,b)$ cannot be, prior to abstraction, distinct from the controlling intension R^2 in the sense that distinct implies a further implicit constituent relation between R^2 and what would be an incoherent 'bare linking'. I refer the reader elsewhere for the expanded argument.[35] This being the case, the cause of the unity of fact :$R^2(a,b)$ must be both unique to it and an internally simple combinator-under-an-intension, i.e., a relation instance R^2_i.

Generalizing then, the important ontological implication of the combinatorial nature of relations is that a *relation-R^n-as-ontically-predicable-of-its-relata* in a fact :$R^n_i(a_1,a_2,...,a_n)$ is an unrepeatable relation instance R^n_i which is a simple entity with the two abstractable aspects of repeatable intension R^n and a particularized unifying agency unique to subject *n*-tuple $<a_1,a_2,...,a_n>$. The unrepeatable predicable aspect of an instance is for ontology a cogent *principium individuationis*, and cuts through the obscurities and problems associated with the alternatives of posited *haecceitas* (Scotus)[36] or bare particulars (e.g., Armstrong, Moreland)[37], the instantiation of specially endowed substance universals (e.g., Loux, Lowe)[38], or simply declaring individuation an unexplainable primitive (e.g., Ockham, Campbell)[39]. The distinction between these two real aspects of individuating combinator and its controlling intension of a nevertheless *non-complex* instance is the scholastics' *distinctio rationis ratiocinatae* (or what Scotus termed the *distinctio formalis a parte rei*).[40] The distinction applied to relation instances refines

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[35] Ibid.

[36] E.g., Scotus, *Ordinatio* II. d.3, part 1, qq. 1-6, in Spade, *Five Texts on the Mediaeval Problem of Universals*, pp.101-02.

[37] Armstrong, *A World of States of Affairs*, pp. 68, 109; James P. Moreland, 'Theories of Individuation: A Reconsideration of Bare Particulars', *Pacific Philosophical Quarterly* 79 (1998): 251-63.

[38] Michael Loux, *Metaphysics: A Contemporary Introduction* (New York: Routledge, 1998), pp. 117ff.; E. J. Lowe, *The Possibility of Metaphysics* (Oxford: Clarendon Press, 1998), pp. 180-83, 197.

[39] 39. For Ockham reference, see note 23; Campbell, *Abstract Particulars*, p.69.

[40] See note 19. For an analogical explanation of the formal distinction see 'Individuation and Instance Ontology'.

and corrects the scholastics' attribution of it to the mutual existence of form (intension + combinator) and matter (individuating subjects) in a substance, or more recently and more accurately Campbell's attribution of the distinction to the individuating and intension aspects of a (combinatorialless) trope[41], or, closer to the scholastics' use, Armstrong's use of it to characterize 'instantiation' as the "distinction without a relation" between a subject particular and its qualifying ontic predicates—the unity of a state of affairs[42]. In sum and importantly, a relation instance is as a single simple entity a 'this-such'—a 'this' because of its unrepeatable unifying agency among a specific set of relata, and a 'such' because of its repeatable intension. Alternately, an instance is both a particular and an ontic predicate. It is these facts that make possible an ontology of particulars that can be both subjects of predication and the predicates themselves, and is the basis for how there can be structures without non-structural object nodes. This will be made clear below.

The results of the above analysis can be summarized into three of four principles that I had previously proposed as complete in properly characterizing an ontology of combinatorial predication—the realist ontology of relation instances. The first principle is the final version of the Ontic Predication Thesis[43]:

> (**O**″) *All plural unifications have as their immediate constituent causes the agency of intensioned combinators—ontic predicates—each a simple unrepeatable instance* R^n_i *with the two aspects distinguishable only in abstraction of a repeatable n-adic intension* R^n *that delimits and orders (if any) other constituents into an extension of subject n-tuples,* $\langle a_1,a_2,\ldots,a_n\rangle$*, and an unrepeatable unification on exactly one of these n-tuples effecting a single fact* $:R^n_i(a_1,a_2,\ldots,a_n)$ *that is, or is a unifying part of, the resultant whole.*

Thesis **N** is assumed in the statement of **O**″, and, as before with versions of **O** and **O**′, **O**″ leaves open the possibility of multiple partial combinators (of various intensions) for a single whole, something now theoretically possible using relation instances. The second principle formalizes the un-

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[41] Campbell, *Abstract Particulars*, p. 56.
[42] Armstrong, *Nominalism & Realism*, pp. 109, 111; and *States of Affairs*, pp. 114-19.
[43] The Ontic Predication Thesis in the form of **O**″ incorporates two theses given separately in my *Moderate Realism*: the Principles of Immanent Instance Realism (**IR**), p. 11, and Instance Predicates (**IP**), p. 26.

repeatability character of ontic predicates as instances—the **Principle of Subject Uniqueness**:

> (**SU**) If $R^n_i(a_1,a_2,...,a_n)$ and $R^n_i(b_1,b_2,...,b_n)$, then $a_1 = b_1$, $a_2 = b_2$, …, $a_n = b_n$.

This asserts that any predicate instance, R^n_i, has only one relata *n*-tuple, i.e., is not repeatable as a universal over multiple sets of subjects. A third principle is what I have called the **Principle of Relata-Linking**:

> (**RL**) No *n*-adic relation instance R^n_i exists except as ontically predicative among, and hence necessarily presupposing, some *n*-tuple of entities which as such it relates.

On the above assay it is intrinsic to the nature of relation instances that they be combinatorial among a set of relata, and hence they cannot exist separated from some such set. A fourth principle not considered above but asserting the non-redundancy of ontic predicates is what I have called the **Principle of Instance Uniqueness**:

> (**IU**) If $R^n_i(a_1,a_2,...,a_n)$ and $R^n_j(a_1,a_2,...,a_n)$, then $R^n_i = R^n_j$.

The assertion under **IU** is that there cannot be two distinct instances of the same intension, R^n, predicable of the same *n*-tuple of subjects, e.g., the ordered pair <3,6> will not have two instances of the relation Prime-Proper-Divisor-of. The argument for **IU** is from ontic economy ('Ockham's Razor') and the fact that there is nothing to differentiate R^n_i from R^n_j here except distinct acts of predicable union, and two such unions per intension/*n*-tuple pair are redundant.

Principles **SU** and **IU** both utilize the global identity relation, =, which is easily definable in a refined predicate logic inherent in the realist instance predication specified in **O″**. Namely,

> (**Id**) Entities *a* and *b* are identical, $a = b$, if and only if, for every monadic property P^1 and every instance P^1_i of P^1, $P^1_i(a)$ if and only if $P^1_i(b)$.[44]

[44] **Id** is given in its PPL formalization (utilizing the device of 'extended binding' by intension quantifiers) in 'The Logic of Instance Ontology' and *Moderate Realism*, p. 213.

Definition **Id** asserts that entities are numerically the same if and only if they have as characterizing properties *numerically the same instances of* numerically the same intension universals. [Note: I argue for a more refined definition of identity in Essay 7 of this volume.] In the tradition the definition of identity without the benefit of the instance refinement (in italics) has been controversial to the extent that it was thought, rightly, not to sufficiently distinguish identity from what, given the available analytic tools, was necessarily the vague notion of 'indiscernibility'. This situation is remedied by instance ontology below.

As noted, at one time I had thought principles **O″**, **SU**, **RL**, and **IU** (or their equivalents) were sufficient to capture what is essential and potent about combinatorial predication and the resulting unit attribute ontology. I have come now to realize that omitted therein was an important principle concerning emergent unity via the proper articulation of multiple constituent instances, what is the correction of thesis **U**, i.e., the correction of the specious thesis that all unity is from a single unifier. Indeed, individual facts $:R^n_i(a_1,a_2,...,a_n)$, each with their constituent trans-relata unifiers, R^n_i, do conform to thesis **U**, yet compound complexes or structures do not. The traditional error has been the false extrapolation of **U** applied to atomic complexes, i.e., facts, to its characterizing compound complexes as well. On the following, it is the ontological refinement of particularized relation instances and the possibilities for their sharing relata and having entire complexes as relata that provides an account of the emergent unity characterizing compound structures.

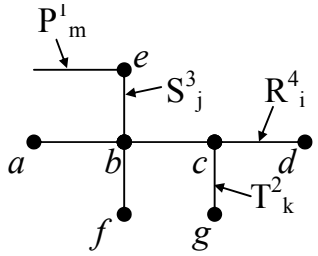
4. Facts and Their Compounds

It is perspicuous on the above assay of facts that any two facts whose relation instances share one or more relata form a compound structure (though not a compound fact). The further insight to be gained is, loosely stated, that if two such pairs share a common fact, then there is a 'transitivity of unification' across all three facts forming a single more complex structure. More specifically, a trans-factual unity, what Peter Simons characterizes in topological terms as 'path connectedness'[45], emerges when pairs of complexes share relata (not necessarily the same) with mediating third complexes, analogous to the connectedness from the first to the last link in a

[45] Simons, *Parts: A Study in Ontology* (Oxford: Clarendon Press, 1987), p. 327.

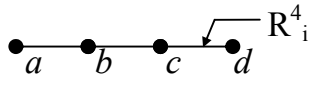
chain without the need, to carry on the analogy, of some additional and single cable running through all the links and joining them. These claims, along with the appropriateness of the rod/node, chain, and lattice analogies used above, are made intuitive by means of spatial diagrams. These diagrams are themselves a subclass of structures whose constituent spatial relation instances are immediately observable. Consider, for example, the diagram:

Compound Complex A:

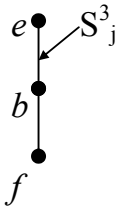


Line segments contained in the same line are to represent the single relation instance named by the terms via the arrows.[46] As a spatial complex, Complex A displays explicitly the intra- and inter-connections among relata established via the composing facts—atomic complexes—that in less perspicuous prefix notation would be given as the conjunction of $:R^4_i(a, b,c,d)$, $:S^3_j(e,b,f)$, $:T^2_k(c,g)$, $:P^1_m(e)$. Graphically, compound Complex A decomposes into the following constituent atomic complexes (facts).

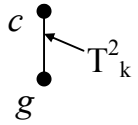
Atomic Complex B:



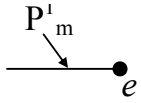
Atomic Complex C:



Atomic Complex D:



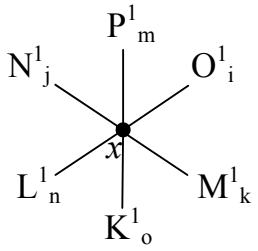
Atomic Complex E:



[46] One disanalogy of spatial diagrams for depicting *n*-adic relations for $n > 2$ is that it gives the impression that all such relations are reducible to conjunctions of dyadic relations. That this is not possible see my 'Peirce: Logic, Categories, and Triads', *Transactions of the Charles S. Peirce Society* XV (1979): 158-75.

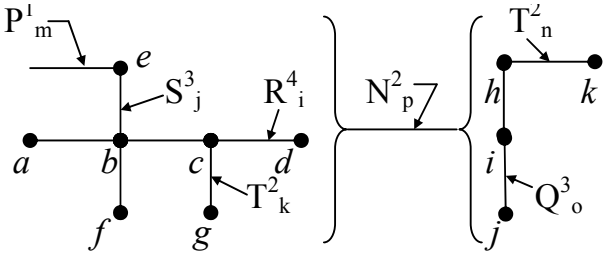
Note that in the constituent Complex E the *monadic* predicate instance $P^1{}_m$ is represented by a line segment appropriately attach at one end to its single subject, *e*. Graphically then, thesis **O″** if it were restricted by the Monadic Intension Thesis, **M**, would yield a Leibnizian universe of individuals—monads—each with its halo of monadic properties, e.g., Complex F, but otherwise absolutely isolated one from the other.

Compound Complex F:



In contrast, this is not the case for the relata making up Complex A. In A, by a transitivity of connectedness via 'road and node', subject *e*, for example, is linked/unified to subject *g* by the segmented route of being a relatum for instance $S^3{}_j$ that also shares a different relatum, *b*, with instance $R^4{}_i$, the latter in turn sharing a relatum *c* with instance $T^2{}_k$ that has as its other relatum *g*. Here we see intuitively the 'path-connectedness' that would characterize any two constituents of a complex that is 'horizontally' composed of its relation instances, i.e., continual connectedness across sequences of facts exclusively by means of shared relata. The emergent transitivity of connectedness here is from the constituents of one fact to those of another via the facts sharing one or more relata, or from the constituents of one fact to those of another via an intermediating fact with which the two facts share one or more relata (not necessarily the same), together with the transitive nature of this connectedness relation. There is, however, in addition to horizontal composition, the important and mostly ignored 'vertical' type of composition involving relation instances having one or more relata that are themselves complexes. Consider Complex G.

Compound Complex G:



Complex G consists of an instance of the dyadic relation N^2 linking Complex A in its entirety and as a single entity as a left relatum with a different complex, say H, taken in its entirety and as a single entity as a right relatum. Analogically, Complex A could represent the structure of a tea cup, H the structure of a saucer, and relation N^2 the Sits-Upon relation, or, more directly, A and H could represent molecular structures and N^2_p an instance of the Has-as-a-Catalyst relation. Importantly, even though between the constituents of A and H there is clearly no 'path-connectedness' by any continual chain of relatum-sharing 'path segments' (that are all constituents of G), nevertheless it is intuitive that constituent *e*, say, is linked to constituent *j* via a 'once-removed' next-level relation instance N^2_p. Instead of being path-connected, we might say that *e* and *j* are 'cross-level-connected'. This is so by what I shall call (despite the particular non-constituent/non-relevant spatial relations of diagram G) the 'vertical connectedness' *e* and *j* have to each other by means of N^2_p, viz., *e*, say, is not a relatum for N^2_p but is a constituent of an encompassing Complex A that is a relatum for N^2_p , *e* being presupposed by but 'once-removed' from the combinatorial agency of N^2_p. So *e*'s connectedness to N^2_p (and to any entity N^2_p is connected to) is inherited via the mediating Complex A. Similarly for *j*. Now, it is easy to imagine this type of vertical combination repeated on Complex G itself—G being a single relatum for other *n*-adic relations, some emergent at this level, and where this vertical structuring can be iterated into hierarchies of increasingly compound complexes. In this hierarchy constituents of the lowest level complexes would be linked or connected to any constituents of complexes at any higher level via a transitivity of unity across chains of vertically and/or horizontally composing relation instances.

Complex G exhibits what are the three and only three types of plural unification, all via relation instances: unity among relata, among relation instances, and among complexes. We can now generalize from Complex G to a full definition of Complexity (or Structure) given recursively in the following axiom for all plural unifications. The axiom is the awaited correction of the Unity-by-the-Unit Thesis, **U**, i.e., corrects the thesis that every plural unity requires a single constituent unifier among all other constituents. We have the **Unity-by-Instances Thesis, I** :

(**I**) All plural unity—complexity or structure—is by the following:

a) A relation instance R^n_i predicable of an *n*-tuple of relata, $<a_1,a_2, ...,a_n>$, is the cause of an individual plural whole, i.e., a fact $:R^n_i(a_1,a_2,...,a_n)$, having R^n_i, a_1, a_2, ..., a_n, as its only constituents.
b) If R^n_i is a constituent of a plural whole *x* and S^n_j is a constituent of a plural whole *y*, and R^n_i and S^n_j, share one or more relata, then there is an individual plural whole *z* that has as constituents all and only the combined constituents of *x* and *y* (horizontal composition).
c) For any fact $:R^n_i(a_1,a_2,...,a_n)$, if for $1 \leq j \leq n$, a_j is a plural whole, then there exists an individual plural whole whose constituents are all and only the constituents of the fact and constituents of a_j (vertical composition)

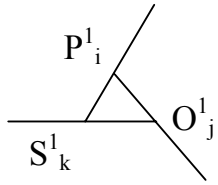
It is now a simple matter to give an identity criterion specific to complexes (utilizing the global identity relation defined in **Id**), a criterion that, importantly, involves only the *internal* and so relevant nature-bestowing components of a complex *qua* complex. It is intuitive that complexes with numerically the same relations (instances) each having corresponding relata that are numerically the same are themselves numerically the same—identical. Under standard ontology where predicates are treated as universals the latter condition would not be guaranteed. However, under the Principle of Subject Uniqueness, **SU**, predicates are particularized to specific relata *n*-tuples, and so instances that are numerically the same have corresponding relata that are numerically the same. Hence, the **Identity Criterion for Complexes**, **ID**, is simply:

(**ID**) For complexes *x* and *y*, $x = y$ if and only if, for every intension R^n and every instance R^n_i of R^n, R^n_i is a constituent of *x* if and only if R^n_i is a constituent of *y*.

Shortly and as promised we shall consider how the ontology of relation instances clarifies the concept of indiscernibility. As a preliminary I shall make good on another important claim repeated above, viz., that at some lowest ontic level it is possible to have only predicable entities (relation instances), i.e., predicates that have as their subjects only further predicable entities. At this base level there are no non-predicable 'substances', but only individual complexes exclusively composed of instance predicates. Consider as a perspicuous example of such a lowest level whole Complex I

which is composed of a chain of monadic instances circularly predicated of one another.

Compound Complex I:



In prefix notation, the three predications here are given in the facts $:P^1_i(S^1_k)$, $:S^1_k(O^1_j)$, and $:O^1_j(P^1_i)$. Clearly, Complex I is a plural whole composed of only predicable individuals—monadic instances O^1_j, P^1_i, and S^1_k—with *no* ***non-predicate*** *subjects*. The same situation is possible for combinations of any *n*-adic instance predicates as long as each of their *n* subjects is itself an instance predicate. Depicting graphically such complexes would be increasingly difficult, requiring the use of curved lines for instances, and best done in three-dimensions. We need not pursue that here. Once we have such basic complexes it is easy to conceive of these wholes extended recursively both horizontally and vertically up through hierarchies of increasingly complex structures. For example, 'instance-only' structures such as Complex I could be the relata *a*, *b*, *c*, etc., in Complex G above, as such vertically connected to G's composing instances. In sum and with an importance for ontology that cannot be overstated, what these examples substantiate is the possibility that all complex individuals whatsoever can be built up exclusively from, and by means of, predicable combinators from the single category of relation instances. Or in the reverse direction of analysis, not all predication necessarily presupposes non-predicable subjects ('substances' or their reducta of bare particulars), but that there can be an atomic ontic level of mutually-sufficing predicable individuals from which all other individuals (compound complexes) are derived—in instance ontology there need not be the regress to absurdity of 'turtles all the way down'. This insight does not contradict the maxim that 'There are no relations (and hence structure) without relata', but corrects the prevailing preconception that a system of relations always presupposes a base level of relata nodes that are 'more substantial' non-relational, non-predicable entities.

5. The Indiscernibility but Non-Identity of Certain Structures

We are now in a position to clarify the concept of indiscernibility, and indeed to illustrate how there can be numerically distinct but indiscernible entities based upon a properly understood 'internalist' criterion of indiscernibility that compares what are their totally composing predicable constituents, i.e., relation instances. Hence, with this we have in a perspicuous manner the falsity of the Principle of the Identity of Indiscernibles. In addition to its importance to ontology generally, the topic of indiscernibility is presently of acute interest in the philosophy of science and concerns the 'loss of identity' or 'metaphysical underdetermination' of sub-atomic entities under quantum mechanics. Operative here is the 'Indistinguishability Postulate' of quantum statistics which asserts that permutations of particles of the same kind are not observable (in making no difference in the probabilities of measurement outcomes).[47] The underdetermination debate has to do with whether quantum entities are 'individuals' (what are often described in this context as sets of intrinsic properties (e.g., rest-mass, charge, spin, etc.) individuated each by a non-qualitative something), or 'non-individuals' (i.e., entities that are in their very identity and nature somehow vague, however this can be understood ontically as opposed to simply modeled formally, e.g., with 'quasi-sets'[48]). In response, (ontological) structural realism has been put forth as, among other things, an alternative that explains the individual/non-individual dichotomy as two ways of conceiving the same structural reality.[49] Here there is a reconceptualization of electrons, elementary particles, etc., in structural instead of individualistic terms, one where the usual relationship of ontic priority between objects and encompassing structures—systems of relations among and together with these presupposed relata objects—is inverted at a foundational level so as to exist between systems consisting exclusively of relations and resultant objects built up from them. We have just seen—with analogs of Complex I—how the latter is possible without a vicious regress of presupposed relata objects. Building upon this analysis we shall see now how a realist instance ontology can provide distinct complex wholes—'objects'—that conform to the Indistinguishability Postulate and yet are each properly characterized as an *individual* in a sense

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[47] Steven French, 'Identity and Individuality in Classical and Quantum Physics', *Australasian Journal of Philosophy* 67 (1989): 432-46.

[48] Steven French and Decio Krause, 'Quantum Objects are Vague Objects', *Sorites* 6 (1996): 21-33.

[49] French and Ladyman, 'Remodelling Structural Realism'.

that corrects the distorting bundle-of-universals-plus-individuator or alternate trope-bundle conceptions, and, moreover, provides a precise criterion based upon internal constituents whereby these distinct individuals are indiscernible. Whether in fact quantum entities can be assayed as such 'objects' is, of course, for the structuralist program in the philosophy of physics to determine. The analysis here is offered as providing the detailed ontological underpinning for such a program.

On the above all entities, with the exception of founding and component relation instances, are complexes in the precise way given, the latter a detailed internal analysis of entities invisible to traditional substance/attribute ontology. In the tradition an ontic analysis of a subject substance *a* consisted of either a single form predicable of the other parts and/or prime matter of *a*, or the monadic properties predicable of *a* bundled together, with or without an additional individuator, to constitute *a*. It was, however, in the cruder context of substance/attribute ontology that our intuitive concept of indiscernibility as *qualitative sameness* was first standardized as the formal criterion $(F)[F(x) \equiv F(y)]$, and in which continues the controversy over the concomitant Principle of the Identity of Indiscernibles, i.e., that indiscernible entities so defined are identical, or symbolically, $(F)[F(x) \equiv F(y)] \supset x = y$. The lack of progress in the latter controversy is, I propose, symptomatic of an error in the standard formal criterion for indiscernibility, and thus in the motivating ontology that can analyze the internal nature of entities only by making essential use of derivative externally predicated attributes of them: their species 'forms' or all their monadic attributes. The intuitive indiscernibility concept of 'qualitative sameness' is synonymous with 'same in every way that is identically repeatable'. If, as was the case in much of the tradition, an ontology recognizes only monadic intensions as numerically repeatable qualitatively characterizing entities, and holds that an entity characterizes a subject by being ontically predicated of it, then indiscernibility between any *x* and *y* does indeed reduce to $(F)[F(x) \equiv F(y)]$. But this makes indiscernibility dependent upon *external* predicates posterior to the subject entities compared, and so confuses the debate over the identity of indiscernibles with tangential and inconclusive arguments why *prima facie* irrelevant external properties like 'is identical to *a*', 'is different from *b*', 'is two units from *a*' are (or are not) indeed irrelevant to

indiscernibility.[50] The intuition that indiscernibility is a matter of the internal constitution of entities is what motivates in this context the attempts to distinguish 'intrinsic' from 'extrinsic' and 'pure' from 'impure' properties, and to make indiscernibility in its strongest form turn upon 'pure intrinsic' properties.[51] The same intuition is found expressed in Leibniz's formulation of the Principle of the Identity of Indiscernibles: "There are never two beings in nature which are perfectly alike and in which it is impossible to find a difference that is *internal* or founded on an *intrinsic denomination*."[52](my italics) For Leibniz the properties of an entity *a* are both predicable of *a* and together compose *a*'s 'complete concept'. By the Identity of Indiscernibles the complete concept of *a* is unique to it since no two individuals can have the same bundle of characterizing properties. Moreover and conversely, the Identity of Indiscernibles follows from the assumption that the universal properties predicable of an entity *a* are all and only the constituents of *a*, together with an intuitive thesis known as the Principle of Constituent Identity: Complete identity in corresponding constituents of *a* and *b* entails numerical identity of *a* and *b*.[53] Of course, the problem here is that an unrepeatable particular cannot be identical to all and only its repeatable properties bundled together since the bundle itself is thus repeatable. What is missing, and telling of the error of the whole analysis, is an individuator but one that, by the same analysis, would in the end have to be a bare particular. In contrast, the other premise—the Constituent Identity principle—is intuitive, and, indeed, when formalized and applied to complexes is our above concluded Identity Criterion for Complexes, **ID**. What would be the corresponding and equally apparent principle for indiscernibility is that: Complete indiscernibility between both corresponding structures (isomorphism) and the corresponding entities structured that jointly make up each of *a* and *b* entails the indiscernibility of *a* and *b*. What is required, then, is that we render precise these pre-critical intuitions con-

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[50] See Bernard Katz, 'The identity of Indiscernibles Revisited', *Philosophical Studies* 44 (1983): 37-44. Also Richard Swinburne, 'Thisness', *Australasian Journal of Philosophy* 73 (1995): 389-400.

[51] This terminology is used in the overview article by Peter Forrest, 'The Identity of Indiscernibles', *The Stanford Encyclopedia of Philosophy* (Summer 2002 Edition), Edward N. Zalta (ed.), URL = <http://plato.stanford.edu/archives/sum2002/entries/identity-indiscernible/>.

[52] Gottfried W. Leibniz, 'The Monadology' in *Goffried Wilhelm Leibniz: Philosophical Papers and Letters*, 2d. ed., trans. & ed. by Leroy Loemker (Dordrecht: Reidel, 1969), p. 467.

[53] Loux, *Metaphysics*, p. 107.

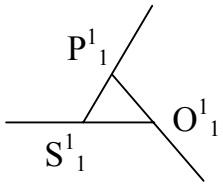
cerning how indiscernibility is a matter of the *internal* nature or constitution of entities, what in the primary sense makes things to be what they are and not something else, and what is presupposed by properties and relations that have these entities as relata (e.g., spatial relations). This is a project now possible in the refined context of structure theory built from instance ontology.

Founding the project of an internal criterion for indiscernibility is the fact that in the realist ontology of relation instances there are atomic entities—individuated ontic predicates—that are absolutely qualitatively the same and yet distinct. Specifically, two distinct instances R^n_i and R^n_j (e.g., $Square^1_1$ and $Square^1_2$) of the same type R^n (e.g., $Square^1$) are qualitatively indistinguishable in the precise sense that each shares as its total qualitative content numerically the same intension R^n, and, importantly, where as such intension R^n is not an ontic predicate of (not external to) the instances it characterizes. What renders R^n_i and R^n_j distinct is not a difference in intension or qualitative content, but rather distinct combinatorial aspects, i.e., distinct predicable 'acts', that are 'formal' in adding nothing to the concomitant intensions of their respective instances. The combinatorial aspect of an instance R^n_i is not a further intension in addition to R^n, but a unifying functionality of R^n_i specified in its range and ordering (if any) by the instance's other aspect and sole intension R^n. It is this pivotal insight that cuts through the unhappy traditional alternatives of entities having to differ either only numerically (*solo numero*), i.e., without any internal difference whatsoever, or in some intensional aspect, or by some posited but unanalyzable constituent individuator (e.g., *haecceitas*, bare particulars). Relation instantces can differ by their non-qualitative but combinatorial aspects. Hence, instances R^n_i and R^n_j of the type R^n are intensionally identical but numerically distinct, and so straightforwardly *indiscernible but not identical*. So, at this point we have a precise notion of indiscernibility based upon internal aspects of entities (i.e., identity of constituent non-predicable intensions) and a refutation of the Principle of the Identity of Indiscernibles, but only, though crucially, for the limiting case of relation instances. Yet, it is the case that instances R^n_i exist only as constituents of facts, $:R^n_i(a_1,a_2,...,a_n)$, and all the plural entities making up reality are facts or their compounds. What is required is extending this analysis to structured entities built up from instances.

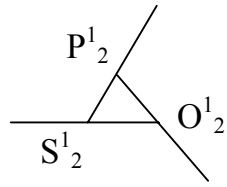
The question is: How is it that *complex entities* built up from relation instances satisfy the intuitive indiscernibility criterion of 'same in every way

that is identically repeatable'? As a first approximation and as noted above, it is apparent that indiscernibility so conceived would mean for structured entities *exact ontic congruence*—an exact matching of constituents that preserves both all formal structure to the last detail and all qualitative aspects of *all* the constituents (viz., the intensions of the respective linking instances and the qualitative content of the respective relata linked). For what is repeatable is both structural form and the qualitative aspects of the entities making up the structure. To reinforce this, consider first the lowest level of complexity, i.e., individual facts. It is evident for facts $:R^n_i(a_1,a_2,...,a_n)$ and $:R^n_j(b_1,b_2,...,b_n)$, whose instances, R^n_i and R^n_j, have the same intension R^n, that they are indiscernible if and only if a_k is indiscernible from b_k, for all k, $1 \leq k \leq n$. That is, because the facts have the same predicate intension *and* because of the isomorphism that exists between the facts' relata n-tuples $<a_1,a_2,...,a_n>$ and $<b_1,b_2,...,b_n>$, due to their being identically ordered by this same intension R^n, the only thing that could qualitatively distinguish these facts internally is some qualitative difference in respective relata. Without this difference, i.e., with indiscernible respective relata, subsuming complexes $:R^n_i(a_1,a_2,...,a_n)$ and $:R^n_j(b_1,b_2,...,b_n)$ are themselves indiscernible, and this is possible in a non-circular way when the correlative relata are indiscernible in the prior manner of relation instances. What is intended here can be seen in the example complexes J and K, which are cases of complex I above.

Compound Complex J:



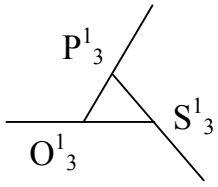
Compound Complex K:



Complexes J and K are isomorphic in form where each constituent instance corresponds to one of the same intension—P^1_1 and P^1_2 having intension P^1, O^1_1 and O^1_2 having intension O^1, S^1_1 and S^1_2 having intension S^1—and more importantly, indeed crucial to any satisfactory analysis of possible indiscernibility for complexes, the corresponding relata for the corresponding instances are themselves instances of the same intension and so, on the above analysis, are guaranteed to be internally indiscernible. Stated otherwise, complexes J and K are indiscernible because they each decompose without remainder into relation instances such that there is a one-to-one

correspondence between them that preserves the relative ordering or structure within each complex, i.e., there is an isomorphism between J and K, and where the corresponding subject or relatum 'nodes' are instances of the same intension or kind. The concept can be clarified by comparing complexes J and K with the following complex L.

Compound Complex L:



There is a one-to-one correspondence between the composing instances of L and those of J and K, but the latter complexes are discernible from L on the basis of the different relative ordering (non-isomorphism) of their instances, e.g., $:P^1{}_1(S^1{}_1)$ in Complex J but $:P^1{}_3(O^1{}_3)$ in Complex L.

We can, of course, have the same indiscernibility on complexes of atomic instances of any *n*-adicity, not just the monadic as in complexes J and K, if the instances composing each complex have other instances of that complex as relata and these predicates corresponds isomorphically across the two complexes. Moreover, it is possible to maintain indiscernibility across vertical compositionality, as long as corresponding substructures are isomorphic and decompose into exactly matching instances of the same intensions, and these instances' corresponding relata are isomorphic and decompose likewise, and so on, until at an atomic level there are only corresponding isomorphic and horizontally composed complexes consisting exclusively of predicable instances having other instances in their complexes as relata, e.g., such as J and K. Consider, for example, two cases of complex G that differ only by subscripts on their respective predicate terms and where corresponding relata, say *a* and *a′*, are indiscernible complexes like J and K.

We can now formulate our post-critical notion of indiscernibility for complexes in its complete generality, i.e., for compounds composed both horizontally and vertically. Namely, complexes are indiscernible if and only if they are isomorphic (structurally congruent), which includes all corresponding substructures taken as single relata, and that the atomic com-

posing instances of every set of corresponding substructures are identical in intension. This educated intuition of indiscernibility for complexes is then made precise with the following formal recursive definition **IND**. The recursive form of the definition guarantees the requisite isomorphisms at each structural level.

(**IND**) Entities x and y are indiscernible if and only if

1) $x = \mathrm{R}^n_i$ and $y = \mathrm{R}^n_j$, where R^n_i and R^n_j are instances of the same intension R^n.
2) $x = :\mathrm{R}^n_i(a_1,a_2,...,a_n)$ and $y = :\mathrm{R}^n_j(b_1,b_2,...,b_n)$ and a_k and b_k are indiscernible for $1 \leq k \leq n$.
3) x and y are complexes such that there is a one-to-one correspondence f between their constituent facts where $f(:\mathrm{R}^n_i(a_1,a_2,...,a_n)) = :\mathrm{R}^n_j(b_1,b_2,...,b_n)$ and where $:\mathrm{R}^n_i(a_1,a_2,...,a_n)$ and $:\mathrm{R}^n_j(b_1,b_2,...,b_n)$ are indiscernible.[54]

Note that as it should be, under **IND** a complex x is indiscernible from itself since clauses 1)—3) allow for the case of $\mathrm{R}^n_i = \mathrm{R}^n_j$. But, of course, the whole point of the previous analysis was to show that, given a realist instance ontology, it is possible for there to be instances R^n_i and R^n_j such that $\mathrm{R}^n_i \neq \mathrm{R}^n_j$ and that they and their corresponding facts satisfy clauses 1)—3). That is, there can be hierarchies of complexes, horizontally and/or vertically composed, and conforming to **IND** that are numerically distinct but indiscernible according to their internal form or structure and respective qualitative/intensional contents of the constituents. Hence the falsity of the Principle of the Identity of Indiscernibles.

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[54] **IND** corrects the definition of indiscernibility that I had given previously (e.g., in *Moderate Realism* and 'The Logic of Instance Ontology'), viz., what is a liberalization on the conditions of **Id**:

> Entities a and b are indiscernible if and only if, for every monadic property P^1, there is an instance P^1_i of P^1 such that $\mathrm{P}^1_i(a)$ if and only if there is an instance P^1_j of P^1 such that $\mathrm{P}^1_j(b)$.

The idea is that entities having as predicates instances all and only of the same properties are indiscernible, and if, for one or more properties, each entity has an instance of it non-identical to that of the other entity, then the entities will be indiscernible but not identical. The problem here, as with the traditional definition of indiscernibility, is finding a principled way to exclude from the range of P^1 trivializing properties like 'is identical to a'. **IND** avoids all this by having indiscernibility turn upon the internal constituents composing entities and making them to be what they are, and not what can be externally predicated of them and presupposing them.

With this we now have an account of how distinct entities can satisfy the Indistinguishability Postulate of quantum physics. If fundamental physical entities, particles or fields, are 'completely relational' in the sense made precise herein and indiscernible according to **IND**, then with the permutation of such complexes within larger subsuming structures, e.g., those including the added system of relations and entities introduced by instrumentation, what will change in the total subsuming structures before and after the permutations is *only* the individuality (i.e., the combinatorial aspects) of the composing instances, not the intensions of the instances or the isomorphisms between the structures. Hence, any instance of a property emergent on such an entity+instrumentation complex and representing a measurement on the entity will differ only individually, i.e., as a different instance, from the instance of that property that will emerge on the indiscernible complex that results with the permutation of an indiscernible entity. Indiscernible measurement structures on indiscernible but distinct complexes entities effects indiscernible but numerically distinct measurement results. On the above, however, this does not imply that indiscernible quantum entities are not-individuals or in their very natures vague entities.

6. Conclusion

It is a theme of contemporary ontology and foundational quantum physics that reality is inherently relational. We have seen how a realist instance ontology of individuated *n*-adic ontic predicates (instances) and their sharable non-predicable intensions can correct traditional theses regarding unification and account for such a world of pure and qualitatively multifarious structure, and this all the way down to an atomic ontic level of only inter-predicable relation instances. No ultimate non-predicable subjects (substances, prime matter, bare particulars, non-predicable relata) need be posited. The combinatorial—agent unifier—aspect of an ontic predicate provides ontology with a non-posited *principium individuationis*, and it together with its formally distinguishable concomitant intension aspect, makes for a category of *intensionally controlled* (as to their compatible subjects) *individuated unifiers*, what are the atomic ontic links (and ultimately what is linked as well) that make up the hierarchical lattice this is reality. It is these property and relation instances that make possible a precise recursive definition of structure or complexity, as well as provide the basis, missing in the tradition, for an internalist or constituent criterion for structural identity

and indiscernibility, the latter definable recursively. The resulting analytic precision makes it possible to display perspicuously how there can be indiscernible but non-identical entities (structures).

3

The Nature and Necessity of Composite Simples, E.g., Ontic Predicates*

1. Introduction

Over the history of philosophy, entities of a number of kinds have been declared *simple* in their being, and, correlative with this, fundamental in some ontological sense. These include God, souls, (at least some) intensions or universals (e.g., Red^1), ontic predicates both as substantial forms or unit attributes (e.g., $Red^1{}_1$, $Red^1{}_2$, …, whether conceived as predicable instances or 'substance-like' tropes), individuating 'bare particulars', spatial points and temporal instances.[1] As typically defined an entity is simple if it *has no proper parts*, *is non-composed*, or *is (actually) undivided and (potentially) indivisible*. Yet, with the exception of simple universals, entities of the above kinds have been analyzed as having *essential but distinct 'aspects'* (controversially so for bare particulars[2]), and this would seem to

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* Original version first published in *Metaphysica: International Journal for Ontology & Metaphysics* 5 (2004): 89-133.

[1] Herein terms naming intensions, e.g., 'Red^1', '$Love^2$', 'Owe^3', will each have superscripts indicating the number of subjects the intension specifies as jointly necessary in order to be characterized or qualified (including being ordered) under that intension. Alternately and in general, the superscript 'n' on the intension term 'R^n' indicates the number n of subjects in an n-tuple $<a_1,a_2,\ldots,a_n>$ necessary in order to form a fact with that intension controlling the ontic predication among a_1, a_2, …, a_n, the fact designated as '$:R^n(a_1,a_2,\ldots,a_n)$'. The colon locution is used to distinguish a fact designated as '$:R^n(a_1,a_2,\ldots,a_n)$' from a corresponding proposition '$R^n(a_1,a_2,\ldots,a_n)$'. Subscripts on intensions, e.g., as in '$Red^1{}_1$', '$Red^1{}_2$', renders each such term a name for a particular and unrepeatable instance of the indicated type, e.g., Red^1. Ontic predication explicated herein is to be understood as what is traditionally identified as 'material' or 'fundamental predication' and concerns the nature of composition among any entities whatsoever, extra-conceptual or conceptual, and is to be distinguished from 'formal', 'linguistic', or 'grammatical predication' which pertains to the linguistic/conceptual syntactical composition of declarative sentences.

[2] D. W. Mertz, 'Individuation and Instance Ontology', *Australasian Journal of Philosophy* 79 (2001): 45-61. J. P. Moreland and Timothy Pickavance, 'Bare Particulars

imply real and internal composition. For example, God is traditionally treated as the coalescence of divine attributes (omniscience, omnipotence, goodness, etc.) and, under the Christian Trinitarian doctrine, identical with three Divine Persons. More generally and found in the tradition has been the thesis that the 'principles of substance'—form and matter—as they compose at least some substances (e.g., Socrates), are one or both simples with yet the dual aspects of being unrepeatable particulars or individuals (e.g., individuated souls, the prime matter of each material substance) having repeatable or universal essences or natures (e.g., Humanity, Non-repeatability). A like analysis was commonly extended to all predicable attributes—properties, accidents, and relations (when admitted)—as each, e.g., this red (Red^1_i), is assayed as a simple ontic predicate composed jointly of an individuating ('thisness') aspect and qualitative/intensional ('suchness') aspect (e.g., Red^1). Unit attributes, particularly in the more telling polyadic form of relations, will be central to the analysis below. Similarly for non-extended simple spatial points that are at best tropes, each with an unrepeatable aspect together with a repeatable spatial-, extension-relevant qualitative essence necessary to non-arbitrarily found the spatial relations that have them as relata. Likewise for temporal instances.

Given, then, any of the above distinctions, it has seemed evident to some philosophers that the 'simples' to which they are attributed must each have an internal or constituent metaphysical diversity necessary to found the distinction, and thus in a strict sense be non-simple. For, the argument would go, if an entity *x* has even one constituent that is not identical with it, then *x* is an ontological composite. And crucially, if *x* is a composite then, *prima facie*, it must have a real internal differentiation and so inherent division—a discreteness/diversity of parts precisely as they each contribute to the reality of the whole—which would render *x* not simple. This would certainly seem to be the case for wholes whose composition included multiple *individuals* or *particulars*, i.e., unrepeatable entities, as, for example, physical wholes like a wall of stacked stones, or abstract wholes such as the natural number 3 together with its properties as instances: $Prime^1_i$, Odd^1_j, etc. With this and the further assumption that all entities whatsoever and at

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and Individuation: A Reply to Mertz', *Australasian Journal of Philosophy* 81 (2003): 1-13. In the same issue see my response, 'Against Bare Particulars: Response to Moreland and Pickavance', 14-20. Also relevant here is Richard Brian Davis, ''Partially Clad' Bare Particulars Exposed', *Australasian Journal of Philosophy* 81 (2003): 534-48.

any level of analysis are individuals, what is the defining thesis of nominalism, then all composites would be non-simple.

What the above argument denies as a premise is the possibility of an entity with a 'virtual differentiation' of constituents, i.e., an entity where there are non-identical constituents but no inherent divisions or 'ontic discontinuity' marking this non-identity. Stated otherwise, denied is an entity that can have an internal non-identity/distinctness of multiple constituents yet among which there is no numerical differentiation/discreteness as they jointly constitute the whole. If to the contrary there were such an entity then any actual differentiation of constituents could only be 'external' and the result of an act of cognitive abstraction, what has been called in the tradition a 'formal distinction'. A principal source of the formal distinction was and is the analysis of entities held to be composed of both repeatable (intensional, qualitative) and unrepeatable (individuating, particularizing) aspects, what would be every entity whatsoever, with the sometimes posited but bogus exceptions of bare particulars at the one extreme and at the other entities treated as bundles of only repeatable properties. William of Ockham, for example, characterized an entity with supposed formally distinguishable aspects as one that would have to have constituents that "while there are not two things, one is not formally the other."[3] Ockham held the impossibility of such a composite, saying that "In created things there is no such thing as a formal distinction. All things which are distinct are really distinct and therefore, different things."[4] What I take Ockham to imply by the 'distinct-implies-really-distinct' requirement on constituents is that *distinct* means *discrete* in the sense of a separation or division inherent to the whole. The clarification below is that all internal division is marked by the requirement that one of the constituents have the special causal status of agent unifier among the remaining constituents (e.g., in the tradition a substantial or accidental form) in order to bridge the division and effect what is a manifold whole. In supposed composites whose constituents are only formally distinct there would be no 'ontic distance' between yet distinct constituents that would require an agent unifier to bridge. But on Ockham's and like philosophers' view, lack of 'ontic distance' implies a coinciding identity and so the absence of real composition. Ockham's view is

[3] William of Ockham, *The Summa Logicae, Part I*, trans. Michael Loux as *Ockham's Theory of Terms* (Notre Dame: University of Notre Dame Press, 1974), p. 82.
[4] Ibid., p. 84.

maintained in contemporary ontology by Herbert Hochberg[5] and J. P. Moreland[6].

The opposing thesis recognizes formal distinctions or 'distinctions of reason that have foundations in things' (*a parte rei* = in reality), what are intermediate between 'real distinctions' on the one side and 'merely conceptual distinctions' on the other, the latter having no extra-conceptual bases.[7] With a formal distinction there is a differentiation—a rendering discrete—by intellectual separation of what is founded in and is partial to a fuller reality undifferentiated *in se*—the internally simple subject of selective abstraction. The recognition of the distinction and concomitant entities, or 'aspects', are found in the scholastics, e.g., with the *distinctio formalis a parte rei* of John Duns Scotus or the *distinctio rationis rationcinatae* and 'modal distinction' of Francisco Suarez and others, a distinction advocated in contemporary ontology by, e.g., Keith Campbell, D. M. Armstrong, and myself.[8] Allowed here are entities that are both in some sense simple—internally undivided—and in some sense composite—having non-identical parts. Scotus advanced the formal distinction in the context of a theory of the union between the repeatable nature (e.g., Man) and the unrepeatable 'individual difference' or '*haecceitas*' ('thisness') that jointly compose a particular (e.g., Socrates). Suarez denied any such distinction in this

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[5] Herbert Hochberg, 'Individuation and Individual Properties: A Study of Metaphysical Futility', *The Modern Schoolman* 79 (2002): 107-35, p. 114.

[6] J. P. Moreland, 'Naturalism, Nominalism, and Husserlian Moments', *The Modern Schoolman* 79 (2002): 199-216, pp. 206ff.

[7] A traditional example of a 'merely conceptual distinction', what was called a *distinctio rationis ratiocinantis* ('distinction of the reasoning reason'), is 'Peter is identical to Peter.' Here Peter is said to be distinguished from himself, which, of course, cannot correspond to any extra-conceptual distinction. For an explanation of these types of scholastic distinctions see Francis Suarez, *On the Various Kinds of Distinctions (Disputationes Metaphysicae, Disputatio VII, de variis distinctionum generibus)*, trans. Cyril Vollert, S. J. (Milwaukee: Marquette University Press, 1947), pp. 18-19.

[8] For Scotus' views see his *Ordinatio II*, d.3, part 1, qq. 1-6, in Paul Spade, *Five Texts on the Mediaeval Problem of Universals* (Indianapolis: Hackett Publishing Co., 1994), pp. 57-113. For Suarez see *On the Various Kinds of Distinctions, Disputatio VII*, pp. 18-19, 27-39. Keith Campbell, *Abstract Particulars* (Oxford: Basil Blackwell, 1990), pp. 56-7. D. M. Armstrong, *Nominalism & Realism: Universals & Scientific Realism*, Vol. I (Cambridge: Cambridge University Press, 1978), pp. 108-111. Mertz, 'Individuation and Instance Ontology', pp. 59-61. In the latter I used the unfortunate term 'non-constituent whole' for what I herein refer appropriately to as a 'continuous composite'. Also see Mertz, 'Combinatorial Predication and the Ontology of Unit Attributes', *The Modern Schoolman* 74 (2002): 163-98, p. 91 [Essay 1 herein].

context but pointed out its necessity between the 'mode' of inherence or *union* of a property and the property (specifically, the intension) as the latter is ontically predicable of a subject.[9] Foundational to the following analysis I shall rehearse an argument of how these contexts are, in fact, the same—that what is ontically predicable is by that very fact unrepeatable, and that though this unifying and particularized aspect is not discrete/divided from a concomitant intension or quality that delimits its agency, neither is it identical to it. Clarified here will be both the necessity for and the nature of the formal distinction in a context in which it continues to be debated: the nature of the union or nexus between the qualitative and individuating aspects of particulars, what historically has been confused by some with the union between subject individuals and their ontic predicates treated as universals. A clarifying thesis argued herein is that ontology's basic particulars are ontic predicates themselves, where each is a union of what are the formally distinct aspects of a qualitative intension and a combinatorial/unifying act among an *n*-tuple of subjects, the latter being as such unrepeatable, i.e., an individuating aspect. Out of the related analysis there will arise a clarification of our pre-critical concepts of the simple, complex, and composite.

Specifically, it will be argued below that the obscurities concerning the concepts of simple and composite, and relatedly of the formal distinction, turn on a failure to distinguish between two types of wholes. These are: a) the commonly recognized and pervasive plural wholes of joined yet discrete elements, what are *complexes* (*structures, systems*), but what here will be more descriptively termed *articulated composites*, and b) theoretically necessitated non-articulated and internally non-differentiated wholes of yet identity-preserving proper constituents, wholes whose assay is more subtle and what are accurately termed *continuous composites*. Both types of wholes are 'composed' but in distinct ways with the result that complexes are non-simple whereas non-articulated composites are simple. The primary analytic tool for clarifying both types of composites will be the predicable unit attribute, or 'relation instance', R^n_i, what I have assayed in

[9] For Suarez's denial of a formal distinction between the individuating and intensional aspects of an entity see Francis Suarez, *On Individuation (Disputationes Metaphysicae, Disputatio V, De unitate individuali ejusque principio)*, trans. Jorge Gracia (Milwaukee: Marquette University Press, 1982), pp. 46-52, and Suarez, *On Formal and Universal Unity (Disputationes Metaphysicae, Disputatio VI, De Unitate Formali et Universali)*, trans. J. F. Ross (Milwaukee: Marquette University Press, 1964), pp. 29-35. For Suarez's assertion that there is a formal or 'modal' distinction between an attribute and its mode of inherence in a subject see his *Disputatio VII*, pp. 27-39.

detail elsewhere as, succinctly, an *individuated intensioned ontic combinator*.[10] Outwardly, when combinatorial (ontically predicable) among one or more subjects, an instance R^n_i is the unifying cause (both 'formal' and 'material') of a resultant *atomic* articulated composite, i.e., a fact or state of affairs, and when conjoined with other relation instances via shared relata is a contributing cause of more complicated structures, this up through hierarchies of complexes that constitute the ordinary objects of experience and of science (e.g., an atom, a living body, the universe, the Natural Number System). A complex is 'articulated' at relata, the 'joints', where the relation instances, the 'connecting rods', meet. Directed externally, relation instances have the ontic role of effecting unity-at-a-distance, i.e., unity among the yet discrete. This analysis explains the fact of structure, what in the Aristotelian tradition and there under the 'tyranny of the monadic' was simply made the effect of a posited 'principle'—monadic *form*, substantial or accidental. The analysis overturns long-dominant and retarding assumptions concerning unification and the nature of polyadic relations, viz., that all elements making up a structured whole must share a *single unifier* as the cause of their collective unity, and, concomitantly, that all ontic predication is *monadic* in intension.[11] Relatedly, relation instances as individuated agent unifiers provide, I propose, non-trivial answers to the Special and General Composition Questions made popular by Peter van Inwagen: respectively, Under what conditions does composition (among the discrete) occur?, and What is composition (among the discrete)?[12] Contrary to what

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[10] Mertz, 'Combinatorial Predication'. Also see Mertz, *Moderate Realism and Its Logic* (New Haven: Yale University Press, 1996).

[11] D. W. Mertz, 'An Instance Ontology for Structures: Their Definition, Identity, and Indiscernibility', *Metaphysica: International Journal for Ontology & Metaphysics* 4 (2003): 127-64 [Essay 2 herein].

[12] Peter van Inwagen, *Material Beings* (Ithaca: Cornell University Press, 1990). The Special Composition Problem—Among what conditions does composition (among the discrete) occur?—is answered by what I have called the **Unity-by-Instances Thesis**: All plural unity—complexity or structure—is by the following:

a) A relation instance R^n_i predicable of an n-tuple of relata, $\langle a_1,a_2,...,a_n\rangle$, is the cause of an individual plural whole, i.e., a fact $:R^n_i(a_1,a_2,...,a_n)$, having R^n_i, a_1, a_2, ..., a_n, as its only constituents.

b) If R^n_i is a constituent of a plural whole x and S^n_j is a constituent of a plural whole y, and R^n_i and S^n_j, share one or more relata, then there is an individual plural whole z that has as constituents all and only the combined constituents of x and y (horizontal composition).

c) For any fact $:R^n_i(a_1,a_2,\ldots,a_n)$, if for $1 \leq j \leq n$, a_j is a plural whole, then there exists an individual plural whole whose constituents are all and

some contend, (articulated) composition is not just a brute fact: the concepts of the so-called 'mereological circle'—of part, sum, and composite—can be analyzed in terms outside of the circle, and this in a way that explains how some entities and not others are 'fastened together'.[13] Indeed, it is not difficult to theorize how a single category of relation instances can effect or 'boot up' all of physical and cognitive reality, starting at what is said to be the purely relational nature of quantum reality—physical simples are instances of quantum and spatial/temporal relations.[14] In a strict and ultimate sense, reality is 'all in the arrangements'. I shall not rehearse the arguments for these extended claims, referring the reader to the given references.

What is relevant herein and founds the above claims is that the analysis demonstrating that, as ontic predicates, relations (including properties in the limiting case) are outward agent combinators, and are individuated as such, also implies an inward nature for relation instances of composite simplicity. Crucial here is the perennial and contradictorially interpreted regress now known as Bradley's Regress.[15] The insight is that relation instances are each 'simple' in more than a crude pre-critical mereological sense of being non-composite. That is, we must give up the naive definition: x is simple $=_{df}$ x has no proper parts. Observed in the limiting case of monadic properties as far back as scholastic ontology, a relation instance R^n_i of any adicity is necessarily assayed as a *continuous composite* of cognitively distinguishable but not discrete constituents, the latter being the correlative aspects of an unrepeatable combinatorial agency (indicated by the subscript 'i') and a specific and delimiting intensional content, R^n (the superscript indicating the number of subjects required jointly for the

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only the constituents of the fact and constituents of a_j (vertical composition)

This is so because the answer to the General Composition Question—What is composition (among the discrete)?—is: The unifying effect of the predicable agency or combinatorial act of a relation (including property) instance with its subjects.

[13] Ned Markosian, 'Brutal Composition', *Philosophical Studies* 92 (1998): 211-249; Van Inwagen, *Material Beings*, pp. 50-51.

[14] For an indication with references of how the single category of relation instances can contribute to an ontology for quantum mechanics see Mertz, 'Instance Ontology for Structure'. Also see in the same volume of *Metaphysica*, 4 (2003), the paper by Michael Esfeld, 'Do Relations Require Underlying Intrinsic Properties? A Physical Argument for a Metaphysics of Relations', 5-25.

[15] A partial historical survey of different interpretations of Bradley's Regress is found in Mertz, *Moderate Realism*.

intension to characterize, what is specified by the intension itself). The uniqueness of the unifying act of a relation instance as predicable of its relata is precisely the 'thisness' (*haecceitas*) aspect distinguished but unexplainable in traditional ontology. In the following it will be explained how it must be the case that, though such a whole is internally undifferentiated, the identities of each of the constituents as constituents are maintained in their full and essence-specific realities, and so the whole is properly a composite. In other words, though such a whole is not a plurality of articulated parts, neither is it homogeneous—it is not the same throughout. It has been called to my attention that such an analysis was one of the "fundamental innovations" by Gustav Bergmann in his posthumous *New Foundations of Ontology* (1992), where he asserts "A simple is a conjunction of two: one is an ultimate sort [certain intensions]; the other an item [an individual]."[16](my inserts) He implies that this composite is nevertheless 'simple' because the ultimate sort and item components are "totally 'inseparable'" in the sense that it is a "'combination'" but where there is no "tie" to hold them together. This is apparently why such composites are "for good reasons called simples"[17], and deserving of the special designation as each a "Two-in-One".

To anticipate, a heuristic analogy for grasping the concept of a composite simple would be a colored disk with colors differing continuously about its center and ranging from red through yellow to green, as in a non-segmented, seamless spectrum color wheel, one of the types used to teach art. The coloring of the whole is not homogeneous yet there are no internal boundaries marking numerically distinct regions of different colors. The disk is, phenomenally, a continuous composite and as such a simple entity. The unity of a continuous whole is a continuum of the yet distinct—a fusion without diffusion, a concretion without an identity-obliterating blending. Characterized as such, the unity of a continuous composite is to be distinguished from what some hold as the only alternative to articulated composition: the erroneous 'absolute unity' attributed by monists to the One.

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[16] Gustav Bergmann, *New Foundations of Ontology* (Madison: University of Wisconsin Press, 1992), pp. 56-58, 90. This reference was brought to my attention by William Vallicella in his 'Bradley's Regress and Relation-Instances', *The Modern Schoolman* LXXXI (2004): 159-83. Vallicella somehow construes Bergmann as implying that bare particulars are simple composites, or 'Two-in-One's', and that there is thus little difference between bare particulars, which I reject, and my relation instances. I propose that Bergmann's relevant texts referenced here show the error of this interpretation.

[17] Bergmann, *New Foundations*, p. 57.

With such an entity the blending of any would-be initial elements is so absolute that the resultant 'reality' has *no composition*, no internal distinction or relations, and where, as Bradley observed, any differentiation by abstraction is necessarily falsification insofar as it supposedly marks a real distinction in the blend.[18] An analogy would be gray paint as the resultant blend of white and black paint, and in which, phenomenally, the latter colors have ceased to exist.

Succinctly then, principal among the insights to be gained in the following are: a) The term 'simple' is properly defined as the absence of any internal differentiation or division—absence of discreteness of constituents or parts *qua* actually contributing to the being of the whole, as opposed to only external differentiation in the intellect by abstraction. b) Discreteness of constituents, what characterizes an articulated whole, is marked by constituent interposing ontic predicates, i.e., relation (including property) instances. For, it is the nature of a relation instance as an ontic combinator existing 'between' and 'among' its distinct relata to be a rigid connector simultaneously bridging and presupposing/enforcing an ontic division of mutually differentiated and discrete subjects, the instance's character as an inter-subject unifier likewise rendering it differentiated and discrete from its relata. Hence, a necessary and sufficient criterion for an entity being simple is the impossibility of any constituent being ontically combinatorial of another constituent. c) There are entities that have non-identical constituents yet have no internal divisions since none of the constituents are themselves ontic predicates, e.g., relation instances. d) And hence, the term 'simple' is to be seen as not the contradictory of 'composite', but rather as equivocal between the non-composite or 'absolutely simple', e.g., the intension Red^1, and the composite, e.g., the relation instance $\text{Red}^1{}_i$, the latter properly termed the 'continuously simple'.

2. Historical Context: Realists vs. Nominalists on Continuous Composites

Historically, the controversy over the possibility of continuous composites stems directly from differing accounts of ontology's central *Triple Aspect*

[18] F. H. Bradley, *Appearance and Reality*, 2d ed. (1897: reprt. ed., Oxford: Clarendon Press, 1966), pp. 404, 467, 509-11, 521; and Bradley's 'Relations' in *Collected Essays*, Vol. 2 (Westport: Greenwood Press, 1970), pp. 648-50, 660, 663-64.

Problem[19]: How is it that apparently unrepeatable ('non-communicable') particulars (whether as ordinary 'substances', e.g., Socrates, or as individuated attributes, i.e., instances or tropes) can *possess* apparently repeatable ('communicable') qualitative contents or intensions that characterize them and make up part of their being? How an ontology interprets predicable 'possession' is correlative with its theses on what of the apparent unrepeatable and repeatable aspects of an entity are real. Every individual is of one or more kinds (types, categories), F, G, H, …, and it is as an individual that it is distinct from every other individual of any kind, and being of kind F it is in some sense the 'same as', and so grouped as like, every other individual of kind F but distinct from every other individual of kind G contrary to F. That we understand this implies that we can at least cognitively distinguish between what is an individuating aspect and one or more qualitative contents or intensional aspects of individual entities. The question is whether there is a real and extra-conceptual distinction in the particular that corresponds to this distinction between abstractions? Essential to their positions, realists are required to admit such real distinctions *a parte rei*, whereas nominalists cannot allow them.

Realists advance a real distinction in recognizing constituent repeatable intensions, but differ on the nature of their union with what individuates the entity they characterize. The standard options exercised by realists have been to construe an ordinary 'thick particular', e.g., an apple, either a) as a bundle of property intensions, b) as intensions predicably attached to an underlying individuating substratum, what must be at some atomic level a bare particular, or c) as a fusion 'tighter' than any ontic predication between the intensional and individuating aspects of an entity. For much of the tradition the union described in b) was between a substantial form and prime matter, with all other properties and accidents predicably attached to the resultant substance. Options a) and b) require articulated composites, where for a) the constituent unifier is the posited 'Compresence^2' relation, and for b) the unification is provided either by the predication of the intensions themselves of the substratum, or, when the intensions are considered combinatorially inert, by the classic mediating relation of 'Exemplification^2' or 'Instantiation^2' linking them to the substratum. Option c) is, first of all, indirectly motivated by strong arguments against a) and b). They are principally: Against a) there is the fact that any bundle of universals is itself universal and so cannot account for a particular's unrepeatability. Fur-

[19] See Mertz, 'Combinatorial Predication'.

ther, bundle theory implies that the Principle of the Identity of Indiscernibles is a necessary truth.[20] Against b) is the following argument. In the context of bare particulars an ordinary thick particular *a* is understood in such a way that at least the intensions P^1, Q^1, …, that are essential to the defining essence of *a* and are ontic predicates of *a*, i.e., where $P^1(a)$, $Q^1(a)$, …, are true, are constitutive of *a* in the sense that *a* is a complex whole consisting of P^1, Q^1, …, as each is non-predicably 'tied-to' the same individuating bare particular p_a, i.e., where it is true that Tied-to$^2(P^1,p_a)$, Tied-to$^2(Q^1,p_a)$, … Significantly, the Tied-to^2 relation implies non-ontic-predication, i.e., for every intension F^1, Tied-to$^2(F^1,p_a) \supset \neg F^1(p_a)$. Now, bare particular p_a itself seemingly has properties essential to it, e.g., Unrepeatability^1, Simplicity^1, etc., so that propositions Unrepeatability$^1(p_a)$, Simplicity$^1(p_a)$, etc., are all true. But now the above analysis applies to particular p_a just as it did to particular *a*, i.e., p_a is a complex consisting of intensions Unrepeatability^1, Simplicity^1, etc., tied-to some bare particular p_a', i.e., Tied-to^2(Unrepeatability^1,p_a'), Tied-to^2(Simplicity^1,p_a'), etc. And like before, Tied-to^2(Unrepeatability^1,p_a') $\supset$ ¬Unrepeatability$^1(p_a')$, and similarly for Simplicity^1, etc. Now, if $p_a \neq p_a'$, there results a vicious infinite regress of further and further bare particulars, p_a'', p_a''', … Alternately, if $p_a = p_a'$, then not only would we have the untoward situation of a bare particular being a constituent of itself, but also we would have contradictions such as Unrepeatability$^1(p_a)$ and ¬Unrepeatability$^1(p_a)$. The last defense is to say that bare particular p_a has no properties essentially, but this is to say that p_a has no essence/nature, and is thus *no*thing, i.e., it evaporates into incoherence.[21]

A related and equally serious problem with bare particulars is their inability to found in a non-arbitrary manner relations (and thus properties) which have them as supposed relata.[22] An additional argument often brought against option b) is Bradley's Regress, though I contend its relevance is in-

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[20] Michael Loux, *Substance and Attribute* (Dordrecht: Reidel, 1978), pp. 117-19, 131-34. Also, Loux, *Metaphysics: A Contemporary Introduction* (New York: Routledge, 1998), pp. 106ff.

[21] For some of the recent dialectic concerning bare particulars see the references in note 2. I note that Davis in ''Partially Clad' Bare Particulars Exposed' argues that Moreland's defense of bare particulars implies that bare particulars must be constituents of themselves.

[22] Ibid. This point is made by Armstrong in regard to the Resemblance^2 relation in the context of criticizing resemblance nominalism. See his *Universals: An Opinionated Introduction* (Boulder: Westview Press, 1989), pp. 43-45.

direct: the regress has to do with the link between the combinatorial agency of an ontic predicate and the predicate's intension that controls/delimits this agency, and the fact that this agency is unrepeatable makes it relevant here, what will be made explicit below. I shall also return below to arguments against bare particulars. The net effect of these arguments is to force realists to conclude that the union between a particular's individuator and its qualifying intensions is one not effected by an interposing ontic predicate, i.e., the union here is not that of an articulated composite. The contradictory nature of these aspects, i.e., unrepeatability vs. repeatability, prevents their identification and requires any entity they jointly make up to be composite, though with a union that can only be a fusion in the manner described above for a continuous composite. Armstrong, for example, concludes that "Obviously, we can and must distinguish between the particularity of a particular, on the one hand, and its properties (and relations), on the other. But it is a distinction without relation."[23] Other realists have called this tighter-than-predicational-unity a 'non-relational tie' (P. F. Strawson[24]) or 'nexus' (Gustav Bergmann, Herbert Hochberg[25]).

Prior to these contemporary views but in stronger and, I propose, more accurate and insightful terms, Scotus described this union between the formalities of a *natura* (*quidditas* ('whatness') or 'specific difference') and a *haecceitas* ('individual difference') making up a particular as a *per se* unity. Here the resultant intension/individuator whole is "one thing which is virtually or pre-eminently as it were two realities."[26] Elsewhere he asserts: "The whole to which this unity belongs is perfect of itself", the two aspects together being "*per se* one", i.e., intrinsically one.[27] In this sense the whole would be simple, what Scotus would seem to imply in distinguishing it in "kind", i.e., as a different species of simplicity, from the "perfect divine simplicity", where, because the attributes of God are each formally infinite, they can include *each other* "through an identity".[28] Importantly, Scotus is explicit in taking the intension/individuator union to be

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[23] Armstrong, *Nominalism and Realism*, p. 111.
[24] P. F. Strawson, *Individuals* (London: Methuen, 1971), pp. 167ff.
[25] Gustav Bergmann, *Realism* (Madison: University of Wisconsin Press, 1967), pp. 9, 42ff.; Herbert Hochberg, 'A Refutation of Moderate Nominalism', *Australasian Journal of Philosophy* 66 (1988): 188-207.
[26] The description is translated and cited by Allen Wolter, *The Philosophical Theology of John Duns Scotus* (Ithaca: Cornell University Press, 1990), p. 46.
[27] Scotus, *Ordinatio II*, in *Five Texts*, p. 101.
[28] Ibid., p. 108.

that of a *composite*, though different from composition "proper" which is between "'thing' and 'thing'".[29] As standard composition the latter is presumably of constituents that remain differentiated and discrete in making up a plural whole, an entity that emerges through the mutual contributions of ontically prior parts *qua* differentiated parts. That is, the external differentiation, discreteness, or otherness—as "'thing' and 'thing'"—of the parts from each other is as much a contribution to or a determinate of the essence of this type of plural composite as is the internal essences of the parts. The further insight urged herein is that a differentiation/discreteness of parts *qua* parts of a whole (a standard composite) mutually implies the existence of at least some parts being unifying ontic predicates among other parts of the whole. Scotus would seem to intimate this thesis when he says that "[ontically predicable] form is more principally that by which something is a [proper] composite than the matter is, so it is more principally that by which a composite is one."[30](my inserts) Succinctly, the point I would urge is that discreteness of parts requires a constituent combinator to bridge the ontic gap between them, and conversely, the absence of this unifier among yet distinguishable parts marks a non-standard composite. Prior to this insight and using Scotus' analytic tool of identity, in describing a whole of discrete parts it makes no sense to speak either of identity between the parts, or of the parts melded into identity in the whole. In contrast, Scotus asserts that the "less proper" composition between an intension and an individuator has these two "realities" as "*quasi per se* parts",[31] in the sense that it "includes both of them through an identity." According to Scotus this identity is not between the nature or intension and the individuating difference, but between them and the including whole. Now, for this to be coherent the identity here can only be between each of these aspects and their respective distinct portion of what, as simple, is nevertheless a non-differentiated (non-divided) 'perfect' whole—what I have labeled above as a continuous composite. Here as with all composite wholes the constituents are ontically simultaneous with the whole, but unlike with articulated composites where the whole emerges from components connected or organized by one or more constituent ontic predicates, in a con-

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[29] Ibid., pp. 107, 108.
[30] Ibid., p. 113.
[31] Ibid., p. 108, where Spade translates the phrase as "as it were *per se* parts", whereas "as *quasi per se* parts" is the translation of R. L. Kilcullen in his 'John Duns Scotus, *Ordinatio*, II, dist. 3, pars 1.', 1996, found at URL = <http://humanites.mq.edu.au/Ockham/wjds.html>. I take "*quasi*" to be Scotus' term but I have not been able to verify it.

tinuous whole the constituents emerge as differentiated/discrete from the whole as the result of external abstraction—the formal distinction. The analogy here is perhaps of two different visual perspectives on a single object, the different content of each representing in a partial way what in itself is one and the same continuous entity. There are no internal demarcations or ontic gaps between what would otherwise be differentiated parts as they make up the whole, and because of this 'non-otherness' among the parts *qua* parts Scotus was lead to describe their union in the whole in terms of 'identity' (*idem* = same) in the sense that 'sameness' is synonymous with 'non-otherness'. As unbroken and continuous the intension/individuator whole can yet be heterogeneous in having internal distinctions—non-identical constituents—as, say, among the colors in the above given example disk that continuously change from red to green across its surface. Though Scotus asserts it in a different context with a different sense, he would have its analog apply here: "[In some wholes] the distinguished [i.e., non-identical constituents] need not be absolutely diverse [discrete as parts]."[32](my inserts)

Nominalists, by contrast, reject the coherence of the very concept of a continuous composite, and, with the rejection of repeatable intensions under their defining thesis that every entity whatsoever is individual, are not theoretically pressured to posit such composites, or so they think. A nominalist can hold without apparent contradiction, and indeed must hold, the thesis that it is possible to make a cognitive distinction differentiating the particularity and qualitative content of a particular *x* and yet this differentiation of aspects correspond to no distinction intrinsic to *x*. That is, a viable nominalism must recognize an atomic ontic level of at least minimally thick particulars—particulars with some qualitative content—that yet have no composition *in re*. The view is explicit in Campbell's defense of nominalistic trope theory: "To avoid such elements [bare or 'thin' particulars], we must deny that in the ontic structure of an individual is to be found any non-qualitative element."[33] Campbell states elsewhere: "We must construct an ontology which does not accord the particularizing role to one sort of being, while attributing sortedness (quality) to another. We require one item with

[32] Scotus, *Ordinatio II*, in *Five Texts*, p. 106.
[33] Keith Campbell, 'The Metaphysics of Abstract Particulars', in *Midwest Studies in Philosophy, Vol. VI, The Foundations of Analytic Philosophy*, ed. P. French, et al. (Minneapolis: University of Minnesota Press, 1981), 477-88, p. 482.

both roles."[34] These atomic items—tropes—do not simply have natures or intensional contents, they are each a particularized nature but without a duality of being.[35] They are necessarily so in order to found the Resemblance^2 relation among some tropes and not others, and so in turn account for the fact that some tropes and not others are non-arbitrarily 'of the same kind'. The founded Resemblance^2 relation, e.g., as in the fact :Resemblance^2(Red^1_i, Red^1_j), is held to eliminate the need to posit with the realists a numerically identical characterizing constituent in each of the resembling tropes, e.g., Red^1 numerically the same in both Red^1_i and Red^1_j.[36]

Supporting the nominalists' necessary rejection of continuous composites is their appeal to both the pre-critical intuitiveness of the contradictory to Scotus' position that the distinguished do not have to be absolutely diverse or discrete, as well as the putative explanatory success of a nominalist ontology without continuous composites. Important here because of their explicit attention to the first claim are the medieval scholastics Ockham and Suarez, Ockham a conceptual nominalist and Suarez a resemblance ('similarity') nominalist in the manner of Campbell.[37] For example, against Scotus' analysis Ockham assets that "In creatures there can never be any distinction outside the mind unless there are distinct things; if, therefore, there is any distinction between the natures and the difference, it is necessary that they really be distinct things."[38] And, "Therefore, one should grant that in created things there is no such thing as a formal distinction. All things which are distinct in creatures are really distinct and, therefore, different things."[39] Later and also in the context of criticizing Scotus, Suarez likewise asserts the contradictory of Scotus' thesis. He states, "All objects which we conceive as two entities are either really the same or are really other. If they are really other they are really distinct"[40], where by 'really distinct' he understands differentiated and discrete as

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[34] Keith Campbell, 'Abstract Particulars and the Philosophy of Mind', *Australasian Journal of Philosophy* 61 (1983): 129-41, p. 129.
[35] Campbell, *Abstract Particulars*, p. 57.
[36] Ibid., pp. 30-34, 59-60.
[37] Evidence of Suarez's similarity nominalism is found at *On Formal and Universal Unity*, *Disputatio VI*, pp. 30, 36, 47-48.
[38] Ockham, *Ockham's Theory of Terms*, p. 82.
[39] Ibid., p. 84. These points are reiterated by Ockham in the *Ordinatio*, translated in Paul Spade, *Five Texts on the Mediaeval Problem of Universals* (Indianapolis: Hackett Publishing Co., 1994), pp. 156ff.
[40] Suarez, *Various Kinds of Distinctions, Disputatio VII*, p.22.

"thing and thing"[41], as "two altogether separate things or entities"[42]. Succinctly then, what Ockham and Suarez are asserting is that any real distinction, any non-identity, internal to a single (created) entity *x* implies *x* is a plural entity of discrete parts. This implies on the extended analysis herein that to be composite at all is to be an articulated composite or complex in the above precise sense. But even prior to this explication it implies that the individuating and qualitative aspects of a particular *x* cannot be distinct (non-identical) in *x*, for otherwise they would be differentiated and discrete in *x* and so requiring, on the one hand, the individuator be a bogus bare particular, and on the other and violating nominalist doctrine, that the intension be repeatable, i.e., a universal, since if it were unrepeatable *x* would have two individuators and hence be two particulars and not one. To the contrary, this latter observation together with the demonstration below that the individuator and qualitative aspects of an ontic predicate cannot be identical will be used to demonstrate the necessity of universals.

The fact that for a nominalist every composite whatsoever is a complex does not mean that the latter would have been defined by the above referenced nominalists (and on the argument below could consistently be defined by any nominalist) in the manner given in the introduction—as networks of entities linked by polyadic and thus interposing relations (even if the relations are treated as individuated attributes or tropes). Such a description was unavailable to Ockham and Suarez, and indeed to most Western philosophers up until recent times. The common assay of entities of yet discrete parts in the influential Aristotelian/scholastic tradition specified that a single constituent be *in act* as a unifier, i.e., as a combinatorial *agent*, relative to the other constituents (patients) that are *in potency* to its agency (*Meta*. 1045a20-25, b16-21), what was identified as either a single substantial or accidental *form*.[43] As Aristotle rightly observed and the tradition concurred, an articulated whole, e.g., a syllable, flesh, a house, a property qualifying a subject, must have, in order to avoid Bradley's vicious regress, a constituent whose ontic role relative to the whole is other than that of just another element to be unified (*Meta*. 1041a6-b33, 1043b5-14; also see 1045a7-19, 1040b7-10). This constituent must have the nature of a *cause* or

[41] Ibid., p. 21.
[42] Ibid., p. 27.
[43] Aristotle, *Metaphysics* 1045a6-b24; Ockham, *Ockham's Theory of Terms*, p. 170; Francis Suarez, *On the Formal Cause of Substance (Disputationes Metaphysicae, Disputatio XV*), trans. J. Kronen & J. Reedy (Milwaukee: Marquette University Press, 2000), pp. 177-78.

principle of the unity among the other elements relative to the whole—it must be an *agent unifier* interconnecting the other separated as differentiated elements. Significant however as a source of error, the form when unifying multiple subjects (e.g., secondary matter, as say bodily organs or parts of an artifact) was *never* conceived as a polyadic relation, this witnessed by the fact that forms were always monadic in intension (e.g., Man, House).[44]

Though the act/potency account of articulated composites given by Aristotle was in a context where a modern would acknowledge real and interposing polyadic relations, for Aristotle and most of the tradition polyadic relations were considered necessarily reducible to monadic properties of their relata, this reduction strategy pursued recently by Campbell.[45] Indeed, the distorted Aristotelian act/potency account is precisely the analytic residue of what is the agent-unifier (combinatorial) nature of relations erroneously reduced to single-subject properties of their relata. Specifically, a dyadic causal relation is taken as equivalent jointly to a monadic property of actuality in an agent correlative with a monadic property of potency in a patient. Telling of the error here is the necessity of using 'correlative' terms which shows that there is no elimination of cross-subject linking, and for this to be non-arbitrary it cannot be a 'bare linking' and therefore it must be controlled by a polyadic intension. More on this below. The monadic reduction of relations was abetted by the equally insidious and classic containment or inherence model of ontic predication, where a subject is conceived as 'containing' its properties analogous to a jar holding its contents (e.g., Aristotle, *Meta*. 1023a7-16; *Cate*. 15b16-30). The model is plausible if ontic prediction is in every case (in every fact) the qualification of only a single subject, i.e., facts of the form $:P^1(a)$, *and* if what is indeed an inert non-unifying (non-predicable) intension P^1 is confusedly identified with the subsuming unifying predicate $P^1(x)$, for then there is no compelling reason why intension 'ontic predicate' P^1 is any more a unifier for fact $:P^1(a)$ than subject *a* would be. Further, given the appealing and historically consequential maxim that 'Unity is by the (shared or common) unit or one', and the fact that multiple properties are unified together as they characterize a single subject *a*, it is easy to mistake this for proving that *a* is the cause of

[44] Jeffrey Brower, 'Medieval Theories of Relations', *The Stanford Encyclopedia of Philosophy* (Summer 2001 Edition), Edward N. Zalta (ed.), URL = <http://plato. stanford.edu/archives/sum2001/entries/relations-medieval/>.

[45] See Campbell's *Abstract Particulars*, pp. 97ff. Campbell admits certain relations are not reducible to foundations in their terms in his 'Unit Properties, Relations, and Spatio-Temporal Naturalism', *The Modern Schoolman* 79 (2001): 151-62.

the unity with each of its properties (like a jar holding multiple stones). The temptation to error in this way is removed when multi-subject polyadic relations are recognized as real and irreducible, for in a relational fact :$R^n(a_1,a_2,\ldots,a_n)$ it is obvious that the polyadic predicate is what is the single 'common unit' among multiple subjects, and so would be by the maxim the cause of the unity effecting the fact. [Importantly, this maxim is false in its scope in that there are also 'horizontal' and 'vertical' compositions of yet discrete entities as explained in Essay 2.] The *locus classicus* for demonstrating the irreducibility of polyadic relations to monadic properties is Bertrand Russell's arguments in *The Principles of Mathematics* (1903)[46], though the full ontological significance of the unreduced interrelata linking nature of polyadic ontic predicates has yet to be generally appreciated.

Now immediately relevant and telling is the fact that, historically, the agent-unifier nature of an ontic predicate was recognized, in the limiting class of monadic properties, and, ironically, even by scholastic nominalists, e.g., Suarez and John Buridan, as forming with its concomitant intension what is termed here a continuous composite. The two aspects were held to exist virtually and to be distinguished only formally by what was variously called a *distinctio rationis ratiocinatae* (a 'distinction of the reasoned reason'), a *distinctio ex natura rei* (a 'distinction from the nature of the case'), or a 'modal distinction'.[47] A modal distinction exits between an entity and its mode, a principal example given being between a property and its *mode of inherence* in a subject. The mode of inherence of a monadic attribute is its agent-unifier aspect as distinguished from its delimiting intension, the former providing the union of the latter with a subject characterized by the attribute. Buridan calls the causal means of inherence of a unit property a 'disposition', and asserts that "Concerning the whiteness and the stone I say that it is necessary that there be an added disposition so that the whiteness may inhere in the stone…", and that, on pain of Bradley's Regress, no further disposition is needed to connect the first disposition to the stone and the instance of whiteness.[48] Further, Buridan maintains that, though

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[46] Bertrand Russell, *The Principles of Mathematics*, 2d ed. (1903: reprt. ed., New York: Norton, 1938), pp. 221ff. I have sought to reinforce these arguments against objections raised by Campbell in my *Moderate Realism*, pp. 163-71.

[47] See Suarez, *Various Kinds of Distinctions, Disputatio VII*, pp. 18ff.

[48] Calvin Normore, 'Buridan's Ontology', in *How Things Are*, ed. J. Bogen & J. McGuire (Dordrecht: Reidel, 1985), pp. 180-203. Normore here translates from Burdian's *Questiones in metaphysicam aristotles*, V, q. 8, fols. 31, 33.

subjects and their properties (which he treats as particulars) can exist apart, not even God can separate the inherence disposition of a predicable property from the property (i.e., intension), for otherwise Bradley's Regress would result.[49] More developed in Suarez, he asserts, "In quantity, for example, that inheres in a substance, two aspects may be considered: one is the entity of the quantity itself [the intension itself], the other is the union or actual inherence of this quantity in the substance."[50](my insert) According to Suarez, here *a particular case* of inherence is a mode of the quantity, i.e., the *union* itself of a property (intension) with its subject is *unrepeatable*, a consequential insight to be developed below. The distinction is "in the real order" but "less than a real distinction", i.e., one "not so great as the distinction between two altogether separate things."[51] That this union between an entity and its mode is very similar to that of a continuous composite as characterized above is evident from Suarez's description: "A mode is not, properly, a thing or entity. Its imperfection is clearly brought out by the fact that it must invariably be affixed to something else to which it is *per se* and directly joined without the medium of another mode, as, for instance, sitting is joined to the sitter, union of the things united, and so of other cases…."[52] Specifically, then, for a property intension and its 'mode of inherence' in (i.e., its ontic predicability of) a subject, they are distinct but 'directly joined without the medium of another mode', i.e., without a further mode of what would be here at least a dyadic (relational) 'inherence'. Suarez also at least implies that if it were otherwise then Bradley's Regress would result.[53] As 'directly joined' the two aspects are seamless and without ontic gap in the manner of a continuous composite, which stands in contrast to their being a composite of 'really distinct' constituents requiring as such a gap-bridging 'medium of another mode', i.e., an ontic predicate, what results in an articulated composite as defined above.

In sum, the important thesis advanced by both Buridan and Suarez is that there are *two types of union involved in ontic predication*: one is the internal union between an intension and its unifying agency ('disposition' or 'mode of inherence') that is 'so tight' as to compose a single undifferentiated entity (i.e., an ontic predicate), and the other is the external union between the latter composite and a subject that is thus characterized/qualified,

[49] Ibid., pp. 197-98.
[50] Suarez, *Various Kinds of Distinctions, Disputatio VII*, p. 28.
[51] Ibid., pp. 26, 27.
[52] Ibid., p. 31.
[53] Ibid., p. 29.

what jointly form a plural differentiated composite that is a fact. And, in forming such a monadic fact :$P^1(a)$ (the only type then recognized), if the former union is confused with the latter, then the mode of inherence is taken as discrete from intension P^1 just as these two jointly are discrete from subject *a*, and this requires that the mode of inherence be either a dyadic unifier having P^1 and *a* as subjects, or that it be a predicably inert subject similar in ontic status in the fact to subject *a* and requiring a further mode of inherence for P^1 to join itself to the first mode of inherence. With polyadic unifiers disallowed, Buridan and Suarez were left to observe that confusing these two types of union precipitates a vicious regress of further and further presupposed (monadic) modes of inherence. The issues at play here will be made more transparent below.

3. The Nature of Ontic Predicates

The above historical survey has tied the existence and nature of continuous composites to both the problem of individuation and the nature of ontic predication. This is no accident, as we shall now see. For, ontic predication properly understood is an *intension-determined unifying agency*, and it is the combinatorial act here that is for ontology a *principium individuationis*, while in composing an ontic predicate the union between a specifying intension and its concomitant unification to and among subjects is that of a simple continuous composite. An ontic predicate is simple in the straightforward sense of having no internal divisions as evidenced by the absence of constituent agent unifiers, the latter otherwise correlative with plural composition and so internal differentiation.

There are, I shall argue, three principles that explicate the intension-relevant unity that is ontic predication. When generalized to predicates of any number of subjects these principles are as follows:

Principle I:

Constitutive of every fact :$R^n(a_1,a_2,\ldots,a_n)$, for $n \geq 1$, is an ontic predicate, $R^n(x_1,x_2,\ldots,x_n)$, that is the agent/cause of the characterizing predicable unity of itself with its relata, a_1, a_2, …, a_n, a unification whose type is to result in a fact, as opposed to a list, set, or mereological sum.

Principle II:

Every ontic predicate $R^n(x_1,x_2,\ldots,x_n)$ has as a constituent an intension R^n whose ontic role is that of delimiting or determining non-arbitrarily the possible *n*-tuples of relata, $<a_1,a_2,\ldots,a_n>$, that predicate $R^n(x_1,x_2,\ldots,x_n)$ can unify into a fact, but the intension of itself has no causal agency whatsoever as a unifier. The abstracted intension is 'predicably inert' and 'substance-like'.

Principle III:

In addition to and distinct from intension R^n, there is constitutive of ontic predicate $R^n(x_1,x_2,\ldots,x_n)$ its actual mode of union, its combinatorial or linking agency, among and to its subjects. The linking aspect of predicate $R^n(x_1,x_2,\ldots,x_n)$ is itself not a further intension in addition to R^n, but a *causal act of unification* that is 'joined' with intension R^n that controls its effects. This joining is the unity of a *continuous composite*, i.e., a union of two distinct entities without the agency of a further interposing ontic predicate or act of unification. Moreover, the unifying act of an ontic predicate is unrepeatable and particular, rendering the containing predicate an individual, i.e., a unit attribute.

The analysis that yields these principles starts first in broadest terms with the fact that a given of our experience is the existence of a myriad of structured wholes—articulated composites—each as such having constituents in one or more *types* or *kinds* of inter-connectedness or organization, e.g., cognitive, physical/mechanical, and social structures. In such complexes, entities *and their mutual qualitative connections ('orderings', relationships, arrangements)* jointly contribute to the existence and nature (specific essence) of the whole. That is, the being of a structure, whether, say, as a dynamic physical system (e.g., an operating engine) or a static formal one (e.g., the Natural Number System), is a function of the mutual qualitative co-relevance of both the intension contents of the constituent unifying relationships and the compatible natures of their respective subjects, and as the former orders the latter. The simplest such or atomic structured whole would be one instance of one kind of intensioned connection or unification among one *n*-tuple of other constituents. This is a fact or state of affairs, $:R^n(a_1,a_2,\ldots,a_n)$, e.g., $:Red^1(a)$, $:Contiguous\text{-}with^2(b,c)$, $:Owes^3(d,e,f)$ (as in '*d* owes *e* to *f*'), whose arrangement-kind is intension R^n, in the examples, respectively, Red^1, $Contiguous^2$, Owe^3. Here the subjects, a_1, a_2, …, a_n, are linked and ordered (if any) into a resultant fact $:R^n(a_1,a_2,\ldots,a_n)$ according to intension R^n, though, on the analysis below, not *by* the intension R^n.

In particular and observed at least as far back as Aristotle (though misconstrued in terms of forms), it is the *prima facie* nature of a polyadic relation that it have the role of a *cause* or *principle—an agent*—of the unity of itself with its relata in forming a fact, and by extrapolation a role likewise but less obvious for the limiting case of a monadic property in its fact. To sharpen this intuition, and to prepare for a reply to a previous challenge, consider first causality in general. An agent/cause is so characterized because it 'brings about', is 'responsible for', or 'produces', the existence and nature of a further and distinct reality beyond (non-identical with) itself—the effect. Other entities or 'patients' (e.g., subject relata) may be needed for the effect (e.g., a structured, intension/essence-dependent unification among relata) but their existences and natures independent of the cause are insufficient for the reality of the effect. This is what is meant by an entity having causal 'power': an agent/cause can 'go beyond itself'—in what is a causal 'act'—and be both a sufficient condition for the existence of, and a conditioning or specifying of the nature of—the qualitative content of—a separate reality. In a temporally neutral sense a cause *qua* cause 'goes beyond itself' to produce something different from itself. This is so whether the act is, for example, a single temporal event or an atemporal state as in the unity of a necessary fact (e.g., :Prime-Proper-Divisor-of^2(3, 6)). A moving billiard ball as cause effects by an act of collision (in the act itself it is a cause in the proper sense) the wholly new reality of the specific motion of a struck billiard ball. In contrast, the 'going beyond itself' nature of a causal agent is absent in the Humean reduction that treats physical causation as mere temporal succession, or causation in general as mere conjunction. A world of radically isolated and other-indifferent/mutually-irrelevant entities, other than being at most temporally juxtaposed (which itself is a relational structure), is contrary to the pervasive given of our experience that involves the productive nature of causation. More particularly and a key point herein, all relatedness, whether physical or 'metaphysical', involves causation in that all relations as ontic predicates 'go beyond themselves' to form trans-subjects unifications, and a Humean universe devoid of causation is a universe without relatedness, which is counter-factual. That motion M_1 is the cause of motion M_2, i.e., that there is a fact :Efficient-Cause-of^2(M_1,M_2), is not reducible to the set $\{M_1,M_2\}$ which lacks any ordering structure, nor is it reducible to the temporal fact :Precedes^2(M_1,M_2), since in the latter the relation Precedes^2 involves an unreduced causation of its own, as detailed presently. The Humean analysis has the plausibility that

it does only in a tradition that reduces relations to properties whose causal nature (as unifying themselves to their subjects) is least obvious.

Consider now specifically what is the necessity and nature of a unifying cause of a plural whole. Just the existence of each of multiple entities is not sufficient for the existence of a whole containing them, contrary to, say, mereological universalism. For, if it were otherwise, since all entities are equal in their status as existents there would be only one whole—the universal whole W—containing everything that exists. Any sub-whole must require something other than just the existences of its elements to differentiate it from W, and hence this something would have to be such as to limit the elements to just those making up the non-global whole. Because extensional existence is not enough to provide it, a whole limited to just certain elements would have to be by other means—what is both a cause of unification among and a delimiting of the union to just these elements. The alternatives are unions by means either a) external to and independent of the qualitative natures of these elements, or b), to the contrary, materially relevant to and so correlative with the internal essences of the elements. In either case, a unifying/delimiting something must 'go beyond itself' in order to link via itself each and all of just the contained constituents. Under a) wholes are the result of arbitrary grouping indifferent to the natures of the grouped entities, what must presuppose as such the free selections and associations (willed or not) of a mind. Here the whole would be either an actual conceptual entity generated by a particular mind, i.e., particular lists, sets, or mereological sums, or the 'Platonic projections' from such: the posited possibilia implied by a formalized theory idealizing the results of these cognitive operations while abstracting away their conceptual origins and extrapolating beyond human limitations, i.e., the entire theoretical realms of such extensional entities implied by set theory and mereology (e.g., see Philip Kitcher[54]). (One might recall here Cantor's appeal to the mind of God to guarantee the realm of infinite sets.[55]) In contrast, under b) a whole exists by a limitation intensionally determined and so internally relevant to the natures of just these entities taken jointly. That is, here there is a particular union controlled in its extent by a specific intension that is qualitatively relevant to the elements mutually and not just singly, and in a way that delimits this relevance to exactly this number of elements. Intuitively, this is pre-

[54] Philip Kitcher, *The Nature of Mathematical Knowledge* (Oxford: Oxford University Press, 1984), pp. 108ff.
[55] See Michael Hallett, *Cantorian Set Theory and Limitation of Size* (Oxford: Clarendon Press, 1984), pp. 34-37.

cisely the categorical nature of polyadic ontic predicates, i.e., relations-as-they-exist-in-facts, facts being the basic structures of the world, both extra-conceptual and conceptual. For example, in the fact :Prime-Proper-Divisor-of^2(3,6), the intension Prime-Proper-Divisor^2 specifies by its very content exactly two relata per its fact-forming role, and where the natures of relata 3 and 6 are mutually pertinent to the intension Prime-Proper-Divisor^2 only as they are paired, and indeed ordered as pair-ed. It is plausible that even the extensional wholes of type a) above exist via cognitive relations of a limiting type that have specific intensions, e.g., Associated-by-Mind-m^2, which are indifferent to the qualitative natures of their relata, as in fact :Associated-by-Mind-m^2(cabbage *a*,square root of 2), again this indifference being a sign of their cognitive status. For the same reason, this cognitive status would also extend to 'trivially essential properties' like Being-Colored-if-Red or Being-Odd-if-Identical-to-3 discussed in the context of bare particulars and where the latter in having such properties supposedly signals an unobserved subtlety in their characterization as 'bare of properties'. Having a conceptual reality only, with no existence *in re*, these properties tell us nothing about the nature of bare particulars nor somehow make their existence palatable.

In general and fundamentally, it is the intuitive nature of relations that in facts they are trans-subjects unifiers—they *act-to-unify* their relata. Even the Aristotelian/scholastic tradition hostile to polyadic relations observed their causal character in 'going beyond themselves' by classifying them as having uniquely an *esse ad*, or 'being toward' quality, what implies equally a character of being 'toward something' (*ad aliquid*). The two together imply a completed 'bridging' between related subjects, the basis for the medievals characterizing a relation as an 'interval' (*intervallum*).[56] Indeed, it seems plausible that the interval nature of a relation as an ontic predicate and among discrete relata implies a holding-apart of its subjects even as it holds them linked (together-at-a-distance), on the analogy of a rigid connecting rod, and what is essential to characterizing the resultant as an articulated composite. In this polyadic predicates would not only mark, but also enforce an ontic division between their relata in jointly forming their respective facts. The idea here is that there is no plurality of entities without a discreteness enforced by interposing relations, as there are equally no multi-subject relations without a plurality of discrete relata to be joined. Without multi-subject relations articulated wholes would at best col-

[56] See Brower 'Medieval Theories of Relations', and Mark Henninger, *Relations: Medieval Theories 1250-1325* (Oxford: Clarendon Press, 1989).

lapse into continuous simples. At any rate, a principal point herein is that a composite free of such internal intermediaries and hence of the divisions they mark is *simple* in a proper sense. Now, as predicable, it is a relation's correlative agent-unifier and bridging roles in a fact that Russell, in response to Bradley, characterized as the 'actually relating' of the relation, and what Bradley termed, respectively, a relation's being a 'together' and a 'between'.[57] Bradley thought these two characteristics were jointly impossible because a relation as a 'between' has no unifying agency and so cannot be a 'together', what is purportedly highlighted by the regress argument that bears his name and that we shall consider below. It is the joint combinatorial and interposing roles among relata that is meant by referring to a relation as an 'ontic predicate', what is symbolized here in general form as '$R^n(x_1,x_2,\ldots,x_n)$'. Given relations thus properly defined, *they can occur only in facts*, $:R^n(a_1,a_2,\ldots,a_n)$, in that they presuppose elements to be unified/bridged, i.e., a subject *n*-tuple $<a_1,a_2,\ldots,a_n>$. Unification presupposes as mutually dependent both unifier and unified. This causal nature of a relation as it occurs in a fact is reinforced by the previously mentioned classic maxim properly modified: All unity among the yet distinct is by a shared chain of connectedness.[58] In the limiting case of a single fact, say with a polyadic relation, e.g., $:Owes^3(d,e,f)$, it is obvious that what is 'shared across' the other constituents as their common 'unifying thread' is the relation—subjects *d*, *e*, and *f* jointly share the relation having intension Owe^3.

It is additional evidence of the causal-agent nature of ontic predicates that they are in themselves incomplete in a way that makes them ontically dependent on other entities. This is precisely the character a causal unifier would have to have in 'going beyond itself' to effect a whole of which it is the unity-contributing constituent. Specifically in regard to a causal unifier, its 'going beyond itself' involves an effected whole of both entities linked ('patients') and the linking agency aspect of the unifier. If in being aware of the whole one abstracts away the entities linked, the cognitive remainder is the unifier with its act of unification, what in itself is incomplete and requiring something else in order to exist, viz., the things that it acts upon to unify. That is, to cognitively focus via abstraction just on the ontic predicate $R^n(x_1,x_2,\ldots,x_n)$ of a fact $:R^n(a_1,a_2,\ldots,a_n)$, as such ignoring the fact's particular subjects, a_1, a_2, …, a_n, but continuing in the recognition of an

[57] Bertrand Russell, 'Some Explanations in Reply to Mr. Bradley', *Mind* 19 (1908): 373-78. Bradley, 'Relations' in *Collected Essays*, pp. 643-44.
[58] See Mertz, 'An Instance Ontology for Structures'.

agent and its agency that is their unification, is to focus on an entity in itself incomplete as to the conditions for its existence. An analogy would be that in regard to the whole that is an act of hand-clapping (strike and recoil), to abstractly focus on the motion of just one hand is to have something essentially incomplete, what is in need of the motion of the other hand in order to constitute the whole on which its existence depends—there is no clapping without two hands. Further and relevant to **Principle II**, to then abstract away and ignore the linking aspect of ontic predicate $R^n(x_1,x_2,\ldots,x_n)$ is to arrive at the analytic residue of the non-predicable intension R^n.

The incompleteness peculiar to ontic predicates has been referenced in the literature as Fregean 'unsaturatedness'[59], Seargent/Armstrong '*ways* that things are'[60], and in part by what the scholastics meant in recognizing an accident as having a type of being that is being-in-another (*ens in alio*), what they further understood as a defective reality relative to a subject substance whose being is being-in-itself (*ens per se*), the latter held necessary to support the former. I have argued elsewhere[61] that, contrary to the latter scholastic view which continues to be prevalent today, incomplete ontic predicates (as relation instances) can have as subject relata—be 'completed' by—other incomplete ontic predicates (as relation instances), and this pervasively at some atomic ontic level, what, for example, is apparently needed in an ontology for quantum physics. The mutual completing/support of incomplete/dependent ontic predicates nullifies a supposed vicious regress based upon the false assumption that ultimately incomplete entities must be sustained by a category of substances each with a 'full and complete' self-sustaining reality. It is evidence that the incompleteness of an ontic predicate stems from the predicate's ontically positive 'activity', i.e., its unifying causality, and so derives from a power representing a richness rather than a deficit of being (see Plato's *Sophist* 247e), as such not requiring it be parasitic upon and sharing in some other type of entity whose 'quantity of being' reaches a threshold of self-sustaining completeness (*ens per se*). Though anomalous to their Aristotelian substance/attri-

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[59] Gottlob Frege, *Translations from the Philosophical Writings of Gottlob Frege*, ed. P. Geach & M. Black (Oxford: Basil Blackwell, 1970), pp. 54-55.

[60] David Seargent, *Plurality and Continuity: An Essay in G. F. Stout's Theory of Universals* (Dordrecht: Martinus Nijhoff, 1985), pp. 113-115, 123. D. M Armstrong, *A World of States of Affairs* (Cambridge: Cambridge University Press, 1997), pp. 30, 99, 123-24, and his *Universals: An Opinionated Introduction*, pp. 96-97, 116.

[61] Mertz, 'An Instance Ontology for Structures'.

bute ontology, the view is in effect the conclusion the scholastics arrived at in analyzing the Trinity in terms of pure relatedness—each Person a relation between the other two, yet necessarily each Relation without a deficiency of being.

Further in support of **Principle I**, and pivotal to the import of Bradley's Regress argument, is the observation that a fact, $:R^n(a_1,a_2,\ldots,a_n)$, e.g., $:Loves^2(a,b)$, is not identical with an extensional whole, e.g., a list, set, or mereological sum, made up of the very same constituents, say, set $\{R^n(x_1,x_2,\ldots,x_n),a_1,a_2,\ldots,a_n\}$, e.g., $\{Loves^2(x_1,x_2),a,b\}$. First, a fact whose ontic predicate has a contingent intension R^n, e.g., $Love^2$, can come into and go out of existence, or never exist at all, independent of the arbitrarily generated existence of the corresponding list and certainly of the corresponding set that is held to exist atemporally 'always'. Further, with certain polyadic relations, e.g., asymmetric and non-symmetric relations, their intensions, R^n, e.g., $Love^2$ or $Prime\text{-}Proper\text{-}Divisor^2$, determine an order among the remaining constituents of a fact, but there is no ordering in the corresponding sets or sums. For example, if both facts obtain for $a \neq b$, then: $Loves^2(a,b) \neq :Loves^2(b,a)$, yet $\{Loves^2(x_1,x_2),a,b\} = \{Loves^2(x_1,x_2),b,a\}$. Finally but most relevant, a fact contains information about the subject relata, $a_1, a_2, \ldots, a_n$, singly and collectively, that the corresponding list, set, or sum does not, viz., that the subject(s) are characterized by the ontic predicate, and in particular when the latter is a polyadic relation that it *jointly* characterizes the subjects. Fact $:Loves(a,b)$ carries the information that *a* loves *b*, whereas the set $\{Loves^2(x_1,x_2),a,b\}$ does not. So, for every fact $:R^n(a_1,a_2,\ldots,a_n)$ we have a corresponding set with exactly the same constituents, $\{R^n(x_1, x_2,\ldots, x_n),a_1,a_2,\ldots,a_n\}$, but where in the set the togetherness of its particular elements is not a function of their essences or any qualitative aspects of them, but requires only their existences, and the cause of their togetherness is not a constituent of the whole but rather is an arbitrary association ignored in abstraction.

Now relevant to Bradley's Regress argument, for any set $\{R^n(x_1,x_2,\ldots,x_n),b_1,b_2,\ldots,b_n\}$, constituent ontic predicate $R^n(x_1,x_2,\ldots,x_n)$ presupposes for its existence an *n*-tuple of relata that it is agent-combinator for, perhaps even $<b_1,b_2,\ldots,b_n>$ whose constituents are elements of the set. However, the unity of the whole that is the set is itself not the unity effected by predicate $R^n(x_1,x_2,\ldots,x_n)$ on $<b_1,b_2,\ldots,b_n>$, for otherwise the resultant whole would not be the set, but rather fact $:R^n(b_1,b_2,\ldots,b_n)$. And obviously, the unity that is the set's is not that effected by the ontic predicate $R^n(x_1,x_2,$

…,x_n) among some other set of relata. Thus, predicate $R^n(x_1,x_2, \ldots,x_n)$ does not have the role of agent unifier for set $\{R^n(x_1,x_2,\ldots,x_n),b_1,b_2,\ldots,b_n\}$. Yet and importantly, this does not mean that $R^n(x_1,x_2,\ldots,x_n)$ does not have this role, and hence the nature of an agent-unifier, in some other whole, i.e., in some fact $:R^n(a_1,a_2,\ldots,a_n)$, and even in a fact $:R^n(b_1,b_2,\ldots,b_n)$. Now, Bradley's regress proceeds on the contrary assumption: that in comparing fact $:R^n(a_1,a_2,\ldots,a_n)$ with corresponding set $\{R^n(x_1,x_2,\ldots,x_n),a_1,a_2,\ldots,a_n\}$, since both wholes have exactly the same constituents and *ontic predicate $R^n(x_1, x_2,\ldots,x_n)$ is not the cause of the unity of the set, then predicate $R^n(x_1,x_2, \ldots,x_n)$ is not the cause of the unity of the fact*. Consequently, since the fact $:R^n(a_1,a_2,\ldots,a_n)$ requires some constituent unifier, on the assumption that this unifier is a further and implicit relation with intension R'^{n+1}, then $:R^n(a_1,a_2,\ldots,a_n) = :R'^{n+1}(R^n(x_1,x_2,\ldots,x_n),a_1,a_2,\ldots,a_n)$. But of course, the same analysis applies to the latter fact, and so on to vicious infinite regress. The alternative is to take the requisite unifier to be intensionless or devoid of qualitative content—a 'bare linking', what philosophers have adopted as a response to the regress under the terms 'non-relational tie' or 'nexus'. I shall rehearse below the incoherence of such a concept. Crucially, what motivates the error leading to either fork, and what brings us to **Principle II**, is the failure to differentiate a non-unifying (non-combinatorial, 'non-predicable') intension R^n, e.g., $Love^2$, from any subsuming unifying ontic predicate $R^n(x_1,x_2,\ldots,x_n)$, e.g., $Loves^2(x_1,x_2)$. An abstracted intension in itself is non-combinatorial in any context—it is causally inert—and if it is identified with the ontic predicate in a fact, then some other constituent of the fact must be found to unify the then 'predicate' (but not 'predicable') intension with the other subjects. This is the road to perdition. However, as we shall see, the prospect of Bradley's regress returns at the sub-ontic-predicate level in assessing the union between a composing intension R^n and a predicable agency, and it is at this point that we see the rationale for composite simples.

Turning now to the warrant for **Principle II**, consider first that though any arbitrary entities whatsoever are said to form a set or sum, only certain limited combinations of ontic predicates and subject *n*-tuples form a fact. This is so because the unity of a fact depends upon the non-arbitrary match or content-determined mutual relevance or qualitative agreement between the predicate's specific intension R^n and the determinate natures of (and order among, if any) the entities in the *n*-tuple. The dyadic predicate expressed, for example, by 'is a father of', i.e., $Father\text{-}of^2(x_1,x_2)$, delimits as its extension pairs including <Philip II,Alexander the Great>, but not <4,5>

or <Apple *a*,Orange *b*>. It is intensions that sort their ontic predicates into contraries and contradictories, and specifies the formal properties of polyadic relations, e.g., the ordering among relata for asymmetric and non-symmetric relations, or their transitivity or not across relata. The same point is established in the negative: if an ontic predicate has no qualitative constituent or intension determining/delimiting the range and ordering of its unifying causation, then it would be a 'bare unifier', analogous to and as illegitimate as a 'bare particular'.[62] An intensionless unifier would be absolutely uncontrolled and without limitation in its agency, both locally in the sense of allowing anything to be unified to anything else, and globally in requiring either nothing or absolutely everything be unifying at once—total reality—without differentiation into any sub-wholes of sets, facts, or complexes. Not only do ordered wholes, e.g., the spatial system that is the fact :Taller-than^2(*a*,*b*), go unaccounted for, but there is no reason why contrary (e.g., Green and Red) or contradictory (e.g., Transparent and Opaque) properties cannot be arbitrarily tied to the same subject, and this is contradictory to the nature of ontic predication.

Further and ontologically crucial is the point that intensions are in themselves non-combinatorial and so are not identical with their subsuming combinatorial predicates. This is seen most clearly in the fact that intensions exist for which there are no corresponding ontic predicates and so facts, e.g., Unicorn^1, Phlogiston^1, or the intensions Spouse^2 or Employer^2 in a world reduced to one extant human that retained the latter as abstractions. Likewise, intensions such as Orbiting^2, One-Meter-Apart^2, and Gravitational-Attraction^2 would exist in a possible universe where all physical/spatial entities are annihilated except one suitably reduced or primitive, but where a single intellect remained retaining the intensions as abstractions. In these examples the intension is in itself either a free creation of a mind or the result of an abstractive act with an existence as separated dependent upon that of a distilling and retaining mind, there being no extant agent ontic predicate of which it is the conditioning content. The processes of abstraction from fact to contained agent ontic predicate, and from the latter to contained agentless intension, are marked by variations on words and phrases in English. We can abstract from a state of affairs or fact, e.g., :Red^1(*a*), :Loves^2(*b*,*c*), :Father-of^2(*e*,*f*), or :Similar-to^2(*g*,*h*), expressed respectively by '*a* is red', '*b* loves *c*', '*e* is the father of *f*', and '*g* is similar to *h*', intensions expressed by abstract nouns, e.g., 'red' or 'redness', 'love',

[62] Mertz, 'Individuation and Instance Ontology'.

'fatherhood', and 'similarity', that have in themselves no combinatorial nuance or 'mode' in the scholastic sense, and that stand in contrast to the intermediate abstractions of ontic predicates proper, e.g., $\text{Red}^1(x_1)$, $\text{Loves}^2(x_1, x_2)$, $\text{Father-of}^2(x_1,x_2)$, and $\text{Similar-to}^2(x_1,x_2)$, expressed in the verb phrases, respectively, as 'is red', 'is in love with', 'is a father of', and 'is similar to'. In addition, the non-predicable nature of intensions is seen from the fact that they do not have the kind of dependence/incompleteness that their subsuming ontic predicates have. Succinctly, this 'substance-like' independence is the prerequisite factual basis for all of the following: the erroneous inference from intensions to Platonic hypostatized Forms; the erroneous assay of ontic predication (corrected herein) as an (inert) intension being a subject, along with the entity(ies) it qualifies, of an (agent) exemplification tie; the initial plausibility of Bradley's Regress; and Russell's correct but undeveloped distinction of contexts where relations 'actually relate' (i.e., are polyadic predicates) and where they do not (what would be the relation intension abstracted from its predicate).

It is, I propose, the non-unifying, inert nature of intensions that renders trope theory deceptively plausible, each trope being an individuated but non-predicable monadic intension. Revealing of its weakness, however, the theory must call upon predicable/combinatorial polyadic relations, and not just the dyadic relations of Compresence^2 and Resemblance^2. Even Campbell in advocating trope theory has maintained that, though Resemblance^2 is (purportedly) monadically reducible as an 'internal' relation, Compresence^2 presents a more difficult case involving at best the 'At^2' relation between a trope and its location which itself is irreducible to properties of its relata.[63] Also identified as irreducible by Campbell is the Referring^2 relation between a term (e.g., 'Paris') and the entity it names (e.g., Paris), and apparently in general any relation of correspondence between elements of a mental state (e.g., the cognitive content that is the 'meaning' of a declarative sentence) and what that state represents (e.g., a fact that is the truth-condition for the declarative sentence).[64] Irreducible relations imply unifying agency by polyadic predicates, though monadic tropes treated as speciously prototypical are not combinatorial/predicable at all, an embarrassing duality for trope theory considering that we have here what is

[63] Keith Campbell, 'The Place of Relations in a Trope Philosophy', *Proceedings of the Colloque International de Philosophie de Grenoble: La Structure du monde; Objets, Propriétés, Etats de choses*, in *Recherches sur la philosophie et le langage*, Universite Pierre Mendes France, Grenoble, 2003.

[64] Campbell, 'Unit Properties, Relations, and Spatio-Temporal Naturalism'.

prima facie one category of entities—characterizing qualities—differentiated only by the number *n* of subjects that are jointly required for them to characterize (their *n*-adicity).

We now have **Principles I** and **II**, and from them follows important and particularly relevant **Principle III**. With **I** and **II** we know that ontic predicates are agent-unifiers among *n*-tuples of subjects and so jointly generate facts, but that the predicates' subsumed/constituent intensions that specify and delimit their linkings have no such agency. This implies that for each ontic predicate there is, in addition to its constituent intension, a non-identical remainder of constituent and intensionless unifying or combinatorial act. The combinatorial acts of ontic predicates are the 'ontoglial' (Greek: 'glue of being') essential to the unity of and marking the diversity in a plural universe. Like an intension relative to its ontic predicate, and indeed the predicate relative to its fact, the unifying act of an ontic predicate is recognized via a process of abstraction, but does not otherwise exist separated. Recall there are no 'bare linkings' without intensions, nor are there ontic predicates without subjects to unify. This now brings us to the principal thesis of the essay: The union between the combinatorial aspect, say unifying act U, and the ontically distinct intension aspect R^n of an ontic predicate $R^n(x_1,x_2,\ldots,x_n)$, the latter providing the intensional unity of some fact $:R^n(a_1,a_2,\ldots,a_n)$, is not a function of an agency of act U, or any other constituent unifier U′, whether U′ is itself an intensionless unifying act or an intensioned ontic predicate. When this is established we will have a composite—ontic predicate $R^n(x_1,x_2,\ldots,x_n)$—consisting of act U and intension R^n but without a constituent unifier, and in particular without a constituent unifier interposing and thus registering an internal differentiation between and so a discreteness of U and R^n. Hence, an ontic predicate is a composite but one 'tighter' than an articulated complex. All of this follows, first, from the fact that agency U cannot cause intension R^n to be linked to it, for otherwise intension R^n would have a status in the fact that is the same as subjects, a_1, a_2, …, a_n, whose unity among themselves is likewise via U unifying itself to them. Here the intension R^n of the ontic predicate $R^n(x_1,x_2,\ldots,x_n)$ is stripped from its correlative unifying act U and then made to be a subject ('patient') of U so that $:R^n(a_1,a_2,\ldots,a_n) = :U(R^n,a_1,a_2,\ldots,a_n)$. What is illicit about the latter is not that it precipitates Bradley's regress, for U is not a further intensioned relation, but rather that U must function as an intensionless unifier or pure unifying act—a bare linking. And, as noted above, a bare linking has no intension in itself to control its agency and so the natures of its relata are indifferent to it, i.e.,

bare linking is the arbitrary linking of a list and is contrary to the union forming a fact. Nor could there be a further constituent unifier of the original fact so that $:R^n(a_1,a_2,\ldots,a_n) = :U'(U,R^n,a_1,a_2,\ldots,a_n)$, for U′ would have to be itself either a bare linking, which is impossible, or alternately, an intensioned unifier, i.e., a predicable relation identical in this regard to $R^n(x_1,x_2,\ldots,x_n)$ in the original fact, and this would effect a Bradley-type vicious regress. In sum, the non-identical but correlative aspects of intension R^n and unifying act U as constituting an ontic predicate $R^n(x_1,x_2,\ldots,x_n)$, the latter the constituent cause of the unity of a fact $:R^n(a_1,a_2,\ldots,a_n)$, form a union without any constituent unifying agent and its agency. This is the unity of a continuous composite, and what makes it a 'tighter' unity than that of an articulated composite whose unity is via constituent agents and their agencies.

In addition to this result two further and significant consequences follow from the above analysis. First, that the agent-unifier/combinatorial-aspect of an ontic predicate $R^n(x_1,x_2,\ldots,x_n)$ is unrepeatable follows in a simple way from the existence of ontic predicates with contingent intensions R^n, e.g., $Support^2$ (in either sense physical or financial), $Above^2$, Owe^3. Assume to the contrary that the act of unification for facts, say, $:Supports^2(a,b)$ and $:Supports^2(c,d)$, for $<a,b> \neq <c,d>$, is, like the intension $Support^2$, repeatable and *numerically the same* in each. Then, if fact $:Supports^2(a,b)$ ceased to exist, i.e., the act of unification between *a* and *b* under intension $Support^2$ ceased to exist, then because it has by assumption numerically the same act of unification between *c* and *d*, fact $:Supports^2(c,d)$ would likewise cease to exist. This is, of course, counter-factual. It follows, then, that the combinatorial aspect of an ontic predicate is unrepeatable, i.e., individual, and so under what Armstrong calls the 'Victory of Particularity' principle the subsuming ontic predicate inherits this particularity. In short, ontic predicates $R^n(x_1,x_2,\ldots,x_n)$ are unit attributes, what I have symbolized elsewhere succinctly as 'R^n_i', 'R^n_j', 'R^n_k', …, where the shared 'R^n' indicates a common intension content and the different subscripts indicate each instance's unrepeatability. In more explicit form, the example facts would be given as $:Supports^2_i(a,b)$ and $:Supports^2_j(c,d)$, and, for $<a,b> \neq <c,d>$, then $Supports^2_i \neq Supports^2_j$. The present argument corrects a thesis advanced independently by Michael Loux and E. J. Lowe[65]

[65] Michael Loux, *Substance and Attribute*, pp. 163ff, and Loux, *Metaphysics: A Contemporary Introduction*, pp. 117ff. E. J. Lowe, *The Possibility of Metaphysics* (Oxford: Clarendon Press, 1998), pp. 180-83, 197.

that individuation is via the instantiation of a repeatable substantial form posited as a type of entity specially endowed with the power to individuate its instances. Observed here is that individuation is via the 'instantiation of any intension' in the sense of following from the predicable or agent-unifier role of relation instances of any kind. Relation instances (including limiting property instances) can in hierarchical fashion jointly account for the existence of all individuals (e.g., 'substances' as iterated complexes of complexes), and hence through them ontic predictability—unifying agency under an intension—is ontology's *principium individuationis*.[66] One of the great virtues of the above analysis and of subsequent relation instances, what gives further warrant to both, is this positive theory of individuation, the alternative to which is an explanatory vacuum in which must be simply posited specious bare particulars.

Secondly, we can now also make good on the promise of an argument for realism: intension or qualitative contents are numerically repeatable—identically the same content in multiple subjects—and thus are universals. This thesis was simply assumed in the above argument for the unrepeatability of predicable agency, *but the argument does not turn upon it*. And as noted, Campbell has argued for a unit attribute conceived as a non-unifying/inert trope that has a qualitative content abstractable from it and distinct in abstraction from the trope's unrepeatability, but a content that is itself unrepeatable and where the trope it characterizes has no internal distinctions. To the contrary and first, it was argued above that for a unit attribute R^n_i its aspects of intension R^n and combinatorial agency U are distinct in composing it for the obvious reason that the latter is a causal entity and the former is not. Now further, if R^n were unrepeatable or individual as is act U, then subsuming instance R^n_i would be composed of two distinct individuals. Then on the principle observed at the beginning of the essay that a whole composed of two or more individuals is internally differentiated/diverse, then some constituent must have the role of unifier among the others, whether this is U or some further implicit entity. But, we have seen the impossibility of these alternatives above. Therefore, intension R^n as a constituent of instance R^n_i is a repeatable entity—a universal.

[66] Mertz, 'An Instance Ontology for Structures'.

4. Results in Context and Replies to Critics

Let us now summarize the major ontological theses advanced herein and their place in the ongoing dialectic, including some attention to the issue of individuated ontic predicates versus bare particulars. First, an ontic predicate now identified as a relation instance R^n_i is a composite continuous simple, whereas its constituent combinatorial act U is absolutely simple, as is its intension R^n in some (e.g., Red^1), if not in all cases. For any instance R^n_i its intension content R^n is *not* ontically predicable of its individuating combinatorial act U, rather the two only jointly as a continuous whole is so predicable of *n* further subjects. That is and contrary to the tradition, an intension R^n is *not* itself an ontic predicate, and it gives qualitative content to a subsuming ontic predicate R^n_i *not* by being predicable of it. Strictly speaking, to characterize an entity, say the number 3, is to be ontically predicable of it, as in :$Prime^1_i(3)$, but the intension $Prime^1$ of ontic predicate $Prime^1_i$ is not an ontic predicate of the latter, i.e., it is false to say that 'Instance $Prime^1_i$ is prime' since intension $Prime^1$ makes sense only relative to characterizing numbers, and not ontic-predicates/relation-instances. An intension R^n is once-removed from ontic predication.

In this regard it is important to be clear on the subtle difference that makes all the difference between individuating combinatorial acts and their theoretical rivals of would-be bare particulars. First and the same for both, whether a particular is taken as individuated by a predicable act (what would be a relation instance) or by a bare particular (what would be an ordinary thick particular), an intension universal, e.g., Red^1, in conditioning that particular is not ontically predicable of its individuator. But contrasting the two, for a relation instance, say Red^1_i, the intension Red^1 conditions the correlative combinatorial act so that it is relevant to the nature of a type of subject, viz., entities that are red (and for some polyadic intensions they order their combinatorial acts as well as specify jointly possible relata, as such having relevance to certain *n*-tuples), whereas with a bare particular p_a the 'predication' of an intension Red^1 of its thick particular *a* reduces to Red^1 being 'tied-to' p_a in a way indifferent and irrelevant to the 'nature' of p_a, what is in effect arbitrary association. Now further, an ontic predicate R^n_i characterizes its *n* subjects *externally* as predicably attached to and among them, in contrast to the traditional inherence model of predication where the predicate's intension is itself the ontic predicate and as such is held to be internally constitutive of the nature of its subject, what as such is necessarily a monadic intension. Importantly, an instance R^n_i predicably at-

taches to its subjects conditioned on its intension R^n being 'mutually qualitatively compatible with' or 'co-relevant in quiddity ('whatness') with' the essences or natures of its subjects (as ordered if relevant), portions or aspects of the latter grounding or providing the foundation for this attachment. This is how an ontic predicate, though external to its subjects, is non-arbitrarily 'true of' and carries information about—is 'telling of'—the internal essences of its subjects. This is a generalization and weakening of what is a specious though widely and implicitly held thesis restricted to monadic predicates, viz., the **Inherence Thesis** (**IT**): In a monadic fact $:P^1_i(a)$, that portion of the being of subject *a* that grounds the predicable attachment to it of ontic predicate P^1_i is itself intension universal P^1. In other words, the universal intension aspect of every ontic predicate 'true of' a subject is a constituent of that subject. Here the essence- or nature-conditioned relevance under the weakened externalist assay becomes identity under the narrower internalist view, what is definitive of the inherence model of predication.

Now for those who adopt **IT** it can serve as a premise for arguments against bare particulars, and indeed I had assumed it implicitly in the past.[67] The arguments are built on the assumption that, in conformity to **IT**, an unrepeatable thick individual *a* is composed of repeatable universals that are constitutive of the ontic predicates characterizing *a*, along with an unrepeatable particular p_a distinct as such from all these universals but to which the latter are joined (e.g., by a Compresence^2 or Tied-to^2 relation) and which serves to account for the individuality of *a*. Now, the arguments against the coherence of p_a starts with the observation that it can have no constituent intensions whatsoever, because otherwise it would be itself a 'thick particular' in need of a further posited individuator, p_a', and so on. But then on **IT**, p_a can have no ontic predicates either, for otherwise their intensions would be constituents of it. On the premise that having no ontic predicates implies having no nature and so no being, then p_a evaporates into *no*thingness. Relatedly, p_a is indeed said to have necessarily certain ontic predicates, e.g., $\text{Unrepeatability}^1_i$, Simplicity^1_j, etc., but even these have repeatable intensions, e.g., Unrepeatability^1 is a universal, which by **IT** would have to be constitutive of p_a, a contradiction. So bare particular p_a dissipates into non-being, and it is in this sense that all bare particulars

[67] E.g., at Mertz, 'Individuation and Instance Ontology', pp. 49-50.

are 'identical'—all are absolutely mutually indistinguishable in their ontic vacuity.[68]

Though these arguments stand in full force against inherence theorists who adopt **IT**, because I reject it on the above analysis I must forgo them. The same analysis, however, shows by other means why bare particulars are untenable. The only way an advocate of posited bare particulars can hope to avoid the above conclusions is by adopting an externalist assay of ontic predication argued herein. This is in effect the tack adopted recently by James Moreland and Timothy Pickavance in dividing ontic predication into two types of relations: the standard 'Rooted-in^2' relation (equivalent to the usual Exemplification^2 relation) between the properties of a thick particular *a* and *a* itself, and, at a lower level, the 'Tied-to^2' relation between the properties of *a* and its bare particular, p_a.[69] All such attempts are, however, doomed to failure. First, I simply note that ontic predication cannot be identified with any particular relation(s), for all relations (including monadic properties) of whatever intensions are all equally cases of ontic predication, and to otherwise make this reduction is to iden-

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[68] This 'identity in vacuity' is the valid point that in 'Individuation and Instance Ontology', p. 52, I garbled badly in an argument that others have been right to criticize (e.g., Vallicella, 'Bradley's Regress and Relation-Instances', and Richard Davis, ''Partially Clad' Bare Particulars Exposed', pp. 541-41). The argument there should have been as follows. First, I understood 'constituent of' as extended in sense to include the 'improper' case of the very essence itself of an entity—the essence of an entity being constitutive of it. Then, the constituent analog to the Identity of Indiscernibles, **CII**, was to include this extended sense: $(x)(y)[(z)(z$ is a proper or improper constituent $x \equiv z$ is a proper or improper constituent $y) \supset x = y]$. If upon analysis bare particulars *x* and *y* have no ontic predicates in the standard sense and therefore have no essences, then the antecedent of **CII** is true and so $x = y$, for all bare particulars *x* and *y*. Hence, absurdly, there could be only one ordinary thick particular. In 'Individuation' I was not explicit about the extended sense of 'constituent of' nor that **CII** was to include it, and made the argument curtly, saying "All bare particulars in having no constituents have exactly the same constituents and so are identical." Whatever the merits of this clarified argument, it does not imply, as critics of the garbled original asserted of it, that all (absolutely) simple entities, in being without proper constituents, would have to be identical. This would be implied only if **CII** were rewritten to concern only proper constituents, but with the variables in the initial quantifiers, '(x)' and '(y)', remaining unrestricted in their range. Moreland and Pickavance, in 'Bare Particulars and Individuation', pp. 12-13, avoid the unwanted implication by holding that the variables in the initial universal quantifiers of **CII** are to be restricted to range over only composite entities, what they contend is the common understanding.

[69] Moreland and Pickavance, 'Bare Particulars and Individuation: A Reply to Mertz'.

tify an aspect of every relation with the whole of a particular relation (or relations). The plausibility of this identification turns on the fact that the chosen relation(s) exercises that very aspect that was to be explained in all relations—a combinatorial act guided by an intension, i.e., ontic predication. In other words, the unsuccessful strategy here is to explicate something exhibited by every element in a class by identifying it with one of the exhibiting elements in the class, a form of vicious circularity. Now specifically in regard to Moreland and Pickavance's externalist strategy to save bare particulars, assume property P^1 (whether as an intension or instance) is externally tied-to a bare particular p_a, what is the individuator for thick particular a, say, a red, round disk. Now, either this means that P^1, e.g., Red^1 or $Round^1$, is non-arbitrarily grounded in a composing nature of p_a, or, to the contrary, P^1 relays no information about p_a and so is arbitrarily associated with it in the manner of items in a list or set. One cannot have it both ways. But on the first alternative this can only mean that P^1 is nature-relevant to a something constitutive of p_a and thus P^1 is rooted-in p_a, what is ruled out by Moreland and Pickavance. On the second alternative any two properties whatsoever can be jointly tied-to p_a, including contrary properties, e.g., $Round^1$ and $Square^1$. Then on Moreland and Pickavance's thesis that Tied-to^2(P^1,p_a) if and only if Rooted-in^2(P^1,a), contrary properties can be ontic predicates of any individual a, e.g., a can, absurdly, be both round and square. Now it might be replied that on my analysis of an ontic predicate R^n_i as a continuous composite I have its intension R^n tied-to its individuating combinatorial act. But on my analysis the latter represents a third type of union distinct from what Moreland and Pickavance intend by the extremes of the Tied-to^2 and Rooted-in^2 relations: unlike the Tied-to^2 relation, the union between a combinatorial act and its correlative intension is not one of mutual indifference but one where the latter aspect controls in extent and order the former, but unlike with the Rooted-in^2 relation the intension is not ontically predicable of—does not characterize—its combinatorial act. As a final point I would only observe that bare particulars are simply posited as ontology's individuating principles for a lack of a known alternative derivable from other ontological considerations—the above provides this alternative.

On another front I would address an argument advanced recently by William Vallicella that a fact can and must have a unifier external to it. This follows from what he would consider to be the failure of the analysis of the unity of a fact given above in **Principles I**, **II**, and **III**, and therefore that "The unity of a fact's constituents cannot be a proper constituent of the

fact"[70], along with the rejection, correctly, that a fact as a plural whole cannot be the cause of the unity of itself, *contra* Armstrong[71]. Vallicella's critique of the former results from a confusion as evidenced by his thinking that it is inconsistent with my theses that "Thus numerically one and the same entity, [universal intension] R, occurs as constituent in both facts [:R(*a*,*b*) and :R(*c*,*d*)]: but R's [agency in] relating *a* and *b* is numerically distinct from R's [agency in] relating *c* and *d*."[72](inserts mine though intended by Vallicella as clear from the context) The supposed inconsistency is said to result from my failure to distinguish between an agent and its agency by identifying a relation R, whether I am treating it as a universal for sake of argument or as an instance, with its concomitant unifying act in a fact. To the contrary, I have argued for this distinction herein, and did so in the works Vallicella cites. Among other criticisms, what is relevant here is that Vallicella rejects the possibility that my ontic-predicates/relation-instances can be simple entities. He finds the concept of a formal distinction and what I have herein called a composite simple to be incoherent on the same grounds as did Ockham and Suarez (when criticizing Scotus). Rejecting my type of analysis Vallicella concludes that the unity of a fact's constituents can only be a function of an external causal 'operator'. What makes Vallicella's view untenable, however, is that a fact :R(*a*,*b*) has its being just as a-fact-producing-type-of-unity-among-the-other-constituents, i.e., the fact :R(*a*,*b*) is not R, *a*, and *b* prior to and independent of their factual union. There is no plural whole without a *constituent* unifying act, as there is no pearl necklace without a unifying string, for a plural whole just is the other constituents (e.g., the pearls) so acted upon (e.g., connected by the string) and nothing less. Now, to declare that the unifying agency of :R(*a*,*b*) is 'external to it' is simply to re-draw the boundaries of the purported fact with just constituents R, *a*, and *b*, to also include the 'external' agency, U, what then is properly an internal constituent of the real fact here as necessarily expanded to parts-properly-unified. This means that the fact mistakenly analyzed as :R(*a*,*b*), i.e., where R is the agent unifier, is properly rendered :U(R,*a*,*b*). But then how is it that fact :U(R,*a*,*b*) avoids the import Vallicella gives Bradley's Regress against purported fact :R(*a*,*b*)? It cannot if U is intensioned, i.e., U is itself a relation like R, and the alterna-

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[70] Vallicella, 'Bradley's Regress and Relation-Instances'. Vallicella's advocacy of the validity of Bradley's Regress can also be found in his 'Three Conceptions of States of Affairs', *Nous* 34 (2000): 237-59, and in 'Relations, Monism, and the Vindication of Bradley's Regress', *Dialectica* 56 (2002): 3-35.

[71] Armstrong, *States of Affairs*, pp. 118-19.

[72] Vallicella, 'Bradley's Regress and Relation-Instances'.

tive is that U is arbitrary association, what presumably would have to be an act of Divine will. The latter would make facts like :Prime-Proper-Divisior-of^2(3,6) and :Left-of^2(*a*,*b*) obtain independently of the natures of any of the relations and relata involved, which is counterfactual.

In regard to Vallicella's rejection of composite simples I offer the following argument, one that expands upon the simple observation that a causal sequence must end somewhere where a cause brings about an effect *immediately*, without otherwise a vicious regress of further causes. It is to establish the point, ironic in regard to Vallicella, that causation itself is in every case a fundamental example of a composite simple. Consider first that causation is *at the causal act*, whether the act is instantaneous (e.g., a collision between inelastic balls), or over a temporal interval (e.g., the unifying act of a contingent fact), or 'eternally' atemporal (e.g., the unifying act of a necessary fact). More specifically, causation proper is at the causal act where cause (agent, 'operator'), patient(s), and effect come together, and only relative to which are each classified such. Now the argument is that at a causal act the agent and the act (the agent's agency), though distinct, form an immediate union tighter that a plural whole, i.e., form a continuous composite. This is so in that there is no constituent of this union, whether agent, agency, or some implicit third, that can have an additional unifying mode or aspect that allows that constituent to go beyond itself and link itself to or among the others, what would otherwise indicate an ontic distinction among the thus united, a characteristic of a looser plural whole, i.e., of an articulated composite. For, if it were otherwise then the constituent would have to be a *cause* of the causal unity between itself and the other constituents in the initial agent/act whole. In other words, it would have to be an agent with a unifying act, act′, having the original causal act as a patient. Clearly, this is the beginning of a vicious regress. Agent and causal act at the act form a continuous composite. So if a unifying act is necessarily constitutive of a whole, its concomitant agent must likewise be. It is by a derivative and misleading 'courtesy of inheritance' that an agent *a* could be said to be 'external' to a causal act, act$_1$, that produces an effect *c*. For, this could only mean that there is a causal relation between *a* and some implicit effect *b*, i.e., fact :Causes^2(*a*,*b*) obtains, where the ontic predicate for the latter has its own causal act, act$_2$, and where fact :Causes^2(*b*, *c*) likewise obtains and the ontic predicate for it has causal act$_1$. In other words, to say that an agent *a* is 'external' to its agency in producing an effect *c* is simply to say that it is a remote cause of *c*.

5. Conclusion: What Can be Understood of Composite Simples

We have seen that in at least the ontologically fundamental category of ontic predicates there are composites that each have two non-identical constituents—an intension and a combinatorial agency—where neither, nor some implicit third constituent, acts as agent unifier relative to the other(s). And, it was argued that all unifications among the yet differentiated/discrete, what I have called articulated composites and what are the ubiquitous structures and complexes of experience and theory, exist if and only if each has one or more constituent ontic predicates that as causal unifiers 'go beyond themselves' to join themselves to and among other constituents. As is obvious in the paradigm case of irreducible polyadic relations, each in forming the 'togetherness' that is a fact is also 'between' its relata in the sense of presupposing an 'ontic distance' between and so discrete otherness of each from the other, and the ontic predicate itself from each relatum. Ontic predicates mark/bridge an ontological division between their subjects, and between themselves and their subjects, in the wholes they serve to unify. What this means is that the criterion for differentiation/discreteness of parts of a composite whole is that each part is either an agent unifier among other parts, or is a patient of such agency. Consequently, with simplicity defined as the 'absence of division' we then have the necessary and sufficient conditions for an entity *x* being simple, viz., if and only if *x* has no constituent which is an ontic predicate of another constituent.[73] It is in this sense that an ontic predicate is simple, and yet with distinct constituents it is internally non-homogeneous making it appropriately termed a 'continuous composite'.

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[73] This answers the 'Simple Question' posed by Ned Markosian in 'Simples', *Australasian Journal of Philosophy* 76 (1998): 213-28. His answer is that entities are simple if they are maximally spatially continuous, a criterion which allows for physical divisibility. Though I think his intuition concerning spatial continuity as involving a simplicity among the yet heterogeneous is in the right direction, the criterion for simplicity offered herein is metaphysically universal and would include spatial simplicity as but one case. I note further that the analysis herein has proceeded in the reverse direction from that suggested by Markosian when he says "If we are to try to figure out how it is that several things can combine in order to compose a single thing [in order to answer van Inwagen's Special Composition Question (see note 12 above)], then we will likely be aided in our investigation if we have some idea of the nature of the basic building blocks that are meant to be combined in order to form [articulated] composite objects."(p. 215; my inserts) I have argued for the insight that it is ontic predicates that are the unifying causes of composites with yet discrete parts, and that consequently it is their absence that is the criterion for being simple.

Consequently, the pre-critical air of paradox concerning the concepts of a composite simple and the 'formal distinction' is removed with the differentiation of ontic division and discreteness from distinctness and non-identity, and the observation that ontic predication is a necessary case of the latter without the former. It was also argued that the union of a causal agent and its agency at a causal act is a case of simple continuous composition. Heuristic to the nature of composite simples I have proposed the analog of a disk whose color changes continuously across its surface from red through yellow to green as in the spectrum of an unpartitioned color wheel. It is continuous in having no inherent boundaries or divisions between colors, and thus is undivided and so simple, yet it is composed of distinguishable colors so known by selective attention. These colors add up to the phenomenal being of the whole—it is not different from them collectively. So it is for any continuous composite, the division and differentiation of the thus discrete parts is posterior to the whole (*post rem*), and though it has distinct/non-identical constituents, their essences as such are not sufficient in themselves to cause a mutual ontic division, what is achieved only by external cognitive analysis. Whereas in an articulated composite the division or mutual discrete otherness of the parts is prior to the whole, and is maintained even as the parts are unified in the whole, a differentiation implied in the ontic predictability of some of the parts relative to the others. Stated otherwise, in both types of unions the existences of the wholes are simultaneous with the 'joint existences' of their parts, where with an articulated composite or complex the union of the parts is a function of the contained parts that remain differentiated as such due to the predicable nature of some among the others (each such whole a *unitas ex intra se*), whereas with a continuous composite this union is a function of the containing whole relative to which the parts are virtual until differentiated externally by abstraction (each a *unitas per se*). An example of the latter is God traditionally conceived as the coalescence of divine attributes, the latter differentiated only in the intellect.

In contrast to an articulated composite, with a continuous composite, because the whole is prior to the parts as subsequently conceptually differentiated, the extra-conceptual existence of these constituents is never independent of (outside the being of) such composites. In regard to ontic predicates (i.e., relation instances), the latter observation is in keeping with the Aristotelian/scholastic thesis that only individuals exist extra-conceptually, and that their characterizing intensions are 'individuated in things', i.e., in-

dividuated as forming in each case a continuous whole with an unrepeatable combinatorial act, but are 'universal in the mind' when conceptually abstracted from these correlative unifying acts. Also, it would seem that, though for articulated composites they can have 'upwardly emergent' and intensionally *sui generis* properties and relations due to the manner of their composition via structuring constituent ontic predicates, e.g., as consciousness emerges with brain complexity, in contrast, with a continuous whole, because there is with it a 'downward emergence' of the parts only mutually divided in abstraction, it can have 'nothing new that is not in the parts'. That is, a continuous whole can have no properties and relations not definable logically from the conjunction of the properties and relations of the parts, since the union here of the parts can add no essence-altering structure to the whole. Hence, composite simples represent an ontological limit, not of analysis, but of system and structure, and in this way they are necessarily atomic to plural structured reality.

It is worth ending on the following observation. It is a symptom of the error of Bradlarian Monism that its analysis of ontic predication requires in the end that not only all discreteness but all distinctness (non-identity) collapse into a homogeneous One. The error is in the assumption that predicable unification is by mutual 'inclusion' or 'absorption', a view abetted by the specious inherence model of predication, and which requires in the end a melding or blending of natures where all distinction among the united is obliterated in a coinciding identity. Continuing the above metaphor, think of the colors on the example disk uniformly blended into a single color homogeneous across its surface. So for such complete 'blends' there is, on the one hand, the requirement that the specific and distinguishing essences of the constituents (e.g., divine omniscience, divine omnipotence, divine goodness) contribute to the cumulative and specific nature of the whole (e.g., the nature of God), and yet on the other, precisely as contributing parts they must lose their content-specifying identities as the blend obliterates all internal distinctions, and with this the whole loses the contributing qualitative essences of the would-be parts. In such bogus blends the natures of the parts disappear and so can make no contribution to the nature of the whole which must then evaporate as an essenceless illusion. The lesson herein is that a whole which analysis reveals must have a unification 'tighter than' that of the usual articulated composite, e.g., an ontic predicate or God, need not collapse into the absurdity of a homogeneous one, but can be a continuous composite.

4

Objects as Hierarchical Structures: A Comprehensive Ontology*

1. Introduction

It is a given of both everyday observation as well as of scientific experimentation and theory that ordinary three-dimensional objects we encounter in daily experience—apples, chairs, computers, trees, humans, etc. —are without exception composites consisting of *parts organized in specific ways*. That is, ordinary objects are *systems, complexes, structures, or networks*, where the various kinds of inter-relations—e.g., spatial and physical/causal, static and dynamic—among the parts are as essential to the nature of the resultant whole as are the related parts. And, in the systematic extension of these observations by instrumentation and theory, our scientific knowledge of material objects is of vastly complex hierarchical structures of structures, where at each level a given structure is itself the single subject for properties and relations that together form structures subsuming it. A chair, for example, consists of parts in certain static spatial and physical-causal relationships (e.g., mechanical or molecular forces at the structural level of artifact), parts that without some of the latter would reduce to a heap of fragments and not a chair. In turn and in wooden chairs, for example, the composing cellulose molecules contribute rigidity and strength to the wood due to their being each a polymerized chain-like structure of glucose molecules, each glucose molecule itself defined by a certain structure between its carbon, hydrogen, and oxygen atoms, and at a lower level still, each of these atoms having definitive characteristics because of various kinds of sub-atomic entities related in certain ways. Living organisms are even more spectacular examples of integrated levels of static and dynamic systems, e.g., of bones and organs functioning in mutually beneficial ways,

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where each organ consists of a particular structure among specialized cells, the latter in turn specified by a particular set of molecules interrelated in certain ways. Perception itself is both possible due to certain types of neural systems and veridical precisely because these systems effect chains of homomorphic signal structures. Emerging at increased levels of living complexity are new 'powers', i.e., the possibility of *sui generis* properties and relations not available at the lower levels, e.g., as in those distinguishing vegetative from sensible life, and as illustrated in the emergences of consciousness and then abstract thinking as functions of certain complexities of brains and nervous systems. This is an important generalizable explanatory point: at some levels of some structures there are emergent and *sui generis* properties and relations, e.g., the dispositional property of Is-a-Chair is an ontic predicate of certain macro-structures but not their molecular micro-structures, or, in the abstract, True and False are emergent properties on (what are conceptual) propositions but not on their subparts, say, individual concepts for subject terms.

Universally, then, analysis reveals ordinary objects to be hierarchies of structures of structures, higher levels having physical properties and relations non-existent at lower levels of structure. This downward iteration of subsumed sub-structures is extended by science all the way to the primary level of quantum entities. Significantly, however, quantum entities represent an apparent lower limit on structure as naively understood. For as realistically interpreted, quantum theory is said to imply that objects or 'substances' at its level dissipate completely into physical systems of *only properties and relations*—pure structures.[1] The proposed proto-ontology, termed 'Structural Realism', is in regard to traditional ontic categories immediately stymied with the problem of how there can be properties and relations without supporting objects as subjects or relata? In the following I shall show how this question is necessitated on ontological grounds alone, and how it can be answered. It will follow that physical micro-reality can be purely structural, as must be all reality at some foundational level. This account is also offered as possibly shedding light on the 'underdetermination' of quantum particles insofar as it provides a perspicuous re-conceptualization of identity and indiscernibility in purely structural terms, one explaining how such

[1] Steven French, 'Symmetry, Structure and the Constitution of Objects', in the *PhilSci Archives* (2001), Center for the Philosophy of Science, University of Pittsburgh at http://phisci-archive.pitt.edu/. Steven French and James Ladyman, 'Remodelling Structural Realism: Quantum Physics and the Metaphysics of Structure', *Synthese* 136 (2003): 31–56.

entities can each have a unique identity (be an 'individual') and can likewise be distinct but indiscernible from others without the need for a simply posited individuator.[2] In all these ways and others to be considered, the account given will have advantages over related trope theory sometimes appealed to in this context.[3]

Now, equally significant for ontology generally but in the opposite direction, this structural characterization extends upward from ordinary mid-size physical objects isolated in our attention for practical reasons to also include more 'scattered' local, global, and cosmically subsuming spatial/physical systems. Moreover and meshing with these systems are abstract cognitive structures, including both contingent relations making up particular psyches as well as necessary relations composing the formal hierarchical systems of mathematics and logic, systems instrumentally essential to our scientific knowledge. There are also ethical and social structures, e.g., the complex and varied systems of relationships that constitute family, corporation, or citizenry. Succinctly then, structure is the ubiquitous given, and ordinary objects are examples of and metaphor for this universal feature. Crucial in this is the fact that relations of various intensions, contingent or necessary, as they exist among subject things are as fundamental in composing the resulting wholes as are the things themselves. What is required, then, to explain this ubiquitous given is a developed and comprehensive *ontology of structure* that as such will include, principally: a) an account of the defining and composing inter-subjects/multi-relata ontic predicates—*polyadic relations*—as they each effect an intensional unification among the yet diverse, i.e., an account of relational facts or states of affairs, monadic properties being the easily distorted limiting case; b) an account of how facts are compounded to form both same-level and hierarchical molecular structural lattices or networks; and c) in order to avoid either intractable problems of traditional ontology or a vicious regress, an account of how at some atomic ontic level there can be pure structures composed exclusively of ontic predicates. I shall give herein what I argue are the princi-

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[2] Ibid. Robert Hilborn and Candice Yuca, 'Identical Particles in Quantum Mechanics Revisited', *British Journal for the Philosophy of Science* 53 (2002): 355-89.

[3] Peter Simons, 'Particulars in Particular Clothing: Three Trope Theories of Substance', *Philosophy and Phenomenological Research* LIV (1994): 553-75. Andrew Wayne, 'A Trope Ontology for Classical and Quantum Field Theory', forthcoming in a volume ed. by W. Myrvold in the *University of Western Ontario Series in Philosophy of Science* (Springer).

ples of such an ontology. It is derived from an analysis of ontic predicates that shows them to have an irreducible substantiality and a primary ontic status not recognized in traditional ontology. Described in Aristotelian terms, ontic predicates are analyzed herein as: 1) each having a particularity or 'thisness', i.e., individuated as a relation instance; 2) like traditional 'forms', they act to intensionally or qualitatively structure their subjects (though this structuring is *inter*-subject, not *intra*-subject as in the tradition); 3) at some atomic ontic level they can be ultimate subject substrata for other instances predicable of them, i.e., have the role of 'prime matter'; and 4) mutually sustaining systems of the latter can found hierarchies of emergent structures that as single subjects endure through the 'accidental' change of certain property and relation instances, and can have 'substantial' change when composing instances of defining properties and relations are destroyed, leaving sub-structures, 'matter', that collectively are not then organized in these defining ways. So described, relation instances answer various criteria for 'substance' Aristotle specified in the *Metaphysics* but could not find one type of entity to satisfy.

As a context motivating the principles of structural ontology, or what I have elsewhere termed more descriptively *network instance realism*[4], I shall first delineate key historical errors concerning the nature of ontic predication. Ontic predication is what the scholastics explicitly referred to as 'material' predication and distinguished from 'formal' or linguistic predication, a distinction going back to but implicit in Aristotle. Linguistic or grammatical predication is itself a type of ontic or material predication, it being generic for a number of syntactic and semantic relations including those among grammatical units forming declarative sentences, or, relatedly, those among conceptual components forming propositions. In general, ontic predication is the qualitatively- or intension-controlled unifying agency among the yet distinct, what is the unity of facts or states of affairs, and is to be primarily contrasted with the arbitrary and nature-indifferent unity of elements in 'heaps', lists, sets, or mereological sums (all the latter being, I propose, formal fictions, useful for modeling but specious when identified with the modeled). Exactly contrary to the tradition, polyadic relations are the instructive paradigm case of ontic predication, monadic properties being the less determined and so easily misinterpreted limiting case.

[4] D. W. Mertz, *Moderate Realism and Its Logic* (New Haven: Yale University Press, 1996). Also Mertz, 'Combinatorial Predication and the Ontology of Unit Attributes', *The Modern Schoolman*, LXXIX (2002): 163-97 [Essay 1 herein].

In particular, a proper understanding of ontic predication is as a unifying cause or agent—a combinator—controlled/determined in its unifying act to specific (but not necessarily distinct) subjects a_1, a_2, …, a_n, by a constituent intension or qualitative content R^n and effecting as a structured whole a fact $:R^n_i(a_1,a_2,...,a_n)$. (The colon locution is used herein to distinguish facts from corresponding propositions.) The unifying act of an ontic predicate is conditioned on a qualitative match or relevancy between intension R^n and the natures of each of a_1, a_2, …, a_n, what makes the resulting fact more than a mere list, and is what answers the classic Bradley's Regress argument.[5] So understood, properties and relations as qualifying or characterizing their subjects join themselves to their subjects *externally*—they do not enter into the composition of each or any of their subjects. In contrast and classically, when monadic properties are considered primary and then easily mis-identified with their constituent and abstracted inert intensions, it becomes speciously plausible that these intensions, or their individuated versions (tropes), are *internal components* of their subjects. This is precisely the case with all the alternatives that follow from what I shall identify below as the tradition's Inert Substrata Thesis. As we shall see, among the failures of these alternatives is the fact that they assign the essential ontic jobs of intensionally determined plural unification and the ordering among entities unified to anemic symmetric 'relations' that, in the case of the 'Compresence' (literally 'Present-Together') relation of trope theory is indifferent to any ordering among their relata, and in the case of the 'Tied-to' relation of bare particular theories is completely indifferent to the natures or intensions of these subjects and thus to any mutual relevance based upon this; i.e., the nature of the Tied-to 'relation' is contrary to the subject(s)-characterization or subject(s)-qualification definitive of all ontic predication. The Tied-to relation is necessarily a completely arbitrary linking of properties to a shared bare particular, and the Compresence relation is likewise arbitrary except perhaps for excluding the linking of contrary and contradictory properties. It is to be noted that, as such, both of these relations are distinct from the formal and once-removed relation of Exemplification (or Instantiation), e.g., Exemplification(*a*,Red), that is itself sometimes mistakenly used as the surrogate for what is the combinatorial aspect of *every ontic predicate*, not just for the Exemplification relation as needed to fulfill its role. Yet, even Exemplification implies a union between its subjects, e.g., *a* and Red, qualitatively controlled by a specific intension now as one of the subjects, e.g., Red. The arbitrariness of the Tied-to unifier and

[5] Ibid.

the near-arbitrariness of the Compresence unifier will be part of the following developed critiques against the alternatives implied by the Inert Substrata Thesis, and so the thesis itself.

2. Historical Errors

In the historically influential Aristotelian/scholastic substance/attribute ontology, structure or complexity was both recognized as essential to the very natures of ordinary objects, whether 'substances' or 'artifacts', and yet by the same theory the concept of structure was doomed to obscurity. This obscurity, which persists more or less into contemporary times, was and is a function of the myopic focus on monadic ontic predication, reinforced at times by the false reductive elimination of polyadic relations.[6] In the Aristotelian/scholastic hylomorphic tradition, structures were differentiated, on the one hand, into those of artifacts (e.g., a statue, a house), and, on the other, into the more spectacular dynamic and internally driven event structures that are the lives of 'natural' substances (e.g., Socrates, a tree). The latter structures were thought to each represent in its enduring totality the fulfillment of an end (*telos*) for that substance, what is an inherent fixed 'program' or nature for that type of entity. To account for the structure of composite wholes (present in every composite except what were considered unstructured 'heaps'), Aristotle and the subsequent tradition posited the two correlative and exhaustive 'principles' of *form* and *matter*. Form, either substantial or accidental, gives structure to a resultant whole by being an *ontic predicate* of a subject or subjects where the latter *precisely in having this role* is matter relative to the former. This matter is either, for substantial forms, ultimate and absolutely undetermined and amorphous prime matter, or, for accidental forms, subjects already informed (i.e., substances as subjects of monadic accidents, e.g., Socrates as being white, or parts ('secondary matter') that a form structures into an artifact.) Importantly, the underlying but hazed insight here is that structure is a function of ontic predication, where an ontic predicate is the duality of an act of unification determined as to its subjects and their mutual 'ordering' by a correlative specific intension or qualitative content, e.g., Man or House. In the words of Aquinas, for example, "Each individual thing is actually a being

[6] Mertz, 'An Instance Ontology for Structures: Their Definition, Identity, and Indiscernibility', *Metaphysica: International Journal for Ontology and Metaphysics* 4 (2003): 127-64 [Essay 2 herein].

through a form, whether in the case of actual substantial being or in the case of actual accidental being. And hence every form is an act, and as a consequence it is the reason for the unity whereby a given thing is one." (*De Spirit. Creat.*, Art. 3)[7] The two aspects of act and intension are of a single entity—the form—that joins itself to a subject or subjects in such a way as to characterize or qualify it or them, essentially or accidentally, and this for multiple subjects in the manner of a structuring among them (See Aristotle, *Meta*. 1041b1-33; 10435-14). The view was that when the subject is prime matter, the single ontic predicate, e.g., Is-a-Man, causes a hierarchical emergence of the sub-structural parts, e.g., bones, organs, tissues, and among these a mutual structural ordering and functioning that is the resultant substance. When the subjects are already informed, as with the parts of a house, the ontic predication of an accidental form, e.g., a form with the intension House, among these ontically prior parts effects a structured artifact, e.g., a house.

Now, it is precisely these examples that show a primary error of the hylomorphic tradition: that the nature of ontic predication so understood requires that all acts of characterizing union and thus structural formation be controlled by *monadic intensions*, e.g., Man, Tree, Statue, House, including those acts that require multiple subjects and that establish an order among them. In this latter and crucial multi-subject case, a monadic property is held to not only attach in a characterizing way to a single subject as an already formed composite, e.g., a man or a house, but also and magically somehow it is to be the immediate cause/agent of the prior structural inter-connections among yet diverse parts that results in this composite as a single subject. In fact, however, the latter inter-connections require multiple intensionally determined ontic combinators each existing simultaneously among multiple subjects, and these are *polyadic relations*, e.g., in the case of a house the static relations such as Supports, Between, Covers, Entrance-to, or, in the case of a human body, dynamic relations such as Moves, Digests, Circulates, Purifies. The error here is abetted by the two further classic errors of the eliminative property reduction of relations and the maxim that all unity is by a shared one (i.e., a single entity). As seen below, the correction of the unity-by-the-one maxim is via observing the unity effected by chains of relation instances pair-wise sharing common relata, or complexes of the latter being single relata for further relations. And, I take it to be definitive

[7] Thomas Aquinas, *On Spiritual Creatures* (*De Spiritualibus Creaturis*, 1267), trans. M. Fitzpatrick and J. Wellmuth (Milwaukee: Marquette University Press, 1949), p. 46.

on arguments by Russell[8] and others[9], and based upon the non-reducible ordering inherent to certain relations (e.g., asymmetric and non-symmetric relations), that polyadic relations are not eliminable in favor of monadic properties of their relata or certain kinds of sets of their relata. More locally, Paul Teller has argued that the apparent fact of superposition or 'entanglement' in quantum mechanics implies the existence of 'inherent' or 'non-supervenient', i.e., irreducible, relations.[10] Indeed, exactly contrary to the insidious reductionism of the tradition where relations dissolve into their relata things, on the analysis herein all things whatsoever dissolve ultimately and without remainder into their composing relations (including properties). The result is a precise and perspicuous *relational holism*, what is often called for as an ontology for micro-physics.

A second error of hylomorphism, though one not peculiar to it, and indeed one deeply ingrained and persistent up into contemporary ontology (e.g., found in the debates over quantum ontology)[11], is the thesis that ontic predicates ('forms') always require non-ontic-predicates (non-'forms') as subjects ('matter'). The pre-critical intuition here is that ontic predicates as intension-determined-combinators are incomplete and dependent entities in that they presuppose for their existences recipients or 'patients' of their unifying acts (each an '*ens ad aliud*' (a being-toward-something-else) or Fregean 'unsaturated'), *and* that these presupposed subjects cannot be further such acts, but rather must be complete in the sense of combinatorially inert, e.g., 'substances' (each an '*ens in se*' (a being-in-itself) and '*ens per se*' (a being-through-itself)), or substance-like entities (e.g., prime matter or Fregean 'objects'). Otherwise stated, the second conjunct asserts that what is inherently dependent requires something inherently independent to sustain it in its being. Figuratively, the situation is thought to be that without the analog of a self-supporting *terra firma* we will have the explanatory failure of 'stacked turtles all the way down'. This view is false, and profoundly so: It is the case that at an atomic level ontic predicates as individuated relation

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[8] Bertrand Russell, *The Principles of Mathematics*, 2d ed. (1903: reprt. ed., New York: Norton, 1938), pp. 221ff.

[9] Herbert Hochberg, 'The Wiener-Kuratowski Procedure and the Analysis of Order', *Analysis* 41 (1981): 161-63, and 'A Refutation of Moderate Nominalism', *Australasian Journal of Philosophy* 66 (1988): 188-207. Also see Mertz, *Moderate Realism*, pp. 163-73.

[10] Paul Teller, 'Relational Holism', *British Journal for the Philosophy of Science* 37 (1986): 71-81.

[11] French and Ladyman, 'Remodelling Structural Realism'.

(including property) instances, R^n_i, can have other relation instances as relata in the manner of a closed circle of combinatorial dependence, and where the resultant structural wholes are themselves non-dependent as non-predicable (each an '*ens in se*', though literally not an '*ens per se*'—not 'a being in virtue of itself'). How this is possible will be reviewed below. Denied this fact, the tradition concluded that in order to avoid an explanatory vicious infinite regress there must be for every structured entity, when subjected to a downwardly iterated analysis of structure into sub-structure, some bottommost level of absolutely unstructured and non-dependent entities, i.e., entities not themselves, or any of their constituents, having the natures of agent combinators, and hence, in this way, not themselves essentially dependent for their existences upon other entities. Or in short: Ontic predicates presuppose for their existence non-ontic-predicates as their subjects. This is the previously referenced *Inert Substrata Thesis*. Logically and in the literature these foundational non-predicable subjects divide according to possible combinations of (at least apparent) repeatability and unrepeatability treated as aspects of them. These possible self-sufficient substrata are accordingly: a) repeatable intensions i.e., abstracted universals, taken as non-combinatorial; b) individuated intensions in the form of substance-like, particularized (and necessarily) non-predicable and *monadic* 'qualities' or tropes, e.g., t-Red_i, t-$Round_j$, etc. ('t' for trope); or c) posited unrepeatable but internally non-qualitatively determined or natureless particulars known as 'bare particulars'. A physical object, or 'thick particular', is analyzed under a) and b) as *a compresent bundle* of either universals or tropes, respectively, and under c) as a plurality of universals *'tied-to' but not ontically predicated of* a bare particular, as such collected into and rendered unrepeatable as a single resultant 'thick' particular. Against each of these theories are serious challenges found in the literature,[12] and though I shall mention some of them briefly in the course of the following, I shall offer other arguments not generally exploited. The point will be that the Inert Substrata Thesis is untenable, making the alternative theory of only atomic mutually sustaining ontic predicates as urgent as I will show it is possible.

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[12] E.g., Michael Loux, *Metaphysics: A Contemporary Introduction* (New York: Routledge, 1988), pp. 87, 93ff. See the relevant essays in Stephen Laurence and Cynthia Macdonald, *Contemporary Readings in the Foundations of Metaphysics* (Oxford: Blackwell, 1998). Fredrik Stjernberg, 'An Argument Against the Trope Theory', *Erkenntnis* 59 (2003): 37-46.

Consider first bare particulars and what I take to be the standard analysis leading to their posit.[13] This analysis will also serve as context for eliminating option a) and the setting up of means for eliminating option b). The underlying theses are as follows (using 'B' to designate their introduction in the context of bare particulars).

Thesis B1: (Pure) monadic ontic predicates F(x), G(x), H(x), …, characterizing an unrepeatable subject individual a (i.e., such that propositions F(a), G(a), H(a), … are true) are or have intensions, respectively, F, G, H, …, that are *constituents of subject a*.

This is the classic containment or inherence model of ontic predication; *praedicatum inest subjecto*.

Thesis B2: An individual a exists if and only if a has at least one monadic ontic predicate P(x), i.e., a exemplifies P, and thus the proposition that P(a) is true.

Thesis B2 is a version of the common assertion that entities cannot exist without being subjects of characterizing properties (and relations) any more than properties (and relations) can exist without subjects to characterize (though the dependencies are of different types).

Thesis B3: Intensions in themselves are repeatable, i.e., universals, in being numerically the same constituents of numerically distinct subjects and thereby accounting for these subjects being of the same kind, and, any collection or bundle of them is likewise repeatable.

Here we have the simple and decisive reason why an ordinary thick *particular* cannot be simply a bundle of universals, and hence the standard observation that option a) must reduce to option c). I note also the arguments against option a) that it would make the Principle of the Identity of Indiscernibles a necessary truth, which it is not, and that intensions in themselves and therefore their bundles are causally inert—they cannot enter into causal relations with other bundles, i.e., there would be no causal relations among thick particulars. It must be the case, then, that:

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[13] James Moreland and Timothy Pickavance, 'Bare Particulars and Individuation: A Reply to Mertz', *Australasian Journal of Philosophy* 81 (2003): 1-13.

Thesis B4: If an unrepeatable entity *a* is composed in part of repeatable intensions, then it must have in addition at least one constituent that is unrepeatable so as to account for the unrepeatability of resultant whole *a*.

The most economical way to satisfy these theses and to account for the unity into a whole of all the constituents is with:

Thesis B5: An ordinary individual *a*, e.g., an apple, consists solely and essentially in—has as its sole identity-bestowing constituents—the repeatable intensions of its monadic ontic predicates *and* a single individuator p_a that unifies the former intensions by each being in some manner tied-to it.

Now, the problem with these theses taken jointly and as is is that they lead to a vicious infinite regress. On the assumption that particular p_a exists, then by Thesis B2 there is some ontic predicate P(x) such that P(p_a). In the literature these properties have been given to include Is-Unrepeatable, Is-Simple, Is-Constitutive-of-One-Object-at-a-Time, Has-No-Other-Properties-Except-These. Then, by Thesis B1, repeatable intension P is a proper constituent of unrepeatable p_a, and this requires by Thesis B4 at least one additional individuator as a proper constituent of p_a itself, p_a'. Clearly this is the beginning of a vicious infinite regress, i.e., p_a' must succumb to the same analysis as did p_a, requiring that p_a' have a further constituent individuator p_a'', which in turn must succumb to the same analysis, and so on.

Advocates of individuating substrata p_a must avoid this regress, and they do so by limiting Thesis B2 so as to exclude them. That is, as sole and saving (*ad hoc*?) exceptions, individuating substrata p_a are held to exist without any *exemplifying* properties in the proper sense—they are *characterizable* by no properties and hence the designation 'bare' particulars. Trading on the intuitiveness of Thesis B2, advocates likewise insist that bare particulars cannot exist without associated properties, but, crucially, the 'association' here must be just that: a nature- or intension-irrelevant conjunction or blank association, e.g., by a 'Tied-to' relation. In the words of J. P. Moreland, "It is open to an advocate of bare particulars to claim that it is a primitive fact that properties are tied to them and this does not need to be grounded in some further capacity or property within them", the

latter as "contained within the inner nature of the bare particular."[14] This character of 'having' properties only by non-descriptive arbitrary association is, as we shall emphasize, a principal nemesis to bare particulars. Preliminary to this, however, note the standard challenges that, first, if a bare particular exemplifies no intensions and so has no properties then it cannot be a relatum for any causal relation whatsoever, and, in particular, we could have no epistemic access to it, i.e., nothing individual *qua* individual would be given in experience, which is counter-factual. Moreover, an entity that does not enter into causal relations is neither destructible nor creatable, and this not only gives bare particulars a metaphysical status that should give one pause but also presents the following problem: What happens to a bare particular p_a when its thick particular *a* goes out of existence? Can it be recycled? It could not by any subsequent thick particular *b* having all the same properties as *a*, for in this case *a* would be numerically identical to *b*. This means that p_a's 'experience' with the set of properties as they jointly went into the making of *a* had to leave a positive mark on p_a preventing it from being associated with these properties again, as in *b*. But such a mark can only be a property of p_a and this contradicts its propertyless status as a bare particular.

Secondly, a bare particular would have to be a natureless entity, a status openly admitted by, for example, Gustav Bergmann: "Bare particulars neither are nor have natures."[15] If it were otherwise a bare particular would be the subject of ontic predicates characterizing its nature and so resulting in the above regress. Yet, something without a nature is *no*-thing—it can not be the 'nature of' a entity to be a natureless entity. Indeed, the intuition behind Thesis B2 would seem to be that an entity exists if and only if it is a *specific something*, and this specificity is a qualitatively determinate nature, relevant as such to intensions of certain ontic predicates (and not others) and because of which these properties (and relations) are combinatorial of and descriptive of it. To have no ontic predicates is to have no nature and so not to exist. Even a bare particular would have to have a specific essence or nature that makes it to be what it is and distinguishes it not only from, say, a tree, an intension, the number three, etc., but also from other bare particulars—what makes p_a's 'thisness' distinct from p_b's 'thisness'. Without these differentiating constituting essences all bare particu-

[14] James P. Moreland, 'Theories of Individuation: A Reconsideration of Bare Particulars', *Pacific Philosophical Quarterly* 79 (1998): 251-63, p. 258.
[15] Gustav Bergmann, *Realism* (Madison: University of Wisconsin Press, 1967), p. 24.

lars would reduce to a single one and hence, absurdly, there would be but one extant thick particular. Thirdly, if a bare particular can exemplify no properties it cannot have what are nevertheless its apparent *prima facie* essential properties of Is-Unrepeatable, Is-Simple, etc. Recently, J. Moreland and T. Pickavance have attempted to nullify this implication by arguing that, in fact, expressions 'Is-Unrepeatable', 'Is-Simple', etc., are linguistic predicates that do not correspond to any genuine ontic predicates.[16] The argument is that these are all less perspicuous versions of negative linguistic predicates, e.g., 'Is-Unrepeatable' is the same as 'Is-not-Repeatable', and as such they mark the extra-linguistic absence of the mentioned positive property. The true proposition Is-not-Repeatable(*a*) asserts that subject *a* *lacks* the property with intension Repeatable, and hence this proposition and negative propositions generally do not require commitment to any nature of *a*. I have argued to the contrary, that true negative propositions require as grounds or 'truth-makers' specific essences for the subjects referenced. Specifically, the properties or relations referenced in these propositions do not obtain among the referenced subjects because the latter have combinatorial of them ontic predicates that exclude the denied attributes, and to have these positive attributes presupposes their subjects have inherent determinate natures founding them. Both of the propositions: that Apple *a* is green, and, that Apple *a* is not green, have true-values determined in part by the nature of *a*. Apple *a* is not green because it has a contrary property, say, of being red, and, for spatial entities *a* and *b*, *a* is not to the left of *b* because *a* and *b* have some other contrary spatial relations, the latter obtaining on at least the condition that *a* and *b* have the natures of extended/spatial-relevant entities. Even the true negative proposition that 2 is not left of 3 turns on the specific natures of 2 and 3, putting them in a category distinct from that of spatial entities. If all of this were otherwise then all negative assertions would be neither true nor false but simply arbitrary denial independent and non-descriptive of reality.

Finally, in addition to these mostly familiar arguments against bare particulars, there are two further arguments, the first being the promised simple and, I propose, more obviously fatal argument that turns on the fact that a bare particular has intensions attached to it, not by characterizing ontic predication, but only by nature-irrelevant arbitrary conjunction, e.g., the Tied-to relation. This undiscriminating unification is the type of unity found among the elements of a list, set, or mereological fusion where the essences

[16] Moreland and Pickavance, 'Bare Particulars and Individuation'.

of the elements are irrelevant to their being linked. The key propositions at issue here are: A bare particular p_a is characterized by no properties, or alternately, exemplifies no intensions whatsoever; and, a thick particular *a* has properties exemplifying intensions F, G, H, ..., if and only if F, G, H, ..., are tied-to *a*'s underlying bare particular p_a. Now, what the completely arbitrary nature of the Tied-to relation implies is that any intensions *whatsoever* can be equally linked to a bare particular p_a, *including contrary or contradictory intensions*, e.g., it could be true that Tied-to(Round,p_a) and Tied-to(Square,p_a). That is, there is nothing inherent to a set of intensions tied to a bare particular that would preclude it from containing contrary or contradictory intensions, anymore that it can be held impossible that intensions Round and Square could be jointly associated with some entity *x* in a set: {Round,*x*,Square}. In order for the linking of an intension P with an entity *x* to preclude the linking with *x* of intensions contrary or contradictory to P, this linking must be that of nature-relevant ontic predication, not that of free association as with the Tied-to relation. Alternately said, for an intension P of *x* to be exclusionary of other intensions of *x*, P must be a component of a property as it is characterizingly predicable of ('says something about the nature of') *x*, and not just arbitrarily juxtaposed with (and so indifferent to the nature of) *x*. Now, what this means is that there is no non-arbitrary reason why in this ontology of bare particulars there could not exist a thick particular *a* resulting from the bundling of contrary or contradictory properties with a unifying bare particular, or more explicitly on the second proposition above, why a thick particular could not exemplify contrary or contradictory properties, and this is absurd. Finally, there is the related argument that if an ordinary thick particular *a* reduces to intensions each arbitrarily tied to bare particular p_a then the distinction between accidental and essential properties of *a* cannot be explained. In sum, the concept of a bare particular is incoherent. Moreover, on the analysis advanced herein the necessity of positing a substratum bare particular to account for either the collective unity of the properties of an ordinary particular or for its individuation disappears.

This leaves us to consider briefly entities under option b)—tropes—as the last of the alternatives required under the Inert Substrata Thesis. Trope nominalists reject repeatable intensions and all *monadic* (note!) ontic predicates as subject-dependent entities, and in this reject as stated all of the prior Theses B1-B5. The strategy of trope theorists is to explicitly admit the qualitative aspect of entities but in such a way that it is consistent with their nominalism; that it avoids the necessity of positing an underly-

ing bare particular; and that it conforms to the Inert Substrata Thesis. This is done by construing *monadic* properties as unrepeatable, non-composite, non-ontic-predicates, i.e., by positing the collapsing together of an apparently repeatable qualitative aspect of single entities, e.g., the quality Red, with an individuating aspect so as to form an absolutely simple, non-composite individuated property that is substance-like in being itself non-combinatorial of any subject. The theses characterizing trope theory are then as follows (using 'T' to designate the relevance to trope theory):

Thesis T1: Given monadic linguistic predicates F, G, H, …, of a prescribed class (usually phenomenal or physicalistic) such that for a particular *a* propositions F(*a*), G(*a*), H(*a*), …, are true, then there exist corresponding to each a non-composite natured individual or trope, t-F_i, t-G_j, t-H_k, …(e.g., t-Red_i, t-$Round_j$, t-$Mass_k$), that are each constituents of *a*.

Thesis T2: A set of tropes each compose a thick particular *a* by being pairwise joined via a Compresence (or similar) relation.

Thesis T3: Tropes may enter into a (exact) Resemblance relation with other tropes, e.g., t-Red_i exactly resembles t-Red_j, where, though the obtaining of the relation is a function of the qualitative content of its relata, it is primitive in the sense that there is nothing numerically identical in each relatum that founds the relation.

For trope theory, then, an ordinary thick particular is a compresent bundle of 'non-bare' yet 'very thin' particulars—each with a single qualitative, though not numerically repeatable, aspect that determines it to fall within a certain resemblance equivalence class, the latter being nominalism's surrogate for an intension universal. Now, as was noted, there are a number of objections to trope theory found in the literature. I will mention two of these. First, equivalence classes or sets of resembling tropes, e.g., the set of all red-resembling tropes or the set of mass-resembling tropes, are claimed to do the work of the realists' shared universals, e.g., Red or Mass, in explaining non-arbitrary classifications. In other words, the commonality that makes, say, a group of tropes to be red-tropes is not explained intensionally by a shared universal, Red, composing each, but rather, in the opposite direction and extensionally, by just these tropes composing a fixed whole—the equivalence class. This class is the single feature that all these and only these tropes have in common, and it defines their 'kind', e.g., their being red. But this tack fails, and it fails even under the ontically

more accurate analysis where the whole is identified with the structure consisting jointly of tropes interrelated by the Resemblance relation. This is so because the whole as either a set or resemblance structure has its constituents necessarily, and would not be the same whole if it had more or less constituents. Hence, the sets or structures that are surrogates for Red or Mass could not have different mutually resembling tropes than they do. In other words, there could not have been more or fewer red things, or, indeed, more or fewer physical objects having mass. Of course, this generalizes to all such equivalence classes or structures: there could not have been more or less of any kind whatsoever. And, this is false. For, just as there is nothing inherent in a contingently exemplified intension, e.g., Red or Mass, that fixes its extension, there is nothing inherent in tropes (each an 'individuated intension'), whether individually or collectively in resemblance classes or structures, that precludes there being more or less of them resembling in the same way, and thus no single such whole could serve as an account of why certain tropes are classified as the 'same kind', e.g., as red. In short, there is no fixed class that could act as a surrogate for a contingently exemplified universal, or, alternately, intensionality cannot be explained in terms of extensionality. Nominalism in whatever guise cannot escape the recognition of shared intension universals.

A second common argument against tropes starts with the observation that tropes themselves have (pure) properties, e.g., trope t-Square$_i$ has the properties Is-Polygonal, Is-a-Shape, Is-Concrete (i.e., is in space and time), Is-Unrepeatable, Is-Qualitatively-Determined (i.e., is a non-'bare' particular). On the same analysis trope theory gives properties of ordinary particulars, viz., construing them as tropes bundled to compose the particulars, likewise properties of tropes would have to be construed as further tropes bundled to compose their subject tropes, and hence, contrary to T1, tropes would be composite. Indeed, with iterations of properties like Is-Unrepeatable, a given trope, e.g., t-Red$_i$, would be composed of a downward infinite regression of contained t-Unrepeatable$_j$ containing t-Un-repeatable$_k$ containing t-Unrepeatable$_l$ containing…. To avoid all of this proponents would have to generate some tortured theory as to why these linguistic predicates, despite all appearances, have no corresponding properties or tropes. The underlying problem here is the assumption that what characterizes an entity must be a constituent of it, as specified in T1.

In addition to these arguments against trope theory, I offer the following: First, as broached above, the Compresence relation cannot be sim-

ply arbitrary or blank association, or we would have the same difficulties as with the Tied-to relation above. The Compresence relation must have as part of its minimal content or 'meaning' a precluding of contraries as relata, e.g., it is necessarily false that Compresence(t-Red_i,t-$Yellow_j$). If it were otherwise then, as with the Tied-to relation, it would be possible for the same complex entity to be, say, both red and yellow. But now there exist complex entities that have contrary properties in the sense of, for example, a metal bar with what here would be trope t-Red_i composing part of one end and trope t-$Yellow_j$ composing part of the other (say, the ends' different responses to a heating of the bar). Now, if tropes and the Compresence relation are the only ontic ingredients making up complex entities in this ontology, and if the bar is such an entity, then, because the Compresence relation is transitive, we would have as true the proposition Compresence(t-Red_i,t-$Yellow_j$). So the alternatives are that we either give up the vast class of entities of which the bar is representative as only illusionally single entities, or admit that such entities are composed of additional things—what could only be relations other than and not reducible to Compresence or other tropes.

Second and relatedly, trope bundles, whether unified by the standard Compresence relation or by a relation expressing some further intension-relevance between its subjects, such as Peter Simons' Husserl-type 'mutual founding' relation,[17] are, because either composing relation is symmetric, virtually without internal order, system or structure. Yet, our initiating point in this essay was that robust internal structure and this at each level in emerging hierarchies is precisely the ubiquitous ontological given and what must be explained. Compresence or Mutual-Founding relations take only tropes as relata, not other bundles, and so cannot generate from the bottom up hierarchies of nested entities. Moreover, it is a given that distinct complexes can have the same parts differently structured, i.e., differently related (either by relations with different intensions or by the same other-than-symmetric relations having relata in different positions), but this is not possible when the only unifying cause of a complex entity is a symmetric relation. What are required are ordering asymmetric and non-symmetric relations, and this ordering generalized to 3-adic, 4-adic, etc., relations (a point made without specifics by Simons[18]). However, once

[17] Simons, 'Particulars in Particular Clothing'.
[18] Peter Simons, 'Farewell to Substance: A Differentiated Leave-Taking', *Ratio* XI (1998): 235-52.

such polyadic relations are admitted into trope theory, we have the following cobbled bifurcated ontology. First, we are reminded that such *n*-adic relations are irreducible to monadic properties of their relata, and so must be admitted as existing fully 'between' and combinatorial of ('actually relating') their *n*-subjects as they qualify these subjects jointly (hence the error of the inherence model of predication). That is, definitionally a relation is an intension-determined-linking of multiple subjects, and as there can be no linking without something linked, there can be no polyadic relation without subjects standing in this relation. A relation in the full sense depends for existence upon the simultaneous existences of other entities and its unifying agency among them—it is a dependent *ens ad aliud* that cannot exist outside of a fact. Assisted by language it is possible to cognitively abstract from a relation in a fact, e.g., :Is-Between(*a*,*b*,*c*) or :Loves(*a*,*b*), a combinatorialless/inert intension, e.g., Between/Betweenness or Love, that when compared to the former are clearly derivative and would be called relations only in a secondary sense. So now in regard to countenancing trope theory we have the following situation: Intrinsic to both properties and relations is the uniform fact of intensions involved in qualitatively characterizing/being-attributable-of one or more subjects, with the only difference being the accidental one of the number of subjects characterized. Further and reinforcing the latter, both properties and relations are seamlessly formalized in our standard logics as equally in the category of predicates. Yet contrary to both this ontic and logical continuity, we have intrinsic to trope theory the ontological bifurcation of monadic predicates treated as non-combinatorial, non-dependent, atomic 'little substances' (i.e., 'subjects' or 'objects' only—each an *ens per se*), and polyadic predicates treated as just the opposite. This bifurcation should strike us as not only suspiciously artificial, but at this point as an error based upon confusing a derivative inert monadic intension, e.g., Red or Mass, with a predicable-of/subject-qualifying and so subject-dependent property, e.g., Is-Red or Has-Mass, and further as an error motivated by—indeed required by—what is the background assumption of the Inert Substrata Thesis. The Thesis applied to an ontology exclusively of attributes requires some class of non-dependent/non-combinatorial entities to support all other dependent/combinatorial entities, and since polyadic relations are clearly the latter, this leaves monadic properties so construed (what are easily misconstrued as the limiting 1-adic case) to fit the bill, viz., predicable properties turned into non-predicable tropes.

In sum, the argument thus far is that all the options a)-c) under the Inert Substrata Thesis, i.e., theories advocating either intensions, tropes, or bare particulars as required ultimate non-predicable substrata, are equally defective. What is needed in response to these negative results is an ontology that actually displays the positive possibility of an alternative to the Thesis. We shall now observe how this is provided in an ontology of network instance realism.

3. Ontic predicates as Individuated Substrata, and Their Compounds

The errors of the Inert Substrata Thesis and the various theories attempting to enforce it are abetted by the naive assumption that monadic ontic predicates—properties—are paradigm and fundamental. Theses B1 and T1 are plausible only on this assumption. As in the tradition the assumption requires that polyadic relations be given either some 'quasi-real' status (Aristotle, *Meta*. 1088a22), e.g., they 'supervene' on their relata or properties thereof but represent no ontic addition, or they reduce without remainder to properties of their relata. Both of these strategies are unsuccessful upon analysis, to say nothing of being *prima facie* contrived and forced. Indeed, when polyadic relations are recognized full and unreduced, with monadic properties the limiting though easily distorted case, there are liberating and profound implications for ontology, implications that correct the above theses and provide an alternative to the Inert Substrata Thesis. I have given a full analysis of polyadic ontic predicates elsewhere[19] and shall here mostly summarize the results. Summarizing general points made above, the perspicuous feature of relations is that they are externally 'between' or 'among' their relata (in medieval terms, each an '*intervallum*' = 'interval'), and, historically less perspicuous (principally because of the distorting bias of the inherence model of predication) though crucial, each is an agent unifier of ('actually relates') its relata, effecting as such a plural whole that is a fact or state of affairs. The latter is the lesson of the classic Bradley's Regress argument. When fully analyzed we have the following detailed principles characterizing ontic predicates:

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[19] Mertz, 'Combinatorial Predication', 'An Instance Ontology for Structures', and 'The Nature and Necessity of Composite Simples', *Metaphysica: International Journal for Ontology & Metaphysics* 5 (2004): 89-133 [Essay 3 herein].

Principle I: Constitutive of every fact :$R^n_i(a_1,a_2,\ldots,a_n)$, for $n \geq 1$, is an ontic predicate, $R^n_i(x_1,x_2,\ldots,x_n)$, that is the external agent/cause of the characterizing predicable unity of itself with its relata, a_1, a_2, …, a_n, a unification whose type is to result in a fact, as opposed to a list, set, or mereological sum.

Principle II: Every ontic predicate $R^n_i(x_1,x_2,\ldots,x_n)$ has as a constituent a single universal intension R^n whose ontic role is that of delimiting or determining non-arbitrarily the possible *n*-tuples of relata, $<a_1,a_2,\ldots, a_n>$, that predicate $R^n_i(x_1,x_2,\ldots,x_n)$ can unify into a fact. However, an intension R^n of itself has no causal agency whatsoever as a unifier (it is 'predicably inert' or 'substance-like').

Principle III: In addition to and distinct from intension R^n, there is constitutive of ontic predicate $R^n_i(x_1,x_2,\ldots,x_n)$ its actual mode of union, its combinatorial or linking agency, among and to its particular *n*-tuple of subjects. The linking aspect of predicate $R^n_i(x_1,x_2,\ldots,x_n)$ is itself not a further intension in addition to R^n, but a *causal act of unification* that is 'joined' with intension R^n that controls its effects. This joining is the unity of a *continuous composite*, i.e., a union of two distinct entities without the agency of a further interposing ontic predicate or act of unification. Of fundamental importance, the unifying act of an ontic predicate is unrepeatable and particular, rendering the containing predicate an individual, i.e., a unit attribute (hence the subscripts, e.g., 'i').

Principle IV: The unifying act among an *n*-tuple of subjects is unique to that *n*-tuple. Hence, an instance ontic predicate subsuming this act is unique to this *n*-tuple of subjects, i.e., if $R^n_i(a_1,a_2,\ldots,a_n)$ and $R^n_i(b_1,b_2,\ldots,b_n)$, then $a_1 = b_1$, $a_2 = b_2$, …, $a_n = b_n$. In the opposite way, ontic economy requires that no *n*-tuple of subjects have more than one instance of the same intension R^n, i.e., if $R^n_i(a_1,a_2,...,a_n)$ and $R^n_j(a_1,a_2,\ldots,a_n)$, then $R^n_i = R^n_j$. Also, because it is intrinsic to an instance ontic predicate to be an agent unifier of an *n*-tuple of subjects, it cannot exist independent of this *n*-tuple except cognitively in selective abstraction.

Henceforth I shall abbreviate individuated ontic predicates or relation instances by dropping the variables designating the subject places, e.g., '$R^n_i(x_1, x_2,\ldots,x_n)$' will simply be '$R^n_i$', this being sufficiently distinguished from 'R^n'

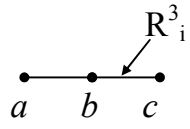
(i.e., without the subscript) used to refer to instance R^n_i's contained and determining intension. Now profound in its consequences, that ontic predicates are individuated to particular *n*-tuples of subjects follows immediately from their natures as unifying acts, and is perspicuous in the case of contingent relations. Assume, for example, that facts :Owes^3(*a*,*b*,*c*) and :Owes^3(*d*,*e*,*f*) both obtain, for $<a,b,c> \neq <d,e,f>$. The combinatorial act linking the elements of $<a,b,c>$ under the intension Owe^3 cannot be numerically the same as the unifying act under intension Owe^3 for $<d,e,f>$, though the intension is numerically the same. This is so because fact :Owes^3(*a*,*b*,*c*) can go out of existence, i.e., *a* can cease to owe *b* to *c*, without fact :Owes^3(*d*,*e*,*f*) ceasing to exist. If it were exactly and numerically the same unifying act for both facts they would have to come into and go out of existence together. It is more appropriate, then, that our facts given as ':Owes^3(*a*,*b*,*c*)' and ':Owes^3(*d*,*e*,*f*)' be designated as 'Owes^3_i(*a*,*b*,*c*)' and ':Owes^3_j(*d*,*e*,*f*)', where, as instance constituents of these facts, Owes$^3_i \neq$ Owes^3_j. In general, fact-effecting acts of predicable unification are as individual and unrepeatable as any other acts, e.g., events. Importantly, what this means is that the combinatorial agency of ontic predicates is ontology's *principium individuationis*—an insight that completely reverses the historical metaphysical role and status of ontic predicates. With this ontology we have a straightforward *account* of individuation without having to resort to simply positing either primitive 'thisness' (*haecceitas*) or incoherent bare particulars.

As an introduction to the implications of Principles I-IV let us contrast them with previous Theses B1-B5 and T1-T3. All of trope theory's T1-T3 are rejected, as are B1 and B5, but with B3 and B4 retained. Thesis B2 is independent of the above principles, yet is, I propose, true when extended as: An individual *a* exits if and only if *a* has at least one ontic predicate P^n_i, i.e., *a* as a subject exemplifies intension P^n, and thus the proposition that P^n_i(..,*a*,..) is true. Crucially and contrary to the misleading inherence model of predication inspiring theses B1 and T1, Principles I and II do not require that an ontic predicate or its contained intension enter into the composition of the subject(s) of the predicate, but rather in characterizing its subjects attaches itself externally to it (or them). The combinatorial act of attachment is a function of a qualitative relevance between the intension of the agent instance and the nature(s) of the instance's subject(s). In general, ontic predicates are not downwardly subsumed parts of their subjects, but rather are the instruments for themselves and their subjects to form upwardly emergent and subsuming wholes. It is the thesis of containment of ontic predicates by their subject individuals that necessitates their being con-

strued either as individual non-combinatorial and only monadic tropes, or as repeatable intensions requiring the posit as a further constituent of an absolutely qualityless individuator. Principle II agrees with B3 and contradicts T1 in admitting intension universals. Principle III details the requirement of Thesis B4 applied to ontic predicates, i.e., a repeatable intension R^n is joined in a non-predicable way with an unrepeatable combinatorial act that determines the particularity of resultant instance R^n_i. Neither intension nor unifying act are aspects or modes of the other, but are each abstractable aspects of the simple instance R^n_i, existing as separate only in the intellect.[20] Likewise, by Principle IV, an instance R^n_i exists separated from its *n*-tuple of subjects, and so from the fact they jointly compose, only in abstraction. Principle IV places conditions on how instances exist relative to *n*-tuples of subjects, conditions essential to the following further principles explicating the ontology of network instance realism.

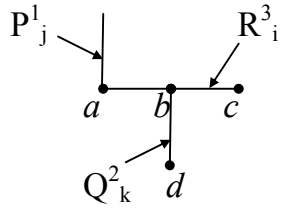
Let us now turn to the central issues of how relation instances characterized by Principles I-IV above can compose hierarchies of structures that are ordinary particulars, e.g., Socrates or a computer, and can at some atomic level be mutually sustaining and collectively complete and non-dependent. Consider first as an example of the simplest type of complex or structure, i.e., single facts, the fact $:R^3_i(a,b,c)$ as modeled with the following diagram:

Complex A:



The horizontal line segment represents the instance R^3_i as the shared unifier among subjects *a*, *b*, and *c*. Now consider two further facts, $P^1_j(a)$ and $Q^2_k(b,d)$, where monadic instance P^1_j shares its only subject *a* (hence a line segment with one subject dot) with triadic instance R^3_i, and dyadic instance Q^2_k shares subject relatum *b* with R^3_i. This would be diagrammed as:

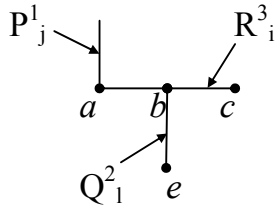
Complex B:



[20] See Mertz, 'The Nature and Necessity of Composite Simples'.

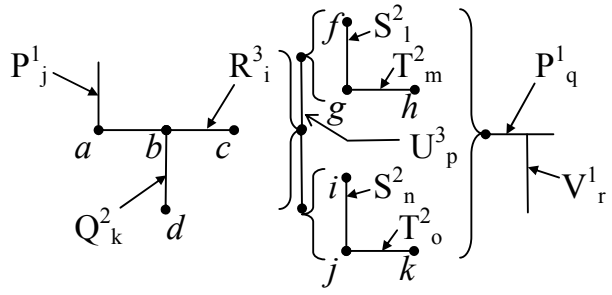
Complex B is a compound or molecular structure, and it is so by what can be called 'horizontal composition', i.e., a 'chain' of connectedness across pairs of relation instances sharing one or more relata, and a transitivity across such pairs via the sharing of an instance, e.g., R^3_i is the shared instance and so common link between relatum-sharing pairs P^1_j and R^3_i, and, R^3_i and Q^2_k. Note that, because instances are unique to their ordered *n*-tuples of subjects, if a relatum is changed then a relation instance of the same intension combinatorial of the replacement and the remaining relata will be numerically different. For example, if *d* is replaced by *e*, $e \neq d$, then instance Q^2_k changes to Q^2_l, where $Q^2_k \neq Q^2_l$. Consider such a change made in the Complex B yielding the following distinct structure.

Complex C:

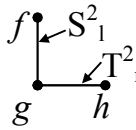


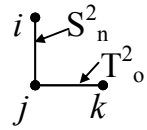
There are two important points to note in comparing Complexes B and C. First and intuitively, though B and C are not identical, they have *exactly the same structure*, i.e., they are isomorphic. Secondly, though a change of one relatum, *d*, to a non-identical relatum, *e*, necessitated a change of instance Q^2_k to Q^2_l, there are no other 'reverberations', i.e., changes, caused within the larger complex. This is not the case for the second type of structural composition, what is hierarchical or 'vertical composition' (notwithstanding the graphic orientation of the examples below). Here entire structures get treated as themselves single relata for further properties and relations, what can be indicated diagrammatically with the use of braces. Consider the following diagram utilizing B as a sub-structure.

Complex D:



Complex D illustrates both horizontal and vertical composition, with two levels of vertical composition. The left-most brace indicates that Complex B on the left of it is, as a whole, a single relatum for relation instance U^3_p, as are each of the isomorphic structures

Complex E: 

and **Complex F**: 

The right-most brace of Complex D indicates that the entire vertical compound to its left is itself a single subject for the property instance P^1_q. One could think of Complex D as representing, for example, the structure resulting from three molecules—Complex B and the two 'identical', i.e., isomorphic, Complexes E and F—structured among themselves by an instance of a triadic inter-molecular relation U^3, this compound in turn and as a whole having an instance of, say, causal property, P^1. Now, it is easy to conceive how this vertical compounding could be continued indefinitely up through further and further levels, and how at certain levels there could be properties and relations, say U^3, whose instances emerge *sui generis*, i.e., do not occur at lower levels and presuppose as at least some of their relata certain types of sub-structures. This fits the bill precisely for an ontology of ordinary objects set as the desideratum in the introduction: ordinary objects are immense though finite hierarchies of horizontally and vertically composed structures generated upwardly from what science determines are the ultimate sub-atomic entities. Similarly, once alerted to these two forms of composition one can see their iterations exemplified in cognitive, mathematical, logical, social, etc., structures. Vertical composition and its distinction from horizontal composition are the conditions *sine qua non* for a proper understanding of emergent properties and relations.

What is now required is that we make precise these intuitive notions of horizontal and vertical composition. This is done iteratively in the following principle, one asserted to characterize all forms of plural unity, starting with and built up from facts as atomic complexes. This in turn will afford refined and differentiated definitions of identity and indiscernibility, that for indiscernibility being particularly promising for solving philosophical problems concerning persistence through change of composition, e.g., the Ship of Theseus problem, and the problem of 'metaphysical underdetermination' for quantum objects.

Principle V: All plural unity—and thus plural wholes (complexes or structures)—is by the following:

a) A relation instance R^n_i predicable of an *n*-tuple of relata, $<a_1,a_2, \ldots,a_n>$, is the cause of an individual plural whole, viz., the fact $:R^n_i(a_1,a_2,\ldots,a_n)$, having R^n_i, a_1, a_2, ..., a_n, as its only constituents.
b) If R^n_i is a constituent of a plural whole *x* and S^n_j is a constituent of a plural whole *y*, and R^n_i and S^n_j, share one or more relata, then there is an individual plural whole *z* that has as constituents all and only the combined constituents of *x* and *y* (horizontal composition).
c) For any fact $:R^n_i(a_1,a_2,...,a_n)$, if for $1 \leq j \leq n$, a_j is a plural whole, then there exists an individual plural whole whose constituents are all and only the constituents of the fact and constituents of a_j (vertical composition).

Principle V is the account of all forms of composition and so of plural wholes whatsoever, and in this regard corrects the erroneous and anemic Theses B5 and T2 above. It likewise serves to highlight what is the debilitating misanalogy of sets or mereological sums used as models for complex entities. Consider next the instance analog of the standard definition of identity [I argue for a refined specification of identity in Essay 7 herein]:

> **Principle VI**: Entities *a* and *b* are identical, $a = b$, if and only if, for every monadic property P^1 and every instance P^1_i of P^1, $P^1_i(a)$ if and only if $P^1_i(b)$.

The more specific identity condition on complexes is given by:

> **Principle VII**: For complexes *x* and *y*, $x = y$ if and only if, for every intension R^n and very instance R^n_i of R^n, R^n_i is a constituent of *x* if and only if R^n_i is a constituent of *y*.

This is so because predicate instances do not exist independently of their relata and, by Principle IV, numerically the same instances have numerically the same relata, combined with the central thesis of this ontology that the being of a complex entity consists solely in its constituent ontic predicates and their relata. Principle VII explicates accurately the intuition that 'constitution is identity', and corrects the common but crude version of 'mereological extensionality' that ignores component (individuated) ontic predicates that are nevertheless essential to every plural whole.

The final principle makes perspicuous the traditionally obscure notion of indiscernibility and how it is derived from the primitive but transparent indiscernibility of relation instances of the same type. It is our precritical intuition that it is ontologically possible for there to exist numerically distinct but absolutely qualitatively the same entities, e.g., two exactly resembling leaves. And more specifically, this indiscernibility has exclusively to do with the internal constitution of the compared entities *a* and *b* such that a property descriptive of a constituent aspect of *a* is also descriptive of a corresponding constituent aspect of *b*, and conversely. But so stated in terms of the *properties of* composing aspects, this is a derivative means of specifying indiscernibility, whereas under the ontic analysis herein we can provide a primary definition in terms of what must be the composing aspects themselves of every composite entity, viz., predicate instances. For if, as we are about to see, at some atomic ontic level relation (including property) instances can be horizontally mutually combinatorial and that all other existents are built up by vertical and horizontal composition on these atomic structures as relata, then indiscernibility can be specified *universally* and recursively as:

Principle VIII: Entities x and y are indiscernible if and only if

a) $x = \mathrm{R}^{n}_{i}$ and $y = \mathrm{R}^{n}_{j}$, where R^{n}_{i} and R^{n}_{j} are instances of the same intension R^{n}.

b) $x = :\mathrm{R}^{n}_{i}(a_1,a_2,...,a_n)$ and $y = :\mathrm{R}^{n}_{j}(b_1,b_2,...,b_n)$ and a_k and b_k are indiscernible for $1 \leq k \leq n$.

c) x and y are complexes such that there is a one-to-one correspondence ϕ between their constituent facts where $\phi(:\mathrm{R}^{n}_{i}(a_1,a_2,...,a_n)) = :\mathrm{R}^{n}_{j}(b_1,b_2,...,b_n)$ and where $:\mathrm{R}^{n}_{i}(a_1,a_2,...,a_n)$ and $:\mathrm{R}^{n}_{j}(b_1,b_2,...,b_n)$ are indiscernible.

Foundational section VIII-a asserts relation instances to be what I propose are the unambiguous counter-examples to the Leibnizian Principle of the Identity of Indiscernibles, viz., instances R^{n}_{i} and R^{n}_{j} (e.g., Is-Between$^{2}_{i}$ and Is-Between$^{2}_{j}$) can differ numerically though their sole qualitative content R^{n} (e.g., Between^{2}) is numerically identical across both. And recall that instances with the same intension differ, not by each having some simply posited and inscrutable *haecceitas* or bare individuator, but by their unrepeatable combinatorial agencies, what is both the intuitive nature of ontic predicates and the requisite *ontoglial* for a plural reality. If other entities are built up from indiscernible atomic instances in accordance with VIII-b

and –c, then we would have structures with complexity to any degree that are *numerically distinct but qualitatively identical*. Or, for illustration in reverse direction consider isomorphic complexes E and F above. They would be indiscernible if under VIII-c and the one-to-one correspondence ϕ where $\phi(:S^2_l(f,g)) = :S^2_n(i,j)$, and $\phi(:T^2_m(g,h)) = :T^2_o(j,k)$, the facts in the pairs $:S^2_l(f,g)$ and $:S^2_n(i,j)$, and, $:T^2_m(g,h)$ and $:T^2_o(j,k)$, are indiscernible. The latter would be the case under VIII-b if corresponding relata *f* and *i*, *g* and *j*, and *h* and *k* are, as paired, indiscernible. The latter would obtain, in turn, if the relata in each pair were again either complexes indiscernible under VIII-c or facts indiscernible under VIII-b. Now this regress for determining indiscernibility would stop if in the downward analysis we reach in each case a bottom level of compound complexes where the composing facts of each have only property or relation instances of its other composing facts as relata—the same demonstration needed to negate the Inert Substrata Thesis and what will be given below. In this situation VIII-a would apply and no entity would be left outside of the scope of the applicability of VIII as a criterion for indiscernibility. Hence, built exclusively of relation instances that differ only numerically, indiscernible complexes so specified would differ only numerically, in whole and in every corresponding part. These complexes would be intrinsically and objectively indiscernible prior to epistemological considerations of re-identification by a knower. It is, then, that we have for indiscernibility the analog of the formal specification in Principle VII for identity, given the specification under Principle V of how instances can be constituents of a complex: Complexes *x* and *y* are indiscernible if and only if, for every intension R^n, an instance R^n_i is a constituent of *x* if and only if an instance R^n_j is a constituent of *y*. [This corrects my previously offered definition of indiscernibility: Entities *a* and *b* are indiscernible if and only if, for every monadic property P^1, there is an instance P^1_i such that $P^1_i(a)$ if and only if there is an instance P^1_j such that $P^1_j(b)$.[21]]

Consider the topic from the side of discernibility. Instances differ other than numerically in two ways: either by having non-synonymous intensions, or, having the same intension, they have different relata *n*-tuples, the exception to the latter being when the *n*-tuples differ only in order of

[21] This definition of indiscernibility was proposed initially in my *Moderate Realism*, and was advocated thereafter in Mertz, 'The Logic of Instance Ontology', *Journal of Philosophical Logic* 28 (1999): 81-111 [Essay 6 herein], p. 92, and in 'An Instance Ontology for Structures'. This definition was also recommended in passing in the original version of this essay.

relata and this is irrelevant to the intension (e.g., for facts :Next-To^2_i(*a*,*b*) and :Next-To^2_j(*b*,*a*), the distinction in *n*-tuples <*a*,*b*> and <*b*,*a*> is irrelevant to symmetric intension Next-To^2, i.e., the facts are identical, but not so if the intension had been, say, the non-symmetric Love^2). Consequently, two hierarchical complexes, say our two leaves, differ other than numerically by having at some level sub-complexes that are not indiscernible, which means formally that for every possible one-to-one correspondence of composing facts of these sub-complexes there exists one or more corresponding composing instances that differ in one of the above ways. In practice, discernible complexes are known to be such because they are known as wholes to be subjects of contrary properties or relations.

Significantly then, including the possibly of resolving current problems of 'particle identity' in quantum mechanics, indiscernible complexes so specified would be epistemically differentiated—known as numerically not the same—only when known as jointly embedded in a further meta-structure composed of them as relata for instances of differentiating irreflexive or non-reflexive relations, e.g., spatial or causal relations. Now consider the following situation. If, say, these indiscernible sub-structures, *a* and *b*, were permuted back and forth several times in the context of a meta-structure that 'remained constant' throughout, i.e., resulting in a temporally extended meta-meta-structure consisting in a connected sequence of these meta-structures chronicling the permutations, then a knower cognizant of the full unbroken sequence, and in this the 'continuous spatio-temporal trajectories' of both *a* and *b*, would, of course, be able to re-identify in the last permutation meta-structure of the sequence which of the permuted indiscernible sub-structures was *a* and which was *b*. That is, *a* would be known as *a* and *b* would be known as *b* throughout and so each would retain its 'identity', or more accurately, its identification, throughout the sequence known in its continuity. However, if for a knower knowledge of the complete sequence of permutations were 'broken'—incomplete or unavailable (e.g., spatio-temporal trajectories from quantum particles are not precisely defined)—then cognizance of the last permutation meta-structure would still be sufficient to discern the *numerical differentiation* of *a* from *b* but not sufficient for their particular identifications, i.e., not sufficient to re-identify which one was which. Now, this would seem to describe the apparent and ontologically challenging situation with the 'vague' entities of micro-physics. Under the 'Indistinguishability Postulate' of quantum statistics, permutations of quantum particles are not counted as representing new arrangements, there being no observational means for distinguishing the per-

mutations.[22] In this way quantum mechanics describes states of indistinguishable but numerically distinct particles, particles said to be cardinally but not ordinally distinct. What has been lacking is an ontological account of this situation, an account that the instance ontology outlined here would seem to provide nicely: if indiscernible complexes specified by VIII (say E and F where their corresponding relata are indiscernible, which rests ultimately on the proof below) are permuted an unknown number of times in a subsuming 'constant' meta-structure-type (including experimental context), then the first meta-structure, say D above, and the last meta-structure, D′, would themselves be numerically distinct but indiscernible, and in this sense there would be no qualitative 'observational difference', i.e., intensionally different composing properties or relations, distinguishing the subsuming contexts, D and D′. Relative to these alterations we could say that the complex type of D and D′ is 'permutation invariant'. Just as it can be said of quantum particles, it is true here of two or more indiscernible entities in the same fixed context/meta-structure, and without a knowable continuous 'trajectory' for each entity, that relative to any possible permutation 'no measurement whatsoever could serve in principle to determine which of the indiscernible entities are which'. In such contexts indiscernible complexes E and F could not be 'named individually', i.e., re-identified, and so in jointly composing the D-type structure would have a cardinality of two but no ordinality.

More generally, quantum particles are said to violate even the weakest form of the Principle of the Identity of Indiscernibles, and thus in not differing by repeatable properties (i.e., construed as intension universals) these particles either differ by some other non-property, non-universal constituent individuators (the options cited being *haecceitas* or bare particulars—known in this context as 'transcendental individuators'), or they differ neither by uniquely possessed intensions nor individuators and are thus some sort of strange 'non-individuals' or 'quanta'. It has been proposed but has remained undeveloped how a 'Structural Realism' might reconcile the individual/non-individual dichotomy by providing a precise formulation of

[22] Steven French and Michael Redhead, 'Quantum Physics and the Identity of Indiscernibles', *British Journal for the Philosophy of Science* 39 (1988): 233-46. Steven French, 'On the Withering Away of Physical Objects', in *Interpreting Bodies: Classical and Quantum Objects in Modern Physics*, ed. Elena Castellani (Princeton: Princeton University Press, 1998), pp. 93-113. French and Ladyman, 'Remodelling Structural Realism'. Hilborn and Yuca, 'Identical Particles in Quantum Mechanics Revisited'.

the relational holism characterizing quantum particles and fields.[23] The ontology presented herein—what I have called network instance realism—details what has promise as such a synthesizing structuralism. It provides a precise specification of indiscernibility showing perspicuously how entities of any degree of complexity can be numerically distinct but qualitatively the same, this for qualities of any adicity and without the need to simply posit a thus suspicious 'transcendental individuator'. It answers the question of how from a level of quantum entities that are all indiscernible but non-identical there can be built-up entities that are discernible/non-identical, i.e., entities whose differences are marked by different monadic properties.[24] This is so simply by the fact that the same kinds of indiscernible structures inter-related in different ways, e.g., by relations with distinct intensions, make for emergent structures themselves with different properties. The instance structuralism given herein demonstrates in what manner an individual can be composed exclusively of attributes, and in this it makes precise the often-made characterization of the quantum world as a realm 'where all is structure'. That is, the analysis takes a Kantian-like view expressed by Cassirer that quantum entities are to be construed exclusively as '"points of intersection" of certain relations' and renders it explanatorily precise and potent by demonstrating in what manner they can be 'mutual intersections of individuated relations'.[25] And in regard to the purely structural nature of quantum entities, a relational hybrid of trope theory is often proposed as a candidate ontology.[26] In contrast to trope theory, however, the above instance ontology retains uniformly the combinatorial nature of ontic predicates of every *n*-adicity, thus providing an account for individuation across the board, and does so without the need for positing non-combinatorial underlying subjects, disarming in this way a persistent objection to Structural Realism—the Inert Substrata Thesis that we cannot have ontic predicates without non-ontic-predicates as subjects. Further, instance ontology has a concomitant formalizable logic that has promise as the sought-after more metaphysically accurate *organon* for de-

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[23] French, 'Symmetry, Structure and the Constitution of Objects'. French and Layman, 'Remodelling Structural Realism'.

[24] Hilborn and Yuca, 'Identical Particles in Quantum Mechanics Revisited', p. 368.

[25] Ernst Cassirer, *Determinism and Indeterminism in Modern Physics* (New Haven: Yale University Press, 1956), p. 180. See French, 'Symmetry, Structure and the Constitution of Objects'.

[26] Simons, 'Particulars in Particular Clothing'. Wayne, 'A Trope Ontology for Classical and Quantum Field Theory'.

scribing micro-reality than current group theory or set theory.[27] To what extent these promises have substance for micro-physics I must leave to the experts.

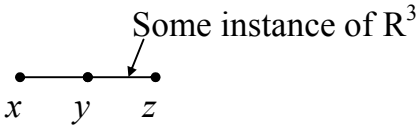
Along this structuralist line it is important to also point briefly to the promise the above instance ontology has for solving more traditional problems of composition, e.g., the Ship of Theseus problem.[28] All physical entities, though enduring, nevertheless change more or less continually, parts being added, removed, or replaced (e.g., the repair of a ship by replacing one plank by another, or of a body by replacing one cell by another). Intuitively, though an entity before such a change of part and the entity resulting from the change are not materially the same—not numerically identical—they can be, depending upon the change, in some legitimate and essential sense 'the same' entity, e.g., the Ship of Theseus before and after every plank in the hull and every other part is successively replaced with one exactly like it. Loosely, the distinction here is between sameness as 'continuity of matter' and sameness as 'continuity of form', where the ship, for example, loses the former but retains the latter. Rea identifies five assumptions involved in classic puzzles over composition and observes that they are jointly contradictory. Central to these and what the above instance ontology rectifies is the assumption that 'sameness' must be numerically identity and this under the 'identity assumption': $(x)(t)[(x$ composes a at time t & x composes b at time $t) \supset a = b]$, where the variable x is taken to range over collections of entities. Then for the Ship of Theseus, with all the parts systematically replaced by exactly similar parts, what would seem in some intuitive sense to be the 'same ship' throughout the replacements, would by the identity assumption be non-identical across them.

The refined precision of instance predicates allows us not only to differentiate composition identity, Principle VII, from indiscernibility, Principle VIII, as two forms of sameness, but also to specify a looser form of sameness: isomorphism. Though I will not give the details of a precise formal definition here, what is proposed can be expressed inaccurately but instructively as: $(R^n)(R^n_i)(R^n_j)[(R^n_i$ an instance of R^n is a structuring element

[27] French and Ladyman, 'Remodelling Structural Realism'. For the logic see Mertz, 'The Logic of Instance Ontology'.
[28] Michael Rea, 'The Problem of Material Constitution', *The Philosophical Review* 104 (1995): 525-52.

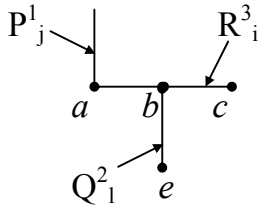
of $a \equiv R^n_j$ an instance of R^n is a structuring element of b) $\equiv a$ is isomorphic to b]. I.e., isomorphism is a corresponding exact similarity of structural components (the 'roads') without the structured relata (the 'nodes') being necessarily similar. Indiscernibility is the strictest form of isomorphism, as is identity the strictest form of indiscernibility. It is, I propose, isomorphism as one-to-one correspondence between instances of identical intensions that is essential to solving at least some of the key problems of composition. Specifically, what I am suggesting is that ordinary objects are definitionally carved out of the dynamic total-structure that is reality by specifying for each a delimited sub-structure that is itself a temporally extended continuous sequence of isomorphic structures, A1-A2-A3-…, and where what endures across all of them is the same isomorphic structure-type A. Let, for example, the form of Complex A above applied to an initial Complex C above be a simplistic model for the specification of the Ship of Theseus. For unrepeatable Complex A its repeatable general form is:

Form A:



where x, y, and z are variables ranging over the categories that intension R^3 delimits, respectively, for each of them. Reproducing Complex C for convenience,

Complex C:



Complex C is the first state, A1, of the ship's existence as here defined, e.g., when, say, Theseus takes ownership (in at least this way there is a conventional element in the identity of the Ship *of Theseus*). Importantly, Complex C has more complexity in its general form than Form A in having properties and relations with relatum-places which Form A does not. As parts of C, a and c might be particular hull-halves, b a particular deck of a particular shape, and relation instance R^3_i an instance of a specific spatial configuration among entities of just these kinds. These parts properly ordered by intension R^3 conform to what is definitionally essential under

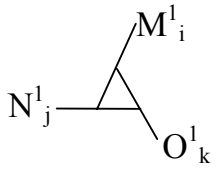
Form A. However, remaining parts of Complex C outside the defining structural form A are as such accidental to the Ship of Theseus; say here, *e* a particular mast and sail, Q^2_1 a relation instance relating positionally this mast and sail *e* to deck *b*, and property instance P^1_j could be the property of a particular defect of particular hull-half *a*. If as the ship changes over time, e.g., hull-halves *a* and *c* are successively replaced, and the deck is replaced in a manner like *b*, each time the replacement and remaining parts are so configured as to conform to intension R^3's delimiting and ordering, then there will result a sequence of A-isomorphic structures starting with A1, i.e., A1-A2-A3-…, and this will be the defined Ship of Theseus—a continuity of form-type of the whole over time. Accidental entities (e.g., *e*), and instances of accidental properties (e.g., P^1) and relations 'attached' to a particular A-form complex in the sequence A1-A2-A3-… may be absent in other complexes in the sequence without rendering the sequence no longer the Ship of Theseus. This would not, or course, be the only form of definitional identity for continuously changing structures. For example, what gives identity to a continuous sequence of particular structures may not be a persistent structural form had by the whole, but rather a structural form had by every sub-structure at some level, and these as related to a subsuming meta-structural form that sustains the formers' existences, e.g., the particular genetic code in every cell making up the body of Socrates, together with this body's metabolic structure that sustains these cells and their contained DNA molecules. Socrates, at least as a biological/physical being, is then the continuous sequence of structures starting with the zygote initiated by his parents and evolving from the dictates of the genetic code of every subsequent cell collectively forming his body and its sustaining metabolic system, a body that in macro-structural form is not constant over time. If Socrates loses a limb, then this sub-structure would no longer be part of *Socrates* since its cells would no longer be part of the subsuming metabolic structure keeping the remaining part of Socrates' body alive. Though introductory, this is, I propose, sufficient to show the promise of this ontology in regard to the traditional problems of composition.

4. Conclusion: No Inert Substrata, No Regress

This brings us to the final but ontologically crucial obligation of demonstrating that, contrary to the Inert Substrata Thesis, instance ontology can rest on a base of only mutually dependent property and relation instances. Contrary to the general tradition, and specifically to some parties in the de-

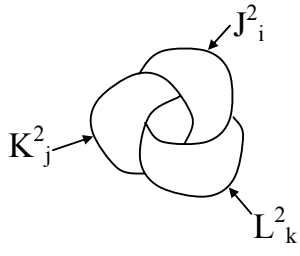
bate over an ontology for quantum particles,[29] the absence of a base of non-dependent entities does not precipitate an infinite regress of dependent entities—as it were, 'turtles all the way down'. Relations (including properties) do not need non-relational relata. The demonstration is at this point in the analysis obvious and simple: Consider first that predicate instances can have as relata other predicate instances, e.g., an instance of a causal relation may be a relatum for instances of spatial relations, or, an instance of Is-Prime^1 would be the subject of an instance of Is-Abstract^1. This is diagrammed, for example, on the right side of Complex D above where instance V^1_r intersects at its end point instance P^1_q, doing so without a shared relata dot indicating that the former is a property directly of the latter, i.e., that fact $:V^1_r(P^1_q)$ obtains. Based upon this it is then possible that there can be closed chains or networks of instances of any adicities having only other instances in the whole as relata. A diagram of one of the simplest such 'closed systems' would be:

Complex G:



This diagram represents the closed chain of horizontally composed monadic facts $:M^1_i(N^1_j)$, $:N^1_j(O^1_k)$, and $:O^1_k(M^1_i)$. All of the composing instances are dependent predicable entities but jointly they form a non-predicable and in this way an independent whole, a 'substance', an *ens in se*. The same mutual support can be seen among dyadic relations in following diagram:

Complex H:



Here we have the closed chain of three dyadic facts $:J^2_i(K^2_j,L^2_k)$, $:K^2_j(L^2_k, J^2_i)$, $:L^2_k(J^2_i,K^2_j)$. It is easily seen that this scheme of mutually sustaining instances can be extended logically to networks composed of any number of relation instances and of any mixture of adicities, as long as each instance

[29] French and Ladyman, 'Remodelling Structural Realism'.

has as subjects in its relata *n*-tuple only other instances of the network. The only constraints in these regards would be via the intension of each composing instance and what it allows as to the natures of and the ordering among its relata. With these observations, then, we prove the falsity of the Inert Substrata Thesis. Concerning absolute indiscernibility, numerically distinct instances of, say, intensions M^1, N^1, and O^1, organized in the same way as those composing Complex G, would compose complexes numerically distinct but indiscernible from G: G′, G″, … Similarly for the intensions involved in the instances composing Complex H, and generally for all other atomic complexes of mutually sustaining instances. Now, if such indiscernible complexes were the respective bottom-most relata for isomorphic meta-structures on them, then the latter would be in a total and absolute sense numerically distinct but qualitatively indiscernible. In this way indiscernibility and its distinction from identity is rendered ontologically precise, and made more perspicuously explanatory of the 'indiscernibility problem' of quantum particles widely described as systems of properties and relations.

In sum, combinatorial ontic predicates, each a dependent *ens ad aliud*, do not presuppose an ultimate substratum of inert non-ontic-predicates, each an independent *ens per se*. The key insight of the agent unifier nature of ontic predicates establishes this and so founds the subsequent and universal ontology of hierarchically structured entities. The unsuccessful theories that would attempt to build structured entities from a base of either intensions, tropes, or bare particulars, become simply irrelevant. Indeed, mutually sustaining relation instances and the networks that emerge from them invert the philosophical tradition: 'substance' is derivative of attributes. We have, then, with the above ontology of individuated ontic predicates not only solutions to traditional problems of substance and a clarification of the logical and ontological concepts of identity and indiscernibility, but also an ontology specifically relevant to micro-physics. In this way the ontology of ultimate entities and their derivatives, and the science of ultimate physical entities and their derivatives, would seem to converge and reinforce each other—plural reality of every kind and at every level, even at its lowest, is structural. In all these ways the network instance realism specified by Principles I-VIII recommends itself as a powerful and economic one-category ontology.

5

Ontic Predicates as Substance*

1. Introduction

It is the role of ontology to give a defensible account of what are the most general kinds or 'categories' of entities and the properties and relations they have in being of these kinds. The given—the 'data'—from which this study must start are apparent facts we intuitively and pre-critically take to be descriptive of general aspects of reality as revealed across our total experience. Important among these *prima facie* facts, and what have been fundamental to the dialectic that is the history of ontology, are:

> Reality consists of a plurality of distinct entities; Some entities are unrepeatable, i.e., individual or particular; All entities are characterized or qualified by attributes—properties and relations—that are or some aspect of which are sharable or repeatable; Attributes in their defining role of qualifying their subjects are dependent for their existences upon these subjects; Among attributes are causal relations whereby some entities exercise kinds of *agency* upon others; And, some entities are themselves structured/organized wholes, i.e., unities where both parts and their particular inter-relationships—their specific 'orderings'—contribute to making the wholes what they are, including the subjects of emergent attributes not had by their parts or substructures.

This last recognition—the existence of structured wholes—will be central in the analysis that follows, as it was in a vaguely understood and misleading form for Aristotelian/ scholastic ontology and the identification there of 'primary being' (Greek: *prōtē ousia*) or what is often though perhaps misleadingly translated as 'substance' (Greek: *hupokeisthai* = Latin: *substare* = to stand under, support).[1] I would propose and will argue for below that this neglect of structure and its absence as a corrective is a source of inade-

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* Original version first published in *Substanz: Neue Überlegungen zu einer klassischen Kategorie des Seienden*, ed. Käthe Trettin (Frankfurt am Main: Klostermann, 2005), pp. 245-71.

[1] See Vasilis Politis, *Aristotle and the Metaphysics* (New York: Routledge, 2004), pp. 12, 192, 207ff.

quacies of subsequent and reactionary ontologies up into contemporary times.

It is the recognition of structures that motivates and guides a proper analysis of attribution, or, closer to classic terminology, what I shall call 'ontic predication'.[2] Ontic predicates include monadic (single-subject) properties, e.g., Is-Red, Has-Mass, Has-Spin-½, Is-True, Is-Even, and polyadic (multiple-subject) relations, e.g., Is-in-Love-with, Owes-…-to, Is-Above, Is-Logically-Equivalent-to, Is-Prime-Proper-Divisor-of. The analysis of structure makes for an accurate and consequential understanding of polyadic (i.e., multi-subject) relations, correcting what are easily made and historical distortions, including especially specious errors regarding the limiting case of monadic properties. These distortions are the source of denials of various of the above *prima facie* facts. For example, if properties are construed according to the classic 'inherence model' as being *constituents* of their subjects, then ontic predication is either identity of subject and ontic predicate, in which case we end up with a monism where what is real is an undifferentiated, homogeneous One, or it is not such an identity, in which case ontic predicates as proper parts of their subjects are no longer dependent upon their subjects, but their subjects are dependent upon them as parts, as in contemporary bundle theories. More important for the following, in the context of structures it is most obvious that polyadic relations have each the nature of an *agent-unifier* among its relata, but this fact is lost when ontic predicates, e.g., Is-a-Triangle or Is-in-Love-with, are mis-identified with their abstracted and repeatable qualitative contents, i.e., their intension universals, e.g., Triangle, Love. This is a common error, what is abetted by the tradition's myopic focus on monadic properties. On this mis-identification, relations as would-be unifying parts of a structure have a non-agent—'inert'—ontic status of but further elements to be unified, requiring further relations to account for the unity, but then with these relations falling to the same analysis. The result is a vicious infinite regress claimed to show the incoherence of relations. This argument, now called Bradley's Regress, is historically reoccurring[3], as one would expect given the common back-

[2] Implicit in Aristotle, medieval philosophers explicitly distinguished between linguistic or 'formal' predication and ontological or 'material' predication, the former referring to the syntactical unity of sentences while the latter refers to the extra-linguistic (though not necessarily extra-conceptual) unity of attributes with their subjects, what results in facts to which true affirmative declarative sentences correspond.

[3] For historical uses of Bradley's Regress Argument and references see D. W. Mertz, *Moderate Realism and Its Logic* (New Haven: Yale University Press, 1996).

ground failure to distinguish between full ontic predicates as *intension-determined agent-unifiers* and the latter's abstracted and 'inert' constituent *intensions* simply. It is to be noted that the proper conclusion of the regress, as F. H. Bradley himself intended[4], is that all ontic predication whatsoever, relation or property, is illusional. And, this denial of most of the above *prima facie* facts should give one cause for a re-analysis of ontic predication, what is most obvious in the case of relations and this in the context of structures. The motivating errors here and what ought to be our response are the same for the consequential fallacy that is the classic Humean reduction of efficient causality to constant temporal conjunction.[5] Relations of efficient causality are the most experientially apparent examples of linking-agency, but with relations 'reduced' to properties and the latter to 'non-relating' intensions, then a cause and an effect can be 'linked' at best by temporal succession. More subtle but telling of the error, temporal succession here must mean real temporal relations each as such with unifying agency under a temporal intension, e.g., Before, linking a cause with its effect. If temporal relations can have this agency then what non-arbitrary reason prevents the more obvious efficient causal relations from having it?

The beginning insight central to the following is that *structures* (*complexes, systems, networks*) are organized composites whose natures turn in an essential way upon how the elements are variously inter-connected, i.e., inter-*related*. One need only think, for example, of what are now known to be the spectacular hierarchical structures, static and dynamic, of living plant and animal bodies; the molecular, atomic, and sub-atomic composition of physical matter; and the functioning complexity of modern machines, e.g., computers. There are equally remarkable abstract formal structures, e.g., the various set theories, number theories, and abstract algebras, as well as 'mixed' structures, e.g., minds, and ethical and social/political structures. For Aristotle, paradigm structures were those of artifacts, e.g., statues, houses, and living creatures, e.g., humans, trees, and in particular, it was the latter as dynamic, internally driven and determined living event structures that were considered most truly 'substance'. These substances were each

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[4] F. H. Bradley, 'Relations' in *Collected Essays*, Vol. 2 (Westport: Greenwood Press, 1970), p. 672. This essay is Bradley's last views on the purported incoherence of ontic predication, including various rehearsings of the vicious regress that bears his name.

[5] David Hume, *Enquiry Concerning Human Understanding*, ed. P. Nidditch (1777 edition; Oxford: Clarendon Press, 1975), Sections IV and VII. Also see Mertz, *Moderate Realism*, pp. 45-47.

seen as developing in a relatively fixed way to the fulfillment of an 'end' (*telos*), viz., a mature one of its kind, as determined by its specific nature, kind, or 'substantial form'. Specifically, then, *structure* consists of a plurality of parts mutually linked in a connected web or network by one or more relations (including properties as the limiting case) of possibly varying intensions (qualitative contents, 'senses'). Here the delimiting specificity—the intensions—of the relations and the mutually relevant specific natures of their subjects jointly contribute to the nature of the whole. If one 'rearranges' parts of a structure (e.g., switches relata, or changes the relations among them), the whole is different, in some cases being exactly isomorphic to the original (as when 'identical' parts are interchanged in a machine), but in most cases being drastically different (think of the usual result of rearranging only a few components in a computer). This type of composition is illustrated even more dramatically in the case of *atomic structures*, i.e., single *facts* or *states of affairs* each consisting of one *n*-adic relation (including what could be a monadic property) and an *n*-tuple of (i.e., an ordered set of *n*) subjects, e.g., the fact that *a* mechanically supports *b*, or that 3 is a prime proper divisor of 9. These facts would be rendered symbolically as, respectively, :Mechanically-Supports(*a*,*b*) and :Is-Prime-Proper-Divisor-of(3,9), (The colon is used to designate a fact :F(a_1, a_2,...,a_n) in contrast to a corresponding proposition F(a_1, a_2,...,a_n)). More complex structures are built up from facts whose relations share parts as relata, what will be further detailed below. The intensions of these last example relations, Mechanical-Support and Prime-Proper-Divisor, each have an aspect of order, the first classified as non-symmetric and the second as asymmetric, and because of this the thought of switching the order of the relata either corresponds to a different fact or no fact at all. Important in this regard, as crucial to correcting historically persistent distortions and to the force of the analysis herein, are the classic arguments by Bertrand Russell and others[6] demonstrating the impossibility of a reductive elimination of these polyadic ordering relations to monadic properties of their relata. What this demonstration does is overthrow the classic assumptions necessitating the reductive strategy for *all* polyadic relations, i.e., the mutually reinforcing inherence model for, and the *intensions*-are-attributes analysis of, ontic predication. With these assumptions eliminated, so is the need to

[6] Bertrand Russell, *The Principles of Mathematics*, 2d. ed. (1903: reprt. ed., New York: Norton, 1938), pp. 221ff. Herbert Hochberg, 'The Wiener-Kuratowski Procedure and the Analysis of Order', *Analysis* 41 (1981): 161-63, and 'A Refutation of Moderate Nominalism', *Australasian Journal of Philosophy* 66 (1988): 188-207. And Mertz, *Moderate Realism*, pp. 163-73.

deny the *prima facie* nature of *all* polyadic relations: each is an external but inter-linking ontic predicate among its relata. That is, a relation is distinct from but has both an ontic status—a being or reality—equal to, and an ontic role of intension-delimited combinator among, its relata. In both of these ways monadic properties are but the limiting single-subject case. The appropriate metaphors here are that relations bridge ontological space, and do so as its 'ontoglial' (Greek: glue of being).

Composed in part of relations so understood, structures stand in contrast to composites whose elements are unified via an intensionless, and hence arbitrary or random association, what is irrelevant and indifferent to the natures of the elements, presupposing only their differentiated existences. These wholes are what Aristotle referred to as 'heaps' and include lists, sets, and mereological 'sums' or 'fusions'. In such a whole any ordering of the elements is either externally imposed conceptually, or, as with spatial relations among objects in a pile or heap, are abstracted away and considered not part of what defines the whole, it being indicative of such wholes that they conform to an axiom of extensionality. These wholes once existing are atomic parts for structural wholes formed from emergent and non-arbitrary formal properties and relations, e.g., Is-an-Element-of, Is-a-Part-of, Is-a-Subset-of, on which the formal sciences of set theory and mereology are built, including the fact that every other property and relation of these theories is definable in terms of them. Structural wholes, in contrast to lists or sets, are more than the sum of their parts—the 'more than' resulting from their specific constituent relations, what is the cause of them having a multitude of *other than formal attributes* that we observe, e.g., Provides-Shelter, Has-an-Entrance, Is-Self-Moving, Is-Metabolizing, Is-Conscious.

Succinctly in regard to structure, the further insights intended here extend in two mutually reinforcing directions: downward to a proper understanding of ontic predication where 'actually relating' relations of *any kind*, including spatial, temporal, causal, ethical, social, these whether conceptual or abstract, static or dynamic, are all real and interconnecting entities among what are then intensionally-linked-but-distinct subjects, and upward to the recognition that structure is a ubiquitous feature of reality where all entities exist in multiple and overlapping networks of various kinds of relations. These facts are the basis of the modern research field of

General Systems Theory[7], and facts that should free us from the naive and retarding bias that only spatial-temporal and physical-causal relations are real and ontically significant. Yet in regard to the latter relations and particularly relevant to the following, there are with a realist interpretation of contemporary micro-physics strong arguments for the thesis that the ultimate physical entities—quantum particles and/or fields—are purely structural, i.e., composed of *only* properties and relations (the latter often theorized as 'tropes' that are 'bundled' into quantum entities) and so without any underlying non-ontic-predicate subjects supporting them.[8] The reaction has been that this is not possible, a response we shall counter below.

The traditional and naive ontological focus has been on relatively isolated subject substances each acting like a hub anchoring an array of characterizing properties. Indeed, the classical/Aristotelian analysis of the latter which we shall detail below leads inevitably to a bottom-most ontic level of absolutely qualityless/natureless subjects—'prime matter' or 'bare particulars'—and there are strong arguments, also given below, that the concepts of such entities are incoherent.[9] The current debate is framed within and stalled, I would propose, because of an assumption common to both the classic and, reacting to it, prevailing contemporary analyses, that requires an ultimate substratum of non-attribute entities. The specious assumption forces a stalemate between, on the one hand, strained arguments

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[7] The active field of General Systems Theory is traceable back to Ludwig von Bertalanffy's *General Systems Theory* (New York: George Braziller, 1969). Also see Ervin Laszlo, *Introduction to Systems Philosophy* (New York: Gordon & Breach, 1972).

[8] Steven French, 'Symmetry, Structure and the Constitution of Objects', in the *PhilSci Archives*, Center for the Philosophy of Science, University of Pittsburgh at http://philsci-archive.pitt.edu/, 2001. Steven French and James Ladyman, 'Remodeling Structural Realism: Quantum Physics and the Metaphysics of Structure', *Synthese* 136 (2003): 31-56. Andrew Wayne, 'A Trope Ontology for Classical and Quantum Field Theory', forthcoming in a volume ed. by W. Myrvold in the *University of Western Ontario Series in Philosophy of Science* (Springer).

[9] For arguments against bare particulars as well as against bundle theories currently seen as the alternative see Michael Loux, *Metaphysics: A Contemporary Introduction* (New York: Routledge, 1998), pp. 87, 93ff. And, see relevant essays in Stephen Laurence and Cynthia Macdonald, *Contemporary Readings in the Foundations of Metaphysics* (Oxford: Blackwell, 1998). Expanded versions of arguments I make herein against bare particulars can be found in Mertz, 'Individuation and Instance Ontology', *Australasian Journal of Philosophy* 79 (2001): 45-61, and 'Against Bare Particulars: A Response to Moreland and Pickavance', *Australasian Journal of Philosophy* 81 (2003): 14-20.

to save qualityless subjects, and on the other, arguments for re-construing some attributes, viz., properties, into non-attribute inert but characterizing 'little substances' as ultimate substrata, e.g., tropes. In contrast, we shall see how reality can be thoroughly structural, and this down to a foundational level of what are *individuated* properties and/or relations—'relation instances'—where their relata which they are necessarily ontically dependent upon *are but further atomic ontic predicates*, and not some categorically different, natureless, non-attribute subjects. With this we can have by closed loops of dependent relation instances the formation of primitive non-attribute and so 'independent' individual structures, what in turn as wholes can be relata for further relation instances that jointly form subsuming structures. Hence the possibility of the above mentioned quantum ontology. The process can be iterated upward into a hierarchy of structures of structures that include the objects of ordinary experience. In this way ontic predicates as instances, i.e., as individuated intension-determined agent-unifiers, make up the category of 'primary being', with instances from certain sub-categories having the role of 'substances' in the sense of being the ultimate components of a foundational ontic substratum. The resulting ontology is what I have termed 'Network Instance Realism'[10], with its heuristic metaphor being that all reality is a single continuous and multi-layered cloth formed of only various inter-connecting threads—relation (including property) instances. Pivotal to all of this are the two principles that ontic predicates are agent-unifiers, and, following upon this, that they are each an individual, particular, or unrepeatable 'this', i.e., each is a unit attribute or 'instance'. Significantly, the implication from the first to the second principle is a solution to ontology's general Problem of Individuation, i.e., a positive account of what makes an entity an individual in contrast to a repeatable 'such ', i.e., intension-only universal. The insight behind the implication is that the agent-unifier nature of an ontic predicate renders it an individual just as any event-agency and so event is an individual.

[10] Arguments for the ontology of Network Instance Realism can be found in *Moderate Realism*, as well in Mertz, 'Combinatorial Predication and the Ontology of Unit Attributes', *The Modern Schoolman* LXXIX (2002): 163-97 [Essay 1 herein]; 'An Instance Ontology for Structures: Their Definition, Identity and Indiscernibility', *Metaphysica* 4 (2003):127-64 [Essay 2 herein]; and in 'Objects as Hierarchical Structures: A Comprehensive Ontology', in H. Hochberg and K. Mulligan, eds., *Relations and Predicates*, *Philosophische Analyse*, Vol. 11 (Frankfurt: Ontos Verlag, 2004), pp. 113-48 [Essay 4 herein].

2. Aristotelian Context: On the Requirement of Independent Substrata

The classic and long dominant 'common-sense' Aristotelian/scholastic ontology sought to accommodate the above given and other related *prima facie* facts and to systematize them into a more detailed and explanatory science of being. The method, as it must be with all extended and systematic knowledge, was to develop an understanding of the key inter-related concepts used in the formulations of these apparent facts, what includes adopting certain principles relating the entities that then fall within their extensions. These key ontological concepts and their equally relevant dichotomous contradictories include in part: attribute|subject, repeatable(universal)|unrepeatable(individual), dependence|independence, determinate|indeterminate, agency|non-agency('inertness'), and structure|non-structure. Importantly, presupposed across all the other of these concepts and in this way fundamental to all of ontology, and in a correlative way instanced in every form of complex experience whatsoever, is the *attribute/subject* dichotomy: reality divides between ontic predicates and the non-ontic-predicate subjects that the former characterize or qualify, e.g., concrete physical objects and abstract logical, conceptual, and mathematical entities. How an ontology construes ontic predication and the correlative notion of subject, and, in particular, how they are said to fall within the above dichotomies, has profound consequences as we shall see. For example, in his early *Categories* (*Cate*. 1a17-3a7), Aristotle involves the first three dichotomies in a cross-classification that identifies primary being with non-ontic-predicates that are ultimate subjects, what is then appropriately termed 'substance'. Substances are then further characterized as independent in both his senses there of being "not in another" and being "not said of another", the latter also involving essentially being unrepeatable or individual. How these classifying phrases are to be further interpreted is unclear. Aristotle adds as an additional criterion for substance that it be an entity that endures as essentially the same through change of non-essential or accidental attributes. The general idea here is that substances are self-sufficient entities that endure over time, and all other entities depend upon them for their existences. In the *Categories* Aristotle takes substances to be concrete, i.e., spatial/temporal, individuals such as particular animals, e.g., Socrates, all left unanalyzed.

Later, much of Aristotle's analysis in the central and at points conflicting books of his influential *Metaphysics* (VII (Z) ,VIII (H), and IX (Θ)) was to further identify primary being in light of the insight there that what

were formerly the pre-critical and naive candidates for substance, e.g., individual animals and plants, are in fact themselves complex entities, and more crucially, *structured* entities. Aristotle carries over from the *Categories* the criteria that primary beings must each be an unrepeatable/individual "this" and an independent or "separable" entity (See, e.g., *Meta*. 1029a28-29; 1017b24-25; 1028a34-35; 1037b27). Importantly, he also repeats (*Meta*. 1017b23-24) but must finally give up as a criterion for primary being (*Meta*. 1028b34-1029a35) the requirement of being an ultimate subject. We will concentrate here upon this latter criterion. According to Aristotle, "Substance [primary being] means that which is not predicable [not an ontic predicate] of some subject."(*Meta*. 1038b15; my inserts) Assuming the classic 'inherence model' of predication—if F is an ontic predicate of a subject *a*, then F is an intensional constituent of ('is in') *a*—then Aristotle asserts elsewhere what he considers to be the equivalent: "By a 'primary' substance I mean one which does not imply the presence of something in something else, i.e., in something that underlies it which acts as matter."(*Meta*. 1037b2-4) Here, as in most of its occurrences in the context of Aristotelian ontology, the term 'matter' is to be understood as the necessary correlative of an ontic predicate, i.e., as a 'subject'. Later the medieval scholastic Thomas Aquinas restates the Aristotelian characterization as: "A substance is a thing to which it belongs to be not in a subject. …Substance is understood as that which has a quiddity ['whatness', i.e., essence] to which it belongs to be not in another."(*Sum. Contra*., I, ch.25, par.10.; my insert)[11]

We can summarize the Aristotelian view of substance with respect to the intersections of the attribute|subject and dependence|independence dichotomies by the following theses. Their combination has been influential throughout Western ontology, both as the principles of all 'substratum' theories and as a constraint on alternate theories reacting to the severe problems of substratum theories. The first is the *Predicate Dependency Thesis*:

> T1) Ontic predicates are inherently dependent entities in their defining role of characterizing or qualifying their subjects, and are thus dependent for their existence upon the existence of these subjects.

[11] Thomas Aquinas, *On the Truth of the Catholic Faith (Summa Contra Gentiles), Book One: God*, trans. by Anton Pegis (Garden City: Doubleday & Co., Inc., 1955), p. 128.

Because of their intrinsically dependent nature, scholastic philosophers described monadic properties as each an *ens in alio* (a 'being-in-another') and polyadic relations as each an *ens ad aliud* (a 'being-toward-something-else'). In modern ontology this inherent incompleteness and dependence of ontic predicates is marked by their characterization as, not things, but 'ways' things are[12], and more famously by Gottlob Frege's term of 'unsaturated'[13]. More significant to all of the tradition is the second thesis, what apparently was and remains, if implicitly, *assumed to be implied by T1*, viz., the *Independent Subtrata Thesis*:

> T2) All dependent entities are dependent, either immediately or mediately through a chain of dependence, upon what are inherently independent entities.

These ultimate independent substances were described by medieval philosophers as each an *ens in se* (a 'being-in-itself') or an *ens per se* (a 'being-through-itself'). The easy assumption made in thinking that T1 implies T2 is that otherwise there would be a vicious infinite regress of ontic dependence. That this is not the case will be demonstrated below. T1 and T2 together with further theses also common to the tradition and that we shall consider next imply that a substance so defined must be an essenceless/natureless entity, and such a posit, whether as Aristotelian prime matter or more contemporary bare particulars, is incoherent, as many philosophers have maintained. Philosophers who reject essenceless substrata, but assume that T1 implies T2, must reject T1 by theorizing a modification of at least some ontic predicates into non-dependent entities, either as repeatable intension universals[14] or unrepeatable tropes[15], what are then required to be

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[12] See David Armstrong, *A World of States of Affairs* (Cambridge: Cambridge University Press, 1997), pp. 30, 38, 98.

[13] Gottlob Frege, 'On Concept and Object' in *Translations from the Philosophical Writings of Gottlob Frege*, eds. P. Geach and M. Black (Oxford: Blackwell, 1970), pp. 42-55, and 'On the Foundations of Geometry' in *Essays on Frege*, ed. E. Klemke (Urbana: University of Illinois Press, 1968), pp. 559-75.

[14] E.g., the view of Bertrand Russell in *An Inquiry into Meaning and Truth* (London: Allen and Unwin, 1940), pp. 98-100, 127-28, and in *Human Knowledge, Its Scope and Limits* (New York: Simon and Schuster, 1948), pp. 292-308.

[15] E.g., Keith Campbell, *Abstract Particulars* (Oxford: Basil Blackwell, 1990), and Peter Simons, 'Particulars in Particular Clothing: Three Trope Theories of Substance', *Philosophy and Phenomenological Research* 54 (1994): 553-74.

'bundled' into ordinary 'thick particulars'. Hence the rationale for all bundle theories.

In his attempt to more precisely characterize and identify what is primary being in the *Metaphysics*, Aristotle utilizes other fundamental ontological theses, those important here making use of the concepts of the further determinate|indeterminate dichotomy. Two of these three theses when joined with T1 and T2 imply the incoherent 'natureless nature' of an ultimate non-ontic-predicate entity posited under T2. The third ontological thesis not included in this set and contradicting its latter implication is the *Determinacy Thesis*:

> T3) For an entity to exist it must be a determinate- (delimited-, specific-) something, i.e., it must have a definite positive and differentiating essence that makes it to be what it is and distinct from other entities.

It is arguable whether Aristotle accepts T3 in its full generality, but I join with a number of philosophers who maintain (e.g., apparently including by implication Aquinas, *Sum*. *Theo*., I, q.66, a.1.) that it is a global condition for all existents, the contrary implying a number of absurdities we shall review below. As a preliminary I would urge the intuition that to be is to be a specific something with a differentiating content, i.e., an essence, what is the cause/source of it being demarcated from and contrastable with everything else, including in the limit homogeneously undifferentiated nothingness. That is, the contradictory of nothingness as the total negative absence of any specificity whatsoever is a positively determinate *such*-a-being. Or alternately, the less an entity is a determinate something the more it evaporates into *no-thing*, i.e., into non-being/non-existence. The implication is that T3 is to be taken in its full generality, though, as we shall see, it then contradicts jointly T1, T2, and the other two Aristotelian theses given presently. Seemingly in accord with T3 Aristotle himself asserts that "In everything the essence is identical with the ground of its being." But then he goes on to say: "Further, the actuality of whatever is potential is identical with its formulable essence."(*De Anima* 415b12-14; Also see *Meta*. 1031b18-19; 1041b2-9) Motivated by statements like the latter, it is controversial whether Aristotle interprets T3 in a less than global sense (see *Meta*. 1030a27-b7), allowing as an exception a purely and completely indeterminate entity, an entity that was characterized as 'pure potentiality'

but that contributes to the being of something determinate when it is the subject of an act of a specifying agency. These various agents are Aristotle's 'substantial forms' (below). The required entity with the indeterminate nature is what the tradition called 'prime matter', T2's ultimate subject of predication (*Meta*. 1029a1-30). Some commentators hold that at one place in the *Metaphysics* (1028b34-29a35) Aristotle himself argued for the absurdity of this notion of a prime matter that is nothing determinate in itself but potentially any specific thing, supposedly rejecting it in favor of a bottom-most ontic level of physical elements: Earth, Air, Fire, and Water, each as such with a determinate essence.[16] What is clear is that at the referenced argument Aristotle rejects as a candidate for *primary being* what he recognizes there would be a totally indeterminate ultimate subject/'matter' on the grounds that primary being must satisfy, in addition to the supposed ultimate-subject/independent-substratum criterion under T2, the further criteria of being 'separable' (independent) and a 'this' (unrepeatable), the latter then necessary determinates inherent to what are the essences of primary beings. At any rate, whatever Aristotle's views in regard to the legitimacy of prime matter, he retains T2 throughout, and T2 together with his other theses independent of T3 jointly imply the existence of an essenceless entity, what indeed is an incoherent notion as will be argued.

Aristotle's other here-relevant theses are, first, the *Intensional Essence Thesis*:

> T4) The essence of an entity *x* consists at least in part of a qualitative content of the being of *x*, what is given in the definition of *x*, and is composed of one or more intensions or universals (*Meta*. 1017b22-23, 1031a12-13, 1035b27-35, 1036a29).

And, the *Universals-as-Predicates Thesis*:

> T5) Intension universals are ontic predicates (*Meta*. 999b35, 1038b15-16).

A more specific version of T5 and one Aristotle arrives at is the *Essence-as-Predicate-of-a-Substratum Thesis*:

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[16] E.g., Mary Gill, *Aristotle on Substance* (Princeton: Princeton University Press, 1989), pp. 19-30, 41ff, 242.

> T5′) If an intension F is the essence of a primary being *x*, then F is an ontic predicate of a further constituent, *y*, of *x*, where *y* is an independent entity (as specified in T2).

I will argue below that the proper version of T5/T5′ is the *Intensions-as-Predicate-Parts Thesis*:

> T5″) If an intension F is a constituent of the essence of an entity *x*, then *x* is an ontic predicate.

Presumably Aristotle would have argued for T4 on his assumptions that knowledge is identity with its objects insofar as it is of the forms that compose the entities known (*De Anima* 431a1-b30), and these forms are the intension-composed essences of their respective entities. Herein T4, along with Determinacy Thesis T3 in its full generality, will be supported, first, on the argument that there are no intensionless entities, and, secondly, on the argument that it is a given of our experience that ontic predicates—properties and relations—each has an intension component, and, on the theory advanced herein, that all reality is composed exclusively of ontic predicates, these making their resultant structural wholes each *what it is*, i.e., composing its essence. The second argument will also establish T5″.

We are now in a position to observe that theses T1, T2, T4, and T5 (or T5′, or T5″) jointly imply that any entity *s* posited under Independent Substrata Thesis T2 is an essenceless/indeterminate entity—one not characterized by any intension—and so contradicts Determinacy Thesis T3 as fully general. Let *s* be an ultimate subject required by T2, i.e., a totally independent entity. We note that *s* so specified must be a simple entity, i.e., have no proper parts. For, if it were otherwise then *s* would not be an independent entity, but rather would be dependent for its existence upon these proper parts. Now *s* either has an essence or it does not. Assume *s* has an essence. Then by Intensional Essence Thesis T4 this essence is composed of one or more intensions, but since *s* is simple then *s* must be identical to what is a single intension. Yet, by the Universals-as-Predicates Thesis T5 (or T5′, or T5″), *s* must then be an ontic predicate, and so by Predicate Dependency Thesis T1 is a dependent entity, thus contradicting its posited independent nature under T2. It follows then that *s* is an essenceless/indeterminate ‘entity’. If, however, this is impossible, i.e., if Determinacy Thesis T3 is true, then one or more assumptions must be rejected. A central point

herein is that Independent Substrata Thesis T2 is to be rejected in light of the warrant for T1, T3, T4, and T5″, together with the demonstration that Predicate Dependency Thesis T1 does not imply Independent Substrata Thesis T2.

We can summarize the above classic and specious strategy as follows: The conceptual road to the result that a T2 ultimate-subject/independent-substratum *s* must be essenceless proceeds from assumptions like those above that require any qualitative/intensional content that would give differentiating specificity to entity *s*, precisely as such, to be or to be a constituent of one or more ontic predicates that are *external* to (not a constituent of) *s* but have *s* as a subject. Hence, and the source of its absurdity, *s in itself* is empty of any characterizing content that would distinguish it from anything else, this having been theoretically removed outside to properties attaching to it. The intuition then is that *s* disappears due to the complete absence of any part of its constituent being that would differentiate it from non-being. Or metaphorically, *s* is like a host on which its properties are parasitic, and the more the properties take into themselves the essence of the host the less determinate being is left for it to be, the host in the end disappearing completely and its properties along with it in having nothing to support them in their 'subject-consuming' parasitic dependency. Ontological pathology!

In addition to these intuitions, however, are the following more detailed arguments. A root problem here is that the 'attachment' of anything to such an entity *s* could not be by means of ontic predication, i.e., *s* could not be the proper subject of any property or relation. This is so because ontic predicates are unified to their subjects, not by random or arbitrary association as in a list, but as a function of a 'fit' or compatibility, however minimal, between their intensions and what must be specific natures of each of their subjects. The classic inherence view was that the union making for the monadic fact :F(a) is due to strict identity where intension F is held to be the same as a constituent of a. If this analysis were true of all ontic predicates then the point would be made that an entity like *s* in having no constituent intensions could have no properties. But, this identity/containment analysis fails for non-eliminable polyadic relations. The correct and general analysis of the predicable union constituent of any fact, :R(a_1,a_2,...,a_n), whether accidental or essential, includes the recognition that it presupposes as a necessary, but not necessarily sufficient, condition only a compatibility or appropriateness—a qualitative 'fit' or relevance—

between the predicate's intension R and the natures of each of its relata, a_1, a_2, …, a_n. In this way, knowledge of a fact tells us something about the subjects not only jointly, i.e., that they are in the R relation, but also singly in each having a specific nature or essence appropriate to its subject-place for intension R. Even with 'accidental predication' as in the fact :Is-Red(*a*), for the ontic predicate Is-Red to have/characterize *a* as its subject, *a* must have the minimal correlative nature of being spatially extended, as opposed to having, for example, the determinate essence of being a set. Similarly with so-called 'external' spatial relations, e.g., Is-Left-of as in the fact :Is-Left-of(*a*,*b*), they obtain between subjects *a* and *b* only on the necessary (but not sufficient) condition that each subject have the specificity of being spatial entities and not, say, numbers, virtues, or concepts. The point here is the basis for absurdities known as 'category mistakes', e.g., the proposition for 'apple *a* is courageous' is absurdly false and we require no empirical evidence to know this because we know that what the ontic predicate Is-Courageous requires of the entities in the category of its possible subjects, viz., intelligence and will, is not had by an apple.

Now given this mutual-appropriateness condition on all ontic predication we have the following arguments against any entity like *s*. First and primarily, *s* as a completely indeterminate entity could not be the subject of any property or relation, and this contradicts its very *raison d'être* under T1 and T2. For this reason and as intuited by the tradition there could be only one such entity, two or more requiring that they differ by at least one property and this is impossible. Further and in particular, the above means that *s* cannot be a relatum for any spatial or temporal relations, nor crucially of causal relations of any type, either as agent or subject of the agency, and in this way can have no effect upon nor be effected by anything else whatsoever. It could not then be as it was intended in the Aristotelian hylomorphic tradition to be pure potentiality to the causal agency of various substantial forms. This has been argued in a more intuitive way by asserting that The-Potential-to-have-a-Property is an apparent higher-order characteristic that *s* must have but as characterless cannot have. In sum, implied *s* would have a god-like status of being neither creatable nor destroyable—though these too seem to be characteristics that *s* could not have—and, as precluded from being a relatum for any relation whatsoever, would be radically isolated from every other entity, consequently unknowable in itself and ontologically useless.

It is to be noted that bare particulars, the modern cousins to ultimate subject *s*, inherit the same difficulties. A bare particular p_a is posited to account for the individuation—unrepeatability—of a particular *a*, e.g., an apple, where every characteristic of *a* is taken to be a repeatable intension universal, F, G, H, …, e.g., :Red(*a*), :Round(*a*), :Nourishing(*a*). The 'thick particular' *a* is then analyzed as the structure consisting jointly of intensions, F, G, H, …, each 'tied-to' collecting individuator p_a. Note that p_a is not itself characterized by, i.e., does not have as ontic predicates, the intensions F, G, H, … —this is its 'bareness'—and hence why the tied-to connection cannot be construed as ontic predication or its surrogate the 'exemplification' relation. At best, the tied-to connection would be a relation whose relata are intensions and bare particulars. But, on the above, this requires p_a have a specific essence—the presupposed correlative of the tied-to connection if it is a relation proper. What is the case, however and absurdly, is that p_a can not consistently have any essence, and derivative of this that the tied-to connection can be but arbitrary association. First, it is the case that, unlike *s*, a bare particular p_a is posited with a minimal essence: as asserted in the literature, it is to be of its very self at least unrepeatable, simple, have no other properties, and the constituent of at most one particular at a time. But the latter are each a determining repeatable intension, each as such with a repeatable contrary, and so, by the same analysis given to the original thick particular *a*, they ought to be external to—not constitutive of—the being of p_a, lest p_a itself be a, albeit considerably thinner, thick particular. In other words, if the intensions contributing to the essence of the original thick particular *a* must be analyzed as external but tied-to a central bare particular p_a, then likewise the intensions contributing to the being of p_a must be analyzed as external to but tied to a further bare particular p_a' constituent of p_a. This, of course, is the beginning of a vicious regress. Hence, p_a must be an essenceless/intensionless 'entity'. Now even if, contrary to fact, this were possible we would have the same situation as with *s* that there could be no more than one bare particular, and so one thick particular. Further, the 'tied-to' connection could not be a relation since a relation presupposes some specific essence in each relatum. Hence, the tied-to connection is nothing but arbitrary association as in a list. But then there is nothing to prevent contrary intensions, e.g., Round and Square, from being tied-to the same bare particular, and so nothing to prevent the same thick particular from having contrary ontic predicates, which is absurd.

We can sharpen the above to a more subtle but I propose insightful argument against intensionless individuators, one that points to the source

of their deceptive plausibility. The essence—what makes it to be exactly what it is—of an *unrepeatable* individual or particular, whether a bare particular or not, cannot, without contradiction, be identified with one or more *repeatable* intensions that in part compose it. The inclusion of an additional something is provided for in the wording of Intensional Essence Thesis T4, and, as expanded upon below and of fundamental importance to ontology, this something is the *combinatorial act* of an ontic predicate as delimited in its agency by the predicate's intension. Now, the fact that a particular must be composed of more than just repeatable intensions is, it seems, what gives specious hope to advocates of intensionless/qualityless individuals by leading them to think that a theory can allow for all the repeatable components of the essence of an individual to be removed to the outside and that there will still be some residual composing essence left—a numerically one and pure intensionless 'this'—which they will identify as a bare particular, and that will be the subject of these externalized intensions. The error here is that any such would-be residual and pure 'this' will itself have to have an essence—what makes it to be what it is—and this will have to be composed in part of the *repeatable* intension Unrepeatability or Thisness—what is the qualitative commonality of all such (possible) pure unrepeatables. After all, to fulfill their ontological role there must be multiple pair-wise non-identical bare particulars, p_a, p_b, p_c, …, yet what as such are all of the *same kind*, i.e., of the kind *bare-particular*. That is, each such entity is essentially determinate as unrepeatable, a determination that contrasts with intensions (each a 'such') that are essentially determined as repeatable, and this shared determination of unrepeatability constitutive of each of p_a, p_b, p_c, …, is itself the repeatable intension Unrepeatability. There is no paradox here anymore than there is in the fact that the intension Concrete is abstract. Now, a theory that posits such entities, what concomitantly requires all associated characterizing qualities be removed from them, is discredited in each case of p_a, p_b, p_c, …, with a vicious infinite regress. For as in the above regress, each such case of 'pure unrepeatability' is not a pure 'this' after all, but decomposes into both a further unrepeatable 'this' and the intension universal Thisness, the latter under the theory becoming an external ontic predicate of the former. But, of course, the last 'this' will likewise decompose, and so on to vicious infinite regress.

Our conclusion is, then, that there can be no intensionless/qualityless entities as the above theses require of ultimate subjects under Independent Substrata Thesis T2. Hence, the warrant for Determinacy Thesis T3 in its full generality and for Intensional Essence Thesis T4, and against T2. How

the Predicate Dependency Thesis T1 does not imply T2, and which version of T5—how intensions are involved in ontic predication—is to be accepted, requires a more detailed analysis of the nature of ontic predication. Here the agency|non-agency('inertness') dichotomy will play a crucial and, when properly understood, profoundly consequential role for ontology.

3. Aristotelian Context: The Intension-Determined Agent-Unifier Account of Structure

We will take our start from Aristotle's recognition and analysis in *Metaphysics* VII (Z) and VIII (H) of what are *structured* entities—organized wholes themselves the single subjects of emergent ontic predicates that are a function of the various qualitatively/intensionally determined 'ways' the parts of the structures are mutually unified or 'ordered'. It is an empirical fact of everyday observation that many entities are structured wholes of more or less complexity, with Aristotle's candidates for primary beings, e.g., living animal and plants, marvelously so. The Aristotelian/scholastic analysis of this fact, distorted under the T2 requirement of an ultimate-subject/independent-substratum, and other theses given below, resulted in the classic and influential *hylomorphic*—'matter-form'—ontology. Under the hylomorphic theory certain monadic properties, i.e., 'forms', some substantial, others accidental, when ontic predicates of their types of matter/subjects, what are, respectively, either ultimate matter or already informed 'secondary matter' (e.g., bodily organs or clay), produce organized wholes that are, respectively, either substances (e.g., a human or a horse), or artifacts (e.g., a house or a statue).

What underlies this theory and is accurate as part of the Aristotelian analysis is the observation that an organized or structured whole is more than the mere "juxtaposition or mixing", or "heap", of its elements, an observation Aristotle repeats five times within the span of a few crucial paragraphs in the *Metaphysics* where he is narrowing in on identifying primary being (1040b5-10, 1041b11-33, 1043b5-14, 1044a4-5, 1045a7-b24; Also see *De Anima* 412a29-b6). In each case the 'more' is marked by emergent ontic predicates that the organized whole has as a single subject and that its parts as unorganized—merely heaped together—would not have, e.g., a certain function or shape. Importantly, Aristotle first observes that what is required is a "principle" (= source) and a "cause" of this 'more' that is a constituent of the being of this structure, and he then demonstrates that this

organizing constituent cannot have the ontic status of just another element relative to the whole. For simply, if it were otherwise then this constituent would contribute to the being of the whole in exactly the same way as the original elements—it and the elements would be alike as each 'inert' relative to the inter-element unification necessary to produce the organized whole—and so collectively would be no more than a mere 'heap' until some further constituent organizer unifies them. Yet, the latter by the same analysis would itself be but a further element to be organized with the others, and so on to vicious infinite regress (*Meta*. 1041b11-33). Consider the analogy of a necklace as a structured whole consisting of multiple beads unified by a shared string running through them. The string has an ontic role of unifier for the necklace, a status that the individual beads do not have, and to give to the string the same status as the beads relative to the necklace, say to withdraw it into a ball and juxtapose it with now what is but a pile of the beads, is to require in order to capture something like the structure that is the original necklace a further string threaded through all of these. If this new thread is in turn withdrawn and made in effect into another bead, a third shared as so unifying thread will be required, and so on. Note that this reasoning is the core of the classic Bradley's Regress Argument used most often historically as an argument against the coherence of polyadic relations, but what is equally relevant to monadic properties when the latter are properly construed as external/non-constitutive of their subjects.

Aristotle rightly interprets the regress argument not as demonstrating the absurdity of ontic predicates, but, to the contrary, as displaying in an obvious way the *agent unifier* nature of structural causes, what he identifies with certain *infima species* as ontic predicates—his substantial and accidental forms. In effect, Aristotle held that these ontic predicates in qualifying their subjects must be each 'an intension-determined agent', what he would intend to expand in sense to something like 'the exerciser of an active power or functionality that sustains a state of qualitatively determined linking or connectedness among distinct elements'. For him each form is an "actuality", a term taken for the Latin *agere* = 'to do', and what is intended to translate Aristotle's technical term *energeia* = 'at work-ness', 'being at work', or 'functioning'. An 'agent', what is necessarily correlative with one or more recipients or 'patients', has as part of its essence an activity or functioning that involves inherently a *going-beyond-itself* to produce some connection with, and in this way has some 'effect' upon,

something else, even if this effect is simply to be so linked by and to the agent. A structuring agent must go beyond itself to unify in an intension-determined way itself to one or more subjects, and Aristotle saw that this is required to be the nature of at least some ontic predicates. A generalizing thesis argued herein is that *all* ontic predicates—properties and relations—are structuring causes in being agent-unifiers of themselves to or among their subjects, each effecting a fact-structure, :R(a_1,a_2,...,a_n). This and the existential dependence of ontic predicates upon their subjects/patients are the grounds for the generality of the Predicate Dependency Thesis T1. A second and equally important generalization is the recognition that a single complex whole can be the result of multiple agent unifiers, specifically, via chains of relatum-sharing facts, facts whose ontic predicates can be of any *n*-adicity. Yet, for specious reasons to be considered shortly, these points are contrary to the Aristotelian/scholastic tradition which held that there is only one such agent per structured whole and it is a *monadic property*. This is the *Monadic Unifier Thesis*:

> T6) Each structured unity is by means of a single monadic ontic predicate of a certain type, i.e., a form, whose nature is to be an intension-determined agent-unifier.

Because forms, when ontic predicates of their respective matters, give defining structure to entities and in this make each to be specifically what it is, Aristotle identifies these forms with the essences of their resulting structures, and, when prime matter is the subject of these forms, he assert that these forms are the primary being of each (*Meta*. 1032b1, 1037a29, 1043a29-b14). We have then with Aristotle a two-tiered ontology of ontic predication—substantial forms when predicated of ultimate matter generate both the determinate parts as well as the structure among them that results in substances, the latter then the subjects of ontic predicates that are as such accidents, with accidental forms having multiple substances as simultaneous subjects and effecting structured artifacts. In this way the dominant role of the dependence|independence dichotomy in the search for primary being is replaced with that of the agency|non-agency dichotomy, though the former dichotomy through defective Independent Substrata Thesis T2 remains influential in requiring the two-tiered theory.

The specious assumptions motivating Monadic Unifier Thesis T6 are both the classic property-reduction of relations, and, perhaps more impor-

tant for its retarding effect upon ontology generally, the persistent and widely held though usually implicit *Unity-by-the-One Thesis*:

> T7) Each plural unification is by a shared one, i.e., by a single entity somehow linking all the other elements.

The impossibility of the reductive elimination of polyadic relations partially corrects T6, but it leaves open the possibility in conformity to T7 that each structured unity is by means of a single *polyadic* ontic predicate having all the other parts as relata. But, on the thesis I propose that relation intensions have fixed 'adicities', i.e., each intension determines a fixed number of relata, this would mean that the addition or subtraction of one or more elements would require a unifying relation of another intension and so make for a structure of an entirely different species—structured entities could not remain the same type of entity and yet change, e.g., could not grow. T7 has the aesthetic appeal of simplicity and proportion, and perhaps the illusion of necessity—unity into a one must be by a one (e.g., *Meta*. 1016b6-10, 1061a17 & 1061b13). And, there are physical models for it, e.g., that above of a single string unifying the beads of a necklace, or a single vessel holding its contents together (e.g., *Meta*. 1023a6-24). Yet, and pointing in a simple way to the error of T7, there are also physical models to the contrary, e.g., a chain of several links. A chain is a unified whole composed of only its links, but the first link is unified with the last not by a single entity shared by all the links. Rather, the unity of the whole is by means of a transitivity across the unions of adjacent inter-connected links. Here it is clear that the unity of the whole is a joint/collective result of constituent relations—topological-physical relations of two mutually containing links—and their relata shared in such a way that there is a transitivity of connectedness between any two constituents of the whole. Once alerted to the possibility of multiple unifiers for a single structured whole, it is apparent that all complex structures analyze like the chain. Hence, Unity-by-the-One Thesis T7 is to be corrected to the *Multiple Unifier Thesis*:

> T7′) Each plural unification is by either a single shared unifier or by multiple unifiers that share subjects in such a way that each element is transitively linked with every other.

Then likewise the Monadic Unifier Thesis T6 is to be modified to conform to T7′. And, on the fact that all ontic predicates are agent-unifiers, we would have the *Polyadic Unifier Thesis*:

> T6′) Each structured unity is by means either of a) a single ontic predicate resulting in the atomic structure of a single fact, or b) multiple ontic predicates among which there are sufficient relatum-sharing pairs that by a transitivity of connectedness every relatum of the structure is linked with every other.

For a more precise statement of T6′ specifying both 'horizontal' and 'vertical'/hierarchical structuring, I refer the reader elsewhere.[17] These two types of structuring will be illustrated below in a context that will reinforce the warrant for T6′.

4. The Ontology of Network Instance Realism: Ontic Predicates as Substance

Let us review briefly. In its analysis of the ubiquitous reality of structure the last section has brought into focus the ontological necessity for and the nature of ontic predicates as intension-determined agent-unifiers. It is this agency that renders ontic predicates dependent entities and so warrants Predicate Dependency Thesis T1. Further and causing us to reject theses T5/T5′ where intensions are ontic predicates, it is this linking nature that distinguishes ontic predicates, e.g., Has-Mass, Is-Wise, and relations, e.g., Is-Father-of, Is-Addition-of-..-and, from their constituent and abstractable non-agent/inert intensions, e.g., Mass, Wisdom, Fatherhood, Addition. Failure to observe this distinction is what makes plausible Bradley's Regress Argument, and other misleading distortions, e.g., abetting the inherence model of predication and standard trope theory. Determinacy Thesis T3, Intensional Essence Thesis T4, and Multiple Unifier Thesis T7′ have been argued for on independent grounds. What is left to do is to a) substantiate the Intensions-as-Predicate-Parts Thesis T5″ and reinforce the Polyadic Unifier Thesis T6′, b) show how T1—the existence of inherently dependent entities—does not necessitate T2—the existence of inherently independent entities, and c) make good on the promise of demonstrating how the agent-unifier nature of ontic predicates implies their individuation as relation instances. Indeed as we shall now see, it is c) that provides the means of accomplishing a) and b).

[17] See my 'An Instance Ontology for Structures' and 'Objects as Hierarchical Structures'.

It is the theses that explicate the individuation of ontic predicates as instances, what I have detailed elsewhere[18], together with the theses defended herein, i.e., T1, T3, T4, T5″, T6′, and T7′, that are at the foundation of the ontology of Network Instance Realism. This ontology is proposed as the coherent alternative to what, under classically assumed thesis T2**,** are the standard and defective 'independent substratum' ontologies: either Aristotelian/hylomorphic or bare particular theories that assume intensions are ontic predicates under T5 or T5′, or contemporary bundle theories that deny the full generality of T5 or T5′.

The demonstration that ontic predicates as agent-unifiers are individuated to unrepeatable instances or unit attributes is straightforward. Consider any contingent relation under an intension R, e.g., Is-Magnet-ically-Attracted-to, and let this relation obtain among distinct relata subjects, e.g., there exist facts :R(*a*,*b*) and :R(*c*,*d*), where *a*, *b*, *c*, and *d* are pair-wise non-identical. The facts :R(*a*,*b*) and :R(*c*,*d*) exist because the relation with intension R in each is the agent-unifier/combinator among the subjects of the respective ordered 2-tuples, <*a*,*b*> and <*c*,*d*>. Now assume that one of the two facts ceases to exist, e.g., that it is no longer the case that *a* is magnetically attracted to *b*. If the relation that is the unifying cause of the whole which is each fact were *one and numerically the same* constituent for both facts, then for this relation to cease to exist as a unifying agent among *a* and *b* would be for it to likewise cease to be the unifying agent among *c* and *d*, and so for fact :R(*c*,*d*) to also go out of existence. But this is absurd. Hence, the ontic predicate with intension R in fact :R(*a*,*b*) cannot be numerically the same as the ontic predicate with intension R in fact :R(*c*,*d*). What is appropriate, then, is 'instance notation': our example facts are given as :R_i(*a*,*b*) and :R_j(*c*,*d*), where R_i and R_j are unrepeatable/individual instances of shared intension R, and $R_i \neq R_j$. As noted above, this result should not be surprising since the unifying agency of an ontic predicate is, or is analogous to, the agency that is definitional of an event, and it is intuitive that events are particulars. In general, then, all ontic predicates are individuated as instances. It is the recognition of the agent-unifier nature of ontic predicates and the implication that this individuates them that is for ontology the achievement of a *principium individuationis* with real explanatory power, what contrasts with the tradition's impotence in being left to simply posit entities to do the job, e.g., bare particulars. With relation instances, R_i, we have a precise explanation of how individuation involves

[18] Ibid.

both specification and differentiation, the former via a repeatable intension R and the latter by a non-repeatable act of unification, a particularity indicated by the subscript 'i'. Instances R_i, R_j, R_k, …, are qualitatively identical in their composition—having as their only intensional component intension R—and yet they differ more than 'solely by number', *solo numero*. The explanatory gain being that each such instance differs from the others by its unique *unifying act* among its unique *n*-tuple of subjects.

Once we have relation instances as intension-determined combinators it is but a short step to observing the possibility of an ontology with an atomic ontic level of only dependent entities, *contra* the classic Independent Substrata Thesis T2. To see this, consider the possibility of three relation instances R_i, S_j, and T_k, composing the facts $:R_i(S_j,T_k)$, $:S_j(T_k,R_i)$, and $:T_k(R_i,S_j)$. Here each instance exists simultaneously as an ontic predicate of the other two instances as its subjects, analogous to the two free ends of a chain being joined to form a closed circle. Though each instance is a dependent entity, this dependency is 'satisfied' with the having of subjects, whether or not these subjects are themselves dependent entities. Hence in the *independent closed system* that is the resultant structure consisting of the three simultaneous facts of $:R_i(S_j,T_k)$, $:S_j(T_k,R_i)$, and $:T_k(R_i,S_j)$, the composing parts are exclusively dependent relation instances. Now it is easy to imagine that this as one type of 'horizontal' composition can be generalized to resultant structures each composed of a set of relation instances of various adicities where the subject *n*-tuples have as relata only other instances from the set. Here closed circles of mutual ontic support answer the worry of otherwise viciously regressing lines of dependence. Further, each such closed system can itself as a single entity be the subject/relatum for other relation instances and in this way form hierarchical structures—a type of composition I have termed 'vertical'. Now clearly it is possible that these types of composition can be iterated up through increasing complex structures,[19] what could include the structured entities given in ordinary experience or theorized by science. In both the horizontal and vertical composition outlined there is a transitivity of connectedness via relation instances, hence the reinforcing of Polyadic Unifier Thesis T6′. Moreover, since the essence of an entity is what makes the entity to be what it is, the essence must be constitutive of it. Hence, if an intension F is part of the essence of an entity *x* then F must be a part of *x*. But here the

[19] For instructive spatial analogies descriptive of these various forms of structuring see my 'An Instance Ontology for Structures' and 'Objects as Hierarchical Structures'.

only entities composing anything are relation instances or wholes that decompose without remainder into relation instances, and so intension F as a constituent of *x* must be a constituent of a relation instance. Hence, Intensions-as-Predicate-Parts Thesis T5″.

In sum, what has been argued herein is the coherence and power of an ontology describing a reality composed exclusively of ontic predicates—properties and relations analyzed as *intensioned combinators*—and the structures that emerge from them. In this way ontic predicates are 'substance', or, more accurately, 'primary being'. The ontology has the virtue of saving us from classic and specious errors that have limited the debate to the two equally unacceptable alternatives of substratum or bundle theories. And, it is comprehensive in being both consistent with our intuitive *prima facie* facts as well as descriptive of the content of our abstract and physical sciences, including what at the physical micro-level is apparently a purely structural quantum reality.[20]

[20] I wish to express my gratitude to Käthe Trettin for her generous comments and assistance toward improving the original version of this essay.

6

The Logic of Instance Ontology*

1. Introduction

The relationship between an ontology and a concomitant predicate logic turns on more than simply the categories of entities posited by the ontology. Equally important is the ontology's theory of ontic or 'material' predication as it is modeled in the grammatical or 'formal' predication of the logic's formation and transformation rules. The example that is most historically influential is the Aristotelian containment model of predication, where a property is 'contained in' ('inherent in', 'immanent in') its subject, which translates formally into a syllogistic logic and a reductionist program for eliminating polyadic relations in favor of monadic properties. Another example is Frege's ontic theory of the rigid distinction between 'unsaturated' concepts and 'saturated' objects and an *n*-level hierarchy of concepts ('functions'), which translates formally into a theory of simple logical types—higher-order logic. A further example, one hitherto neglected but far-reaching in its implications for logic, is the ontology of unit or particularized attributes—instance ontology. The refined system of logical relationships that follow from a version of this doctrine as specified below, what I have called *Particularized Predicate Logic* (PPL), is remarkable in its power to make distinctions and solve problems that cruder standard predicate logics cannot. Not the least of these advantages is PPL's character as a consistent 'extended' predicate logic, i.e., what is in effect a higher-order logic without predicates stratified into orders or types. In my *Moderate Realism* I provided a prototype for PPL and displayed its power in solving problems in the foundations of logic and mathematics.[1] However, this pilot formalization contained a number of omissions and minor inaccuracies, particularly acute with regard to the crucial axiom (schema) of

* Original version first published in the *Journal of Philosophical Logic* 28 (1999): 81-111. Used here with the kind permission of Springer Science and Business Media.

[1] D. W. Mertz, *Moderate Realism and Its Logic* (New Haven: Yale University Press, 1996). Those portions of *Moderate Realism* included here are with the permission of Yale University Press.

comprehension. Also in regard to the latter axiom, the formulation given contained a complicating and distracting redundancy. Below I shall rectify these deficiencies with a precise and more perspicuous version of PPL, and display further its power with a new example consisting of an illuminating predicate instance analysis of Grelling's Paradox. On the basis of its demonstrated and potential utility alone, the refined logic of PPL warrants further study, and in this lies the motivation for disseminating the following improved version.

Instance ontology is the doctrine that relations (including monadic properties) exist as unrepeatable individuals, and not as shared universals as they are commonly and pre-critically characterized. The doctrine asserts, for example, that there are distinct instances of the same relation *Less-than* in the facts '2 is less that 3' and '3 is less than 4'. Stated symbolically, for $\text{Less}_i(2,3)$ and $\text{Less}_j(3,4)$, $\text{Less}_i \neq \text{Less}_j$. Apparent reference is made to these instances, or 'particularized predicates', in ordinary language. Peter Strawson gives the example 'The wisdom of Socrates is preserved for us by Plato'.[2] It is the wisdom specific to Socrates and not that of someone else, nor the common property of Wisdom, that Plato is preserving. Symbolically, $(\exists \text{Wis}_i)[\text{Wis}_i(s) \cdot (\exists \text{Pre}_j)\text{Pre}_j(\text{Wis}_i,p)]$. A theory of events, and particularly the issue of their individuation, is much clarified when events are construed in a straightforward way as particularized predicates. An event reported in 'Cain hit Abel with a stick at noon' would be a specific hitting, Hit_i, and it would be this individual and not the universal, Hit, nor Cain nor Abel, that has the properties of being done with a stick and at noon. Formally, $(\exists \text{Hit}_i)[\text{Hit}_i(c,a) \cdot (\exists \text{Stk}_j)\text{Stk}_j(\text{Hit}_i) \cdot (\exists \text{Noo}_k)\text{Noo}_k(\text{Hit}_i)]$. In regard to such examples, PPL provides a perspicuous inference engine for arguments involving them and a refined analytic tool for ontic analysis of the contained concepts, results that standard predicate logics are too gross to effect.

Instance ontology is ancient and perennial. There is evidence for it in the writings of Plato and Aristotle, it was popular among the scholastics, and it has received increased attention this century, both in its nominalist ('trope theory') and realist forms. In advocating trope theory, contempo-

[2] Peter Strawson, *Individuals* (London: Methuen, 1971), pp. 168–169, n. 1. Strawson also gives the examples: 'His anger cooled rapidly' and 'His cold is more severe than Mary's'.

rary philosophers (e.g., Keith Campbell, John Bacon)[3] have argued for its problem solving power in the areas of ontology, causation, perception, belief, and the philosophy of mind. However, the problem solving power these authors identify is independent of their nominalist thesis. Indeed, I have argued that a 'moderate realist' version of instance ontology not only avoids the classic as well as newly identified weaknesses of nominalism, but also solves additional problems in ontology and the theory of predication, the self-referential paradoxes, the foundations of mathematics, and in identity theory.[4] Moderate realism's advantage over trope theory in effecting these results is due to its superior analysis of ontic predication and the identification of principles that follow from it. Briefly stated, ontic predication is the combinatorial mode an intension has among or between a set of relata. It is the ontically defining role of a relation in a fact (e.g., 2 is less that 3) to combine or link its relata (2 and 3) under a specific content or intension (Less-than). A relation, insofar as it has this ontically defining status of 'actually relating' its relata, is proprietary to this specific relata *n*-tuple. A relation as a content abstracted from any specific relata sets is a nonpredicative abstraction, and precisely as such, a repeatable *universal*. The realism here is moderate, as opposed to Platonic, because the intension R, when abstracted from its predicative mode as an aspect of any particular instance R_j, has a conceptual existence only. Specifically, of the principles characterizing moderate realism, the two directly relevant to predication and hence to the paralleling predicate logic are:

Principle of Immanent Instance Realism (IR)

For distinct instances $\mathbf{R}^n_i$, $\mathbf{R}^n_j$,…, there exists an intension $\mathbf{R}^n$ which is a numerically identical aspect of each of the instance $\mathbf{R}^n_i$, $\mathbf{R}^n_j$,…, i.e., $\mathbf{R}^n$ is a universal.

Principle of Instance Predicates (IP)

Only unrepeatable relation instances, $\mathbf{R}^n_i$, are *ontic predicates*—that is, only instances exist as *predicative* among specific subject *n*-tuples; the universal $\mathbf{R}^n$ is not ontically predicative.

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[3] Keith Campbell, *Abstract Particulars* (Oxford: Blackwell, 1990); John Bacon, *Universals and Property Instances: The Alphabet of Being* (Oxford: Backwell, 1995).

[4] See the essays herein and my *Moderate Realism*.

Though additional principles characterize moderate realism,[5] **IR** and **IP** are principal in implying a refined system of logical relationships that is formalized in PPL. In particular, **IR** and **IP** jointly require, in addition to a domain of individuals, domains each of common intensions and their unrepeatable instances. In compliance with **IP**, only instances, $\mathbf{R}^n_i$, can occupy predicate positions in well-formed formulas of the paralleling logic. Of more interest is a logical device implied by **IR**, one I refer to as 'extended binding', and that allows intension quantifiers, $(\mathbf{R}^n)$ or $(\exists\mathbf{R}^n)$, to bind instance variables $\mathbf{R}^n_i$, and so instance quantifiers, $(\mathbf{R}^n_i)$ and $(\exists\mathbf{R}^n_i)$, within their scope that have the same intension $\mathbf{R}^n$ as content. I shall expand upon this in the sequel. Extended binding is intuitive and easily mastered. As one might suspect, this ease corresponds directly with the complications necessary to formalize extended binding within a set of formation and transformation rules. The reader will find it easier going if the concept of extended binding is made intuitive from the beginning. More specifically, PPL as developed below is a three-sorted *impredicative intensional logic* with identity. It has constants referring to, or variables ranging over, entities within one of the respective domains of individuals, intensions, or instances. In addition, global variables will be introduced that range over all three domains. In a common and derivative sense of the term, PPL is an impredicative predicate logic in that constants and variables of all three domains are allowed in subject position, and there is no stratification of predicates into a hierarchy of types of any kind. We shall see how this is possible while still maintaining consistency by introducing a well-motivated modification of the standard axiom (schema) of comprehension. Indeed, one of the powerful motivations for studying this logic is its clarification of the self-referential nature of impredicative definitions and the distinction between legitimate and illegitimate forms. Using Graham Priest's refinement of Russell's characterization of the mechanism inherent in all of the classic self-referential paradoxes, it will be seen how PPL diagnoses and

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[5] In addition to **IR** and **IP**, I had at the time of *Moderate Realism* identified the following principles as characterizing a moderate realist ontology:

(**SU**) Principle of Subject Uniqueness: If $\mathbf{R}^n_i(\mathbf{a}_1,\mathbf{a}_2,\ldots,\mathbf{a}_n)$ and $\mathbf{R}^n_i(\mathbf{b}_1,\mathbf{b}_2,\ldots,\mathbf{b}_n)$, then $\mathbf{a}_1 = \mathbf{b}_1, \ldots, \mathbf{a}_n = \mathbf{b}_n$.

(**IU**) Principle of Instance Uniqueness: If $\mathbf{R}^n_i(\mathbf{a}_1,\mathbf{a}_2,\ldots,\mathbf{a}_n)$ and $\mathbf{R}^n_j(\mathbf{a}_1,\mathbf{a}_2,\ldots,\mathbf{a}_n)$, then $\mathbf{R}^n_i = \mathbf{R}^n_j$.

(**RL**) Principle of Relata-Linking: No n-adic relation instance $\mathbf{R}^n_i$ exists except as ontically *predicative* among and hence necessarily presupposing some n-tuple of entities which as such it relates. [I have subsequently identified other principles refining moderate realism. See the essays herein.]

solves these antimonies. In this regard, I propose PPL is an approximation to what Gödel once described as an important but programmatic 'theory of concepts', what would be an intensional complement to extensional set theory.[6] It is the pivotal refinement of instance predicates and the logical devices that it entails that gives PPL its advantage in intuitiveness over other proposed impredicative intensional logics, such as those of Cocchiarella[7] or Bealer[8], where the provable consistencies of these systems rest upon *ad hoc* maneuvers.

2. Formalization of a Particularized Predicate Logic (PPL)

The following formal language is a refinement on standard predicate calculi, as found, for example, in Church's *Mathematical Logic*[9]. For brevity, the propositional calculus is not included. The system without an axiom of comprehension, *Restricted PPL*, will be first detailed and proved consistent, and then shown to be consistently modified to *Full PPL* by the addition of a proper axiom of comprehension.

Primitive symbols:

Improper: ( ) $\neg$ $\supset$ $\exists$ $\vee$ $\cdot$ $\equiv$

Proper:

(1) **Individual symbols**: Infinite lists of

Variables: $x\ y\ z\ x_1\ y_1\ z_1\ x_2\ y_2\ z_2\ \ldots$

Constants: $a\ b\ c\ a_1\ b_1\ c_1\ a_2\ b_2\ c_2\ \ldots$

(2) **Intension symbols** (constants that name or variables that range over universals):

Variables: an infinite list of one-place variables

$P^1\ Q^1\ R^1\ \ldots\ Z^1\ P'^1\ Q'^1\ R'^1\ \ldots\ Z'^1\ P''^1\ Q''^1\ R''^1\ \ldots$

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[6] Gödel's view of logic as a 'theory of concepts' is reported in Hao Wang, *Reflections on Kurt Gödel* (Cambridge: MIT Press, 1987), p. 297. Also see Gabriella Crocco, 'Gödel on Concepts', *History and Philosophy of Logic* 27 (2006): 171-91.

[7] Nino Cocchiarella, *Logical Investigations of Predication Theory and the Problem of Universals* (Naples: Bibliopolis, 1986).

[8] George Bealer, *Quality and Concept* (Oxford: Clarendon Press, 1982).

[9] Alonzo Church, *Introduction to Mathematical Logic* (Princeton: Princeton University Press, 1956).

An infinite list of two-place variables

$$P^2\ Q^2\ R^2\ \ldots\ Z^2\ P'^2\ Q'^2\ R'^2\ \ldots\ Z'^2\ P''^2\ Q''^2\ R''^2\ \ldots$$

Likewise for three-place, four-place, etc., variables. The superscripts represent the number of relata (i.e., the adicity) of the intensions over which the respective variables range.

Constants: an infinite list of one-place constants

$$A^1\ B^1\ C^1\ \ldots\ O^1\ A'^1\ B'^1\ C'^1\ldots\ O'^1\ A''^1\ B''^1\ C''^1\ \ldots$$

Similar infinite lists are assumed for two-place, three-place, etc., constants. Included among the intension constants of PPL is Id^2, that under the intended interpretation refers to the identity relation.

(3) **Instance symbols** (constants that name or variables that range over, to one of two extents, instances of intensions):

Full instance variables: an infinite list of one-place instance variables for each of the one-place intensions (predicate-type) variables above,

$$P^1{}_i\ P^1{}_j\ P^1{}_k\ \ldots\ Q^1{}_i\ Q^1{}_j\ Q^1{}_k\ \ldots\ Z^1{}_i\ Z^1{}_j\ Z^1{}_k\ \ldots\ P'^1{}_i\ P'^1{}_j\ P'^1{}_k\ \ldots\ Q'^1{}_i\ Q'^1{}_j\ Q'^1{}_k\ \ldots$$

The list continues indefinitely on subscript letters by the use of prime marks, e.g., $P'^1{}_{j'}$ The same is assumed for the following. An infinite list of two-place instance variables for each of the two-place intension variables above,

$$P^2{}_i\ P^2{}_j\ P^2{}_k\ \ldots\ Q^2{}_i\ Q^2{}_j\ Q^2{}_k\ \ldots\ Z^2{}_i\ Z^2{}_j\ Z^2{}_k\ \ldots\ P'^2{}_i\ P'^2{}_j\ P'^2{}_k\ \ldots\ Q'^2{}_i\ Q'^2{}_j\ Q'^2{}_k\ \ldots$$

Similarly for the three-place, four-place, etc., intension variables above. The range of a full instance variable $\mathbf{P}^n{}_i$ is all instances of $\mathbf{P}^n$, where $\mathbf{P}^n$ is a variable ranging over all intensions of adicity *n*.

Limited instance variables: an infinite list of one-place predicates for each of the one-place intension constants above,

$$A^1{}_i\ A^1{}_j\ A^1{}_k\ \ldots\ B^1{}_i\ B^1{}_j\ B^1{}_k\ \ldots\ C^1{}_i\ C^1{}_j\ C^1{}_k\ \ldots$$

Similarly for two-place, three-place, etc., intension constants above. The range of a limited instance variable is restricted to the instances of the *fixed* intension constant component of the variable symbol, e.g., 'A' of 'A_i'.

Instance Constants: an infinite list of one-place instance constants for each one-place intension constant,

$$A^1{}_1\ A^1{}_2\ A^1{}_3\ \ldots\ B^1{}_1\ B^1{}_2\ B^1{}_3\ \ldots\ C^1{}_1\ C^1{}_2\ C^1{}_3\ \ldots$$

Similarly for two-place, three-place, etc., intension constants above, which includes instances of the identity relation 'Id^2', i.e.,

Id^2_1 Id^2_2 Id^2_3 …

Finally, Gothic font Latin letters will be used for variables that range globally over entities of all three ontic categories of intensions, instances, and individuals.

(4) **Global variables**:

𝔞 𝔟 𝔠 … 𝔷 𝔞′ 𝔟′ 𝔠′ … 𝔷′ 𝔞″ 𝔟″ 𝔠″ …

Global variables have an important use in stating truths of identity, e.g.,

$(\mathfrak{a})(\exists Id^2_i)Id^2_i(\mathfrak{a},\mathfrak{a})$

which says that for any entity whatsoever there is at least one instance of the identity relation that relates the entity to itself. This truth of identity will be seen to follow from the axiom of PPL characterizing the identity relation, an axiom innovative in its refinement over the standard treatment of identity and its distinction from indiscernibility, as I shall indicate. [I have subsequently refined these formulations of identity and indiscernibility. See Essays 4 and 7.]

In order to talk efficiently about PPL as the object-language, we shall use as the informal metalanguage a fragment of English modified to include certain Greek and bold Latin letters with or without subscripts. The latter are syntactic variables which will take object-language single terms or complex expressions as substituenda. Object-language symbols will be taken over into the metalanguage as names of themselves. For ease of reading, braces and brackets will at times be used as abbreviations for parentheses.

Formation Rules:

(1) If $\mathbf{R}^n_i$ is a full or limited instance variable or an instance constant, and if $\mathbf{a}_1$, $\mathbf{a}_2$, …, $\mathbf{a}_n$ are variables or constants of any kind, then $\mathbf{R}^n_i(\mathbf{a}_1,\mathbf{a}_2,\ldots,\mathbf{a}_n)$ is a wff, and it or its negation is said to be a *literal* of PPL. Literals contain at most one occurrence of negation, ¬, those containing ¬ are negative literals; otherwise they are positive literals. For wff $\mathbf{R}^n_i(\mathbf{a}_1,\mathbf{a}_2,\ldots,\mathbf{a}_n)$, $\mathbf{R}^n_i$ is said to occupy the *predicate position*, and $\mathbf{a}_1$, $\mathbf{a}_2$, …, $\mathbf{a}_n$ each occupy a *subject position*.

(2) If **A** is a wff, then ¬**A** is a wff.

(3) If **A** and **B** are wffs, then (**A** ⊃ **B**), (**A** ∨ **B**), (**A** · **B**), and (**A** ≡ **B**) are wffs.

(4) If **A** is a wff, and **a** is any variable, then (**a**)**A** and (∃**a**)**A** are wffs.

A wff **B** is a *subformula* of wff **A** if and only if and only if **B** is a part of **A**.

Note that by (1) only instance symbols (full or limited instance variables or instance constants) occupy predicate positions in wffs of PPL, though they may occupy subject positions as well. Symbols for intensions, individuals, and global variables may occur only in subject positions. These restrictions conform to the ontic Principle of Instance Predicates (**IP**).

For ease of use and understanding I have included in PPL the existential quantifier whose relation to the universal quantifier is given in the object-language by the 'definitional' (i.e., noncreative axiom) schema

D1. (∃**a**)**A** ≡ ¬(**a**)¬**A**

Likewise for perspicuity and ease of use I have included the logical constants of conjunction, disjunction, and biconditional in the object-language, where they are assumed to be defined in terms of ⊃ and ¬ by axiom schemata of the standard forms.

There are two aspects of PPL that constitute its analytic refinement over standard predicate logics: the increased expressive power that the combination of intension and corresponding instance quantifiers provide, and, building on this, the device of *extended binding* by an intension quantifier of corresponding instance variables and instance quantifiers within their scope. In standard predicate logics there are only two modes of quantification, e.g., (P)…P(*a*) or (∃P)…P(*a*). When standard quantifiers are restricted to minimum scope, as in (P)P(*a*) or (∃P)P(*a*), the latter correspond, respectively, to the PPL expressions:

(P)(∃P_i)P_i(*a*) [i.e., for every property P there exists at least one instance of P, P_i, such that P_i(*a*)], and

(∃P)(∃P_i)P_i(*a*) [i.e., there exists at least one property P and at least one instance of P, P_i, such that P_i(*a*)].

However, in PPL there remain two further quantifier combinations as in:

$(P)(P_i)P_i(a)$ [i.e., for every property P and every instance of P, P_i, $P_i(a)$], and

$(\exists P)(P_i)P_i(a)$ [i.e., there exists at least one property P such that for any instance of P, P_i, $P_i(a)$]

The first example says that every instance of every property belongs to *a*, which is absurdly false. The second example asserts that there is at least one property P that is instantiated by only the one subject *a*. An example of such a property would be 'is a prime divisor of 2'. More relevantly, when the scope of (P) is broadened and negation is introduced to yield the form $(P)\ldots(P_i)\neg P_i(a)$, we then have the crucial 'impredicative' situation involved in the self-referential paradoxes, as I shall show below. It will be a non-*ad hoc* restriction added to this configuration that will provide a solution to the paradoxes.

Concomitant with the refinement of including both intension and instance quantifiers, and along with them a distinguishing feature of PPL, is the device of *extended binding* by intension quantifiers. Extended binding, together with its use in the rule of universal instantiation below, is the implication for PPL of the Principle of Immanent Instance Realism, **IR**. Loosely, extended binding by an intension quantifier, (**P**) or (∃**P**), allows it to bind not only occurrences of **P** within its scope, but also occurrences of corresponding instances $\mathbf{P}_i$, including as operator variables of quantifiers ($\mathbf{P}_i$) or (∃$\mathbf{P}_i$), within its scope. Here the scope of quantifiers is defined in the usual way: where **a** is any type of variable and **A** is a wff, then every symbol in (**a**)**A** [(∃**a**)**A**] is within the *scope* of (**a**) [(∃**a**)]. Binding by quantifiers in PPL is then defined as:

(1) If **a** is an individual or global variable, or a full or limited instance variable (i.e., if **a** is a non-intension variable), then (**a**) [(∃**a**)] binds all occurrences of **a** within its scope with the exception that (**a**) [(∃**a**)] does not bind occurrences of **a** within any subformula of **A** of the form

(**a**)**B** or (∃**a**)**B**, and

(2) If **a** is an intension variable, $\mathbf{R}^n$, then

(i) (**a**) [(∃**a**)] binds all occurrences of $\mathbf{R}^n$ within its scope, except those occurrences within a subformula of **A** of the form ($\mathbf{R}^n$)**B** or (∃$\mathbf{R}^n$)**B**, and

(ii) [*Extended binding*] (**a**) [(∃**a**)] binds all and only the occurrences of the instance variables $\mathbf{R}^n{}_i$, for any subscript *i*, within its scope

that occur in any subformulas of **A** of the form $(\mathbf{R}^n_i)\mathbf{B}$ or $(\exists\mathbf{R}^n_i)\mathbf{B}$, except when these subformulas occur within subformulas of **A** of the form $(\mathbf{R}^n)\mathbf{C}$ or $(\exists\mathbf{R}^n)\mathbf{C}$.

A variable not bound in a wff is *free* in that wff. A wff with no free variables is *closed*, and a quantifier that binds no variables within its scope is *vacuous*. The exception clauses in (1) and (2) are necessary because of the liberal formation rules allowing the same wff to have quantifiers with the same operator variables and overlapping scopes, e.g., $(P)[(\exists P_i)P_i(a) \cdot (\exists P)[(\exists P_j)P_j(b) \cdot b \neq a]]$.

The device of extended binding defined in 2(ii) is relatively simple. Consider the following example where the symbols below the quantifiers are repeated under the variables they bind.

$$(Q)(Q_i)(\exists x)[B_3(Q,Q_i) \supset (\exists Q_j)Q_j(x)]$$
$$\begin{array}{cccccccc} & & \cdot & & & & & \cdot \\ & * & & & * & & \circ & \circ \\ \wedge & \wedge & & \wedge & \wedge & & \wedge & \wedge \end{array}$$

By the transformation rules given below we will be able to instantiate Q by, for example, constant A, to yield

$$(A_i)(\exists x)[B_3(A,A_i) \supset (\exists A_j)A_j(x)].$$

A subtlety of 2(ii) is displayed in the following example, with underlying symbols again indicating quantifier binding.

$$(\exists P)(\exists P_i)(P)P_i(P)$$
$$\begin{array}{cccc} & & \cdot & \cdot \\ & * & & * \\ \wedge & \wedge & & \wedge \end{array}$$

The variable P_i of the subformula $(P)P_i(P)$ is not bound by the contained quantifier (P) because, by 2(ii), an intension quantifier only binds a corresponding instance variable when the latter is bound by an instance quantifier whose variable is bound in turn by the intension quantifier. In general, an intension quantifier (**P**) binds corresponding full instance variables $\mathbf{P}_i$ within their scope only through the mediation of a full instance quantifier $(\mathbf{P}_i)$ or $(\exists\mathbf{P}_i)$.

The device of extended binding reflects the ontological fact, or at least as I have argued[10], that the relation between an intension $\mathbf{P}^n$ and any of its instances $\mathbf{P}^n_i$ is distinct from the relation between an instance $\mathbf{P}^n_i$ and

[10] In *Moderate Realism* and in the essays in this volume.

the subjects $\mathbf{a}_1$, $\mathbf{a}_2$, …, $\mathbf{a}_n$ that are unique to it. The latter relation is that of ontic predication, whereas the former is the relation of a common non-predicative aspect to a predicative, and because of this, individuated whole. A strong argument can be made that it is the predicative, combinatorial status of an instance $\mathbf{P}^n_i$ among its relata that renders it unrepeatable.

Transformation Rules:

For wffs **A** and **B** of PPL,

Rules of inference:

R1. If $\vdash \mathbf{A}$ and $\vdash \mathbf{A} \supset \mathbf{B}$, then $\vdash \mathbf{B}$.

R2. If **a** is any variable and $\vdash \mathbf{A}$, then $\vdash (\mathbf{a})\mathbf{A}$.

Axiom schemata:

A1. $\mathbf{A} \supset (\mathbf{B} \supset \mathbf{A})$.

A2. $(\mathbf{A} \supset (\mathbf{B} \supset \mathbf{C})) \supset ((\mathbf{A} \supset \mathbf{B}) \supset (\mathbf{A} \supset \mathbf{C}))$.

A3. $(\neg\mathbf{A} \supset \neg\mathbf{B}) \supset (\mathbf{B} \supset \mathbf{A})$.

A4. $(\mathbf{a})(\mathbf{A} \supset \mathbf{B}) \supset (\mathbf{A} \supset (\mathbf{a})\mathbf{B})$, where **a** is a variable of any kind, and (**a**) does not bind a variable in **A**.

The next axiom is that of universal instantiation. Section A5(II) will complete the rationale for extended binding. What is in need of explanation are the conditionals of A5(III) with antecedents of the form '$(\exists\mathbf{a})(\mathbf{a} = \mathbf{b}) \supset$'. This device is used to parry the effect of existential presupposition in regard to instances of intension constants. That is, we want to avoid what is, without the given conditional, the requirement that for every intension constant, e.g., A, there is at least one instance of its kind, A_i . This would follow from conditionals like $(A_i)A_i(c) \supset (\exists A_i)A_i(c)$, which in turn are derivable from A5(III) if not prefaced as indicated. Yet, we want PPL to be able to handle uninstantiated intensions, e.g., 'is phlogiston', 'is a unicorn', 'is a perfect material sphere'.

A5 (I) $(\mathbf{a})\mathbf{A} \supset \mathbf{B}$, where:

a is (1) an individual variable or (2) a global variable or (3) a full instance variable $\mathbf{R}^n_i$ and **B** results for **A** by replacing all free occurrences of **a** in **A** by a term **b** which is, respectively,
(1′) an individual variable or constant,

e.g., $(x)(\exists B_j)B_j(x) \supset (\exists B_j)B_j(c)$;

(2′) a variable or constant of any type other than a limited instance variable or instance constant,

e.g., $(\mathfrak{a})(\exists R)(\exists R_i)R_i(\mathfrak{a}) \supset (\exists R)(\exists R_i)R_i(S)$;

(3′) a full instance variable $\mathbf{P}^n_j$,

e.g., $(P_i)(A_j)[P_i(x,A_j) \supset A_j(P_i)]$

$\supset (A_j)[Q_i(x,A_j) \supset A_j(Q_i)]$.

In all cases, **b** is such that it must not become bound in **B** at a place where **a** is not bound in **A**.

(II) $(\mathbf{a})\mathbf{A} \supset \mathbf{B}$ where **a** is an intension variable, $\mathbf{R}^n$, and **B** is the result of substituting as **b** an intension variable or constant, $\mathbf{P}^n$, for each free occurrence of $\mathbf{R}^n$ in **A** and substituting a full or limited instance variable $\mathbf{P}^n_j$, respectively, for each occurrence of an $\mathbf{R}^n_i$ bound by the initial quantifier (**a**). What is substituted must be so chosen as not to become bound where the variable substituted for was not.

By the device of extended binding, (II) makes it possible to go from antecedents of the form

$(P)[(P_i)P_i(P) \supset (\exists P_j)P_j(P)]$

to a consequent of

$(A_i)A_i(A) \supset (\exists A_j)A_j(A)$

or to a consequent of

$(Q_i)Q_i(Q) \supset (\exists Q_j)Q_j(Q)$

where P is instantiated with intension constant A and intension variable Q, respectively.

The last examples display the conditional implying existential presupposition that was discussed in the preface to this axiom and that require in the following cases the use of the prefatory clause '$(\exists \mathbf{a})(\mathbf{a} = \mathbf{b}) \supset$'.

(III) $(\exists \mathbf{a})(\mathbf{a} = \mathbf{b}) \supset [(\mathbf{a})\mathbf{A} \supset \mathbf{B}]$ where **a** is (1) a full or (2) a limited instance variable $\mathbf{R}^n_i$, or (3) a global variable, and **B** results from **A** by replacing all free occurrences of **a** in **A** by a term **b** that is, respectively, (1′) either a limited instance variable $\mathbf{P}^n_j$ or instance constant $\mathbf{P}^n_r$, r a natural number; (2′) a limited instance variable $\mathbf{R}^n_j$ or an instance constant $\mathbf{R}^n_r$, r a natural number; or (3′) any limited instance variable or instance constant, e.g.,

$$(\exists \mathrm{R}_i)(\mathrm{R}_i = \mathrm{B}_j) \supset [(\mathrm{R}_i)\mathrm{R}_i(a) \supset \mathrm{B}_j(a)], \text{ or}$$

$$(\exists \mathfrak{a})(\mathfrak{a} = \mathrm{A}_i) \supset [(\mathfrak{a})(\exists \mathrm{P}_j)\mathrm{P}_j(\mathfrak{a}) \supset (\exists \mathrm{P}_j)\mathrm{P}_j(\mathrm{A}_i)]$$

As before, what is substituted for **a** in **B** must not become bound in **B** at places where **a** is not bound in **A**. In all cases, if (**a**) is vacuous for **A**, then **B** is the same as **A**.

The last axiom defines the relation of *identity*:

$$\text{A6. } (\mathfrak{a})(\mathfrak{b})\{(\exists \mathrm{Id}^2_i)\mathrm{Id}^2_i(\mathfrak{a},\mathfrak{b}) \equiv (\mathbf{P}^1)(\mathbf{P}^1_j)[\mathbf{P}^1_j(\mathfrak{a}) \equiv \mathbf{P}^1_j(\mathfrak{b})]\}.$$

Axiom A6 is a ‘creative’ axiom in that it asserts the existence of instances of the identity relation Id^2 when the conditions of the definiens are satisfied. For ease of expression, I shall in the following abbreviate instances of the schema

$$(\exists \mathrm{Id}^2_i)\mathrm{Id}^2_i(\mathbf{a},\mathbf{b})$$

as simply

$$\mathbf{a} = \mathbf{b},$$

and instances of the schema

$$\neg(\exists \mathrm{Id}^2_i)\mathrm{Id}^2_i(\mathbf{a},\mathbf{b})$$

as

$$\mathbf{a} \neq \mathbf{b}.$$

I note briefly the significant point that the specification of the identity relation in A6 has a refined advantage over its counterpart in standard predicate logics. The only form that the definiens of the identity relation can take in prevailing non-instance logics is the undifferentiated ‘$(\mathrm{P})[\mathrm{P}(x) \equiv \mathrm{P}(y)]$’ or its logical equivalent, and this is insufficient to differentiate between strict numerical identity and the looser indiscernibility. [In Essay 7 in this volume I argue for further refinements on the specifications of iden-

tity.] Yet in PPL, indiscernibility, Ind^2, can be perspicuously defined and distinguished from identity by

$$(\mathfrak{a})(\mathfrak{b})\{(\exists \mathrm{Ind}^2{}_i)\mathrm{Ind}^2{}_i(\mathfrak{a},\mathfrak{b}) \equiv (\mathbf{P}^1)[(\exists \mathbf{P}^1{}_j)\mathbf{P}^1{}_j(\mathfrak{a}) \equiv (\exists \mathbf{P}^1{}_k)\mathbf{P}^1{}_k(\mathfrak{b})]\}.$$

This says that entities are indiscernible when for each instance that characterizes one there is an instance of the same property that characterizes the other, though these instances need not be numerically the same. This contrasts with the specification for identity that requires instances of shared properties be numerically the same. [A corrected analysis of indiscernibility is given in Essay 4 of this volume.]

The instance logic thus far delineated is restricted PPL. We shall now demonstrate its consistency.

3. The Consistency of Restricted PPL

The consistency of restricted PPL is established by the appropriate modifications of a standard syntactical consistency technique. The technique uses the extended propositional calculus, EPC, which is the standard propositional calculus extended to include quantifiers over propositions. Each wff of PPL has a set of associated formulas of the EPC, or 'afeps', which are identical in logical form, differing at most in the choice of proposition variables or constants. An afep for a given wff **A** of PPL is determined as follows. First, in **A** delete all subject position symbols and their enclosing parentheses. Next, delete all quantifiers with intension variables, and then all other quantifiers that at this point in the reduction are vacuous. At this point in the reduction what will remain of **A** are predicate instance constants and variables, and quantifiers with the latter as operator variables. As a final step, replace each remaining distinct instance variable and constant with a distinct lower case Latin letter proposition variable or constant, respectively. The letters 'p'…'z' and 'a'…'o', with or without numerical subscripts, will represent proposition variables and constants, respectively. An afep of a wff **A** will be designated by '**A′**'.

Consider the example where **A** is

$(\exists \mathrm{R})(\mathrm{R}_i)(\exists \mathrm{Q}_j)[\mathrm{C}_2(\mathrm{Q}_j) \supset (\mathrm{R}_i(x,\mathrm{Q}_j) \vee \neg \mathrm{R}_i(x,\mathrm{Q}_j))]$.

Delete all but predicate terms in predicate positions and the instance quantifiers binding them:

$(R_i)[C_2 \supset (R_i \vee \neg R_i)]$.

Next, replace R_i with a proposition variable, say r, and instance constant C_2 with a proposition constant, say c. The result is the afep of **A**, i.e., **A′** is

$(r)[c \supset (r \vee \neg r)]$.

Because afeps of a wff **A** will differ only in the choice of proposition variables or constants, any afep of **A** will go proxy for the others in the following analysis.

To prove the syntactical consistency of PPL we will first define what it is for an afep of a wff **A** to be valid. The strategy is then to show that all instances of the above axioms have valid afeps and that any propositional form derivable by the rules of inference from propositional forms with valid afeps also has valid afeps, but that not every expression of PPL has valid afeps. For example, afeps of the form $\mathbf{A} \cdot \neg \mathbf{A}$ are not valid. The consistency of PPL is thus established, since if it were inconsistent then every proposition would be derivable, including those with invalid afeps.

An afep is valid if and only if it comes out true, *T*, for all the assignments of *T*s and *F*s made to proposition variables by the following specifications. For EPC expressions of the form $(\mathbf{p})\phi(\mathbf{p})$, $(\mathbf{p})\phi(\mathbf{p})$ is valid if and only if $[\phi(T) \cdot \phi(F)]$ is valid, where $[\phi(T) \cdot \phi(F)]$ is the conjunction consisting of ϕ with all occurrences of **p** replaced with *T* conjoined to ϕ with all occurrences of **p** replaced with *F*. Expressions $\phi(\mathbf{p})$ of EPC with **p** as a free variable are treated as if they are of the form $(\mathbf{p})\phi(\mathbf{p})$, and so $\phi(\mathbf{p})$ is valid if and only if $[\phi(T) \cdot \phi(F)]$ is valid. Afeps of the form $(\exists\mathbf{p})\phi(\mathbf{p})$ are valid if and only if $[\phi(T) \vee \phi(F)]$ is valid. Afeps that are propositional constants **c** or their negatives $\neg\mathbf{c}$ are treated as if they are of the forms $(\exists\mathbf{p})\mathbf{p}$ or $(\exists\mathbf{p})\neg\mathbf{p}$, respectively, for some proposition variable **p**. With repeated applications of these rules for substitution of *T*s and *F*s for variables in a given afep, what I call a *truth-value expansion* on each variable, the result will be an expression containing only *T*s and *F*s and logical connectives. Finally, an expression containing only truth-values and logical connectives is valid if and only if it results in the value *T* using the standard truth-table definitions of the connectives. The process of deriving a single truth-value from a truth-value expansion of an expression of EPC will be called a *truth-value resolution* of the expression.

The first step in proving that the axioms of PPL have valid afeps is to observe that afeps of any instance of the axiom schemata A1–A6 have, respectively, the following forms:

a1. $\mathbf{A}' \supset (\mathbf{B}' \supset \mathbf{A}')$.

a2. $(\mathbf{A}' \supset (\mathbf{B}' \supset \mathbf{C}')) \supset ((\mathbf{A}' \supset \mathbf{B}') \supset (\mathbf{A}' \supset \mathbf{C}'))$.

a3. $(\neg\mathbf{A}' \supset \neg\mathbf{B}') \supset (\mathbf{B}' \supset \mathbf{A}')$.

a4. (i) If **a** does not bind a predicate variable in **B**, then afeps of A4 have the form

$(\mathbf{A}' \supset \mathbf{B}') \supset (\mathbf{A}' \supset \mathbf{B}')$.

E.g., $(P_j)[A_3(x) \supset (\exists S_i)S_i(P_j)] \supset [A_3(x) \supset (P_j)(\exists S_i)S_i(P_j)]$ has an afep of

$[a \supset (\exists s)s] \supset [a \supset (\exists s)s]$.

If (**a**) does bind a predicate position variable in **B**, then **a** is an instance variable or an intension variable, hence:

(ii) If **a** is an instance variable, then afeps of A4 have the form

$(\mathbf{p})(\mathbf{A}' \supset \mathbf{B}') \supset (\mathbf{A}' \supset (\mathbf{p})\mathbf{B}')$,

where **p** is the proposition variable substituted for **a**. $\mathbf{A}'$ will not contain a free occurrence of **p**.

E.g., $(R_i)[(\exists T_i)T_i(x) \supset R_i(x,a)] \supset [(\exists T_i)T_i(x) \supset (R_i)R_i(x,a)]$. It has an afep of the form

$(r)[(\exists t)t \supset r] \supset [(\exists t)t \supset (r)r]$.

(iii) If **a** is an intension variable, then the afeps of A4 have the form

$(\mathbf{A}' \supset \mathbf{B}') \supset (\mathbf{A}' \supset \mathbf{B}')$.

E.g., $(R)[(\exists T_i)T_i(x) \supset (\exists R_i)R_i(x,a)] \supset [(\exists T_i)T_i(x) \supset (R)(\exists R_i)R_i(x,a)]$ has an afep of the form

$[(\exists t)t \supset (\exists r)r] \supset [(\exists t)t \supset (\exists r)r]$.

a5. (i) If (**a**) does not bind a predicate position variable in **A**, then afeps of A5 have the form

$\mathbf{A}' \supset \mathbf{A}'$.

E.g., $(S_i)R_j(S_i,y) \supset R_j(S_k,y)$ with afeps of the form r $\supset$ r. If (**a**) does bind a predicate position variable in **A**, then **a** is an instance variable or an intension variable, hence:

(ii) If **a** is (1) a full instance variable or (2) a limited instance variable such that (**a**) binds predicate position variables in **A**, then the afeps of A5(I) or A5(III) are either of the form

$(1')$ $(\mathbf{p})\mathbf{A}' \supset \mathbf{B}'$

or

$(2')$ $\langle$afep of $(\exists\mathbf{a})(\mathbf{a} = \mathbf{b})\rangle \supset [(\mathbf{p})\mathbf{A}' \supset \mathbf{B}']$,

where **p** is the proposition variable substituted for **a** in **A**, and $\mathbf{B}'$ differs from $\mathbf{A}'$ at most in containing a proposition variable **q** or constant **a** at all positions where $\mathbf{A}'$ contains **p**.

E.g., $(P_i)[B_j(P) \vee P_i(y)] \supset [B_j(P) \vee S_i(y)]$ has afeps of the form

$(p)[b \vee p] \supset [b \vee s]$.

Or, $(\exists A_i)(A_i = A_2) \supset [(A_i)(A_i(b) \vee \neg A_i(b)) \supset (A_2(b) \vee \neg A_2(b))]$ that has afeps of the form

$\langle$afep of $(\exists A_i)(A_i = A_2)\rangle \supset [(p)(p \vee \neg p) \supset ((\exists q)q \vee (\exists q)\neg q)]$.

Here A_2 has first been replaced with a propositional constant **a**, and then **a** has been replaced with an existentially quantified variable.

(iii) If **a** is an intension variable, the afeps of A5 have the form

$\mathbf{A}' \supset \mathbf{B}'$,

where $\mathbf{A}'$ and $\mathbf{B}'$ are identical in form with the possible exception of having different but corresponding proposition variables, but in this case the different proposition variables will be bound by the same type of quantifier. This follows from the fact that by the formation rules an intension variable only binds an instance variable through the mediation of an instance quantifier.

E.g., $(R)[(A_2(R) \vee R_j(a)) \cdot (\exists R_i)R_i(x)] \supset [(A_2(Q) \vee R_j(a)) \cdot (\exists Q_i)Q_i(x)]$ has afeps of the form

$[(a \vee s) \cdot (\exists r)r] \supset [(a \vee s) \cdot (\exists q)q]$.

In the example, R_j is not bound by the initial quantifier (R) (since there is no mediating instance quantifier) and so is not subject to the effects of instantiating R with Q.

a6. $(\exists \mathbf{p})\mathbf{p} \equiv (\mathbf{q})[\mathbf{q} \equiv \mathbf{q}]$,
where **p** and **q** are any proposition variables.

For most of the afep schemata in a1–a6 it is immediate that they are valid. The only non-obvious cases are those afep forms involved in a5(ii), a5(iii), and a6. First consider a6. The truth-value expansion of any afep of this form would be:

$$(T \vee F) \equiv [(T \equiv T) \cdot (F \equiv F)].$$

This resolves to $T \equiv T$, and then to T, and is thus valid. In regard to identity it is important to remember that $\mathbf{a} = \mathbf{b}$ is an abbreviation for $(\exists \mathrm{Id}^2_i)\mathrm{Id}^2_i(\mathbf{a},\mathbf{b})$ and hence that the afeps corresponding to $\mathbf{a} = \mathbf{b}$ are of the form $(\exists \mathbf{p})\mathbf{p}$ and so resolve to T. Correspondingly, as an abbreviation for $\neg(\exists \mathrm{Id}^2_i)\mathrm{Id}^2_i(\mathbf{a},\mathbf{b})$, $\mathbf{a} \neq \mathbf{b}$ has afeps of the form $\neg(\exists \mathbf{p})\mathbf{p}$ which resolves to $\neg T$ and then to F.

In regard to a5(ii)(1′), the truth-value expansion of $(\mathbf{p})(\mathbf{A}' \supset \mathbf{B}')$ will result in a finite set of conjuncts each of which having one of the two forms: '$(\mathbf{p})[\ ] \supset T$ ' or '$(\mathbf{p})[\ ] \supset F$ ', where T or F represent the results of a truth-value resolution of $\mathbf{B}'$, and '$(\mathbf{p})[\ ]$' represents a partial resolution of $\mathbf{A}'$ consisting in a logical matrix of truth-functional connectives among truth-values and **p**, all other proposition variables and constants having had truth-values substituted for them. Consider, for example, the afep

$$(\mathrm{p})(\mathrm{r} \cdot \mathrm{p}) \supset (\mathrm{r} \cdot \mathrm{q}),$$

whose resolution proceeds downward as

$$[(\mathrm{p})(T \cdot \mathrm{p}) \supset (T \cdot \mathrm{q})] \cdot [(\mathrm{p})(F \cdot \mathrm{p}) \supset (F \cdot \mathrm{q})],$$

$$[(\mathrm{p})(T \cdot \mathrm{p}) \supset (T \cdot T)] \cdot [(\mathrm{p})(F \cdot \mathrm{p}) \supset (F \cdot T)] \cdot [(\mathrm{p})(T \cdot \mathrm{p}) \supset (T \cdot F)] \cdot$$
$$[(\mathrm{p})(F \cdot \mathrm{p}) \supset (F \cdot F)],$$

$$[(\mathrm{p})(T \cdot \mathrm{p}) \supset T] \cdot [(\mathrm{p})(F \cdot \mathrm{p}) \supset F] \cdot [(\mathrm{p})(T \cdot \mathrm{p}) \supset F] \cdot [(\mathrm{p})(F \cdot \mathrm{p}) \supset F].$$

Now for all conjuncts of the form '$(\mathbf{p})[\ ] \supset T$ ', they resolve to T independently of the truth-value of '$(\mathbf{p})[\ ]$'. Conjuncts of the form '$(\mathbf{p})[\ ] \supset F$ ' also resolve to T due to the following. Since $\mathbf{A}'$ and $\mathbf{B}'$ are identical in form, with the exception that $\mathbf{B}'$ contains a proposition variable **q** or constant **c** where $\mathbf{A}'$ contains **p**, then any conjuncts in the resolution process of the form '$(\mathbf{p})[\ ] \supset F$ ' derive from a conjunct of the form '$(\mathbf{p})[\ ] \supset \{\ \}$', where [ ] and { } are identical truth-functionally, { } containing only truth-functional connectives among truth-values, and the two differing only in the

fact that { } will have a truth-value substituted for **q** or **c** in all the places where [ ] retains **p**. The case we are now considering is when { } is *F* on this truth-value assignment. Now, when (**p**)[ ] is resolved to the conjunction [.*T*.] · [.*F*.], the result of substituting first *T* and then *F* for **p** in [ ], one of these conjuncts will be identical to { } and thus have the value *F*. Thus, the antecedent of '(**p**)[ ] ⊃ *F* ' is *F* which renders the whole conditional *T*. Hence, in the resolution process each conjunct making up the resolution of forms like '(**p**)[ ] ⊃ *F* ' has truth-value *T*, as does each conjunct making up the resolution of forms like '(**p**)[ ] ⊃ *T* ', and thus the entire conjunction is *T*. In sum, a5(ii)(1′) has only valid afeps.

Based upon these same considerations, the afeps of a5(ii)(2′) that have the form ⟨afep of (∃**a**)(**a** = **b**)⟩ ⊃ [(**p**)**A**′ ⊃ **B**′] will also be valid, since the antecedent, ⟨afep of (∃**a**)(**a** = **b**)⟩ resolves to *T* and, as just shown, the consequent likewise resolves to *T*.

Finally, the afeps of a5(iii) are seen to be valid as follows. The afeps of a5(iii) have the form **A**′ ⊃ **B**′ where **A**′ and **B**′ are identical in form with the possible exception of containing different proposition variables, though in this case the variables will be quantified identically. Recall the given example:

$$(R)[(A_2(R) \vee R_j(a)) \cdot (\exists R_i)R_i(x)] \supset [(A_2(Q) \vee R_j(a)) \cdot (\exists Q_i)Q_i(x)]$$

which has afeps of the form

$$[(a \vee s) \cdot (\exists r)r] \supset [(a \vee s) \cdot (\exists q)q].$$

Because all such afeps are conditionals with antecedents and consequents that are identical, with the possible exception of the choice of variable of quantification, e.g., (∃r)r and (∃q)q, then upon a complete truth-value resolution these conditional afeps will resolve to a finite conjunction of conditionals where each of the latter conditionals has an antecedent and a consequent with the same truth-values, and so each afep resolves to *T*. Thus, all afeps of a5(ii)(2′) are valid.

Hence, all the creative axioms A1–A6 have valid afeps. Likewise, it is a simple matter to determine that the noncreative 'definitional' axiom schema D1, (∃**a**)**A** ≡ ¬(∃**a**)¬**A**, as well as the other unstated schemata for conjunction, disjunction, and the biconditional in terms of ⊃ and ¬, all have valid afeps.

The second step in proving the consistency of restricted PPL is demonstrating that the rules of inference of PPL preserve the property of having a valid afep—that wffs inferred from wffs with valid afeps have themselves valid afeps.

First, for inference rule R1,

if $\vdash \mathbf{A}$ and $\vdash \mathbf{A} \supset \mathbf{B}$, then $\vdash \mathbf{B}$,

if $\mathbf{A}'$ and $(\mathbf{A} \supset \mathbf{B})'$ have valid afeps then so must $\mathbf{B}'$. Assume that upon their truth-value resolutions both $\mathbf{A}'$ and $(\mathbf{A} \supset \mathbf{B})'$ have the value *T*. Because $(\mathbf{A} \supset \mathbf{B})'$ is identical in logical form to $\mathbf{A}' \supset \mathbf{B}'$, differing only possibly in the choice of proposition variables or constants, then $\mathbf{A}' \supset \mathbf{B}'$ will have the value *T* upon its truth-value resolution. Now, because $\mathbf{A}'$ and $\mathbf{A}' \supset \mathbf{B}'$ both resolve to *T*, this implies that $\mathbf{B}'$ must also resolve to *T*.

For inference rule R2,

if **a** is any variable and $\vdash \mathbf{A}$, then $\vdash (\mathbf{a})\mathbf{A}$,

when (**a**) does not bind a variable in predicate position in **A**, then by the rules for constructing afeps, $\mathbf{A}'$ and $((\mathbf{a})\mathbf{A})'$ are identical, and hence validity is preserved. For example, for **A** as $(P_i)P_i(x)$ and (**a**)**A** as $(x)(P_i)P_i(x)$, both have afeps of (p)p. If (**a**) does bind a predicate position variable in **A**, then **a** is either an instance variable or an intension variable:

(i) If **a** is an instance variable, e.g.,
for **A** as $P_i(..)$ and (**a**)**A** as $(P_i)P_i(..)$,
then the truth-value expansion of $\mathbf{A}'$ and $((\mathbf{a})\mathbf{A})'$ are identical in form; hence validity is preserved, since **a** in **A** is a free variable in predicate position which, by the rules for constructing afeps, requires the proposition variable substituted for **a** in **A** to be universally quantified. In the example, constructing $\mathbf{A}'$, we go from $P_i(..)$, to p to (p)p, which is identical in form to $((\mathbf{a})\mathbf{A})'$, the form of afeps of $(P_i)P_i(..)$.

(ii) If **a** is an intension variable, e.g.,
for **A** as $(P_i)P_i(..)$ and (**a**)**A** as $(P)(P_i)P_i(..)$,
then again the truth-value expansions of $\mathbf{A}'$ and $((\mathbf{a})\mathbf{A})'$ are identical in form; hence validity is preserved, since the construction of afeps requires the deletion of intension variables. In the example, $\mathbf{A}'$ has the form of (p)p, which is identical to that of $((\mathbf{a})\mathbf{A})'$, the form of afeps of $(P)(P_i)P_i(..)$.

In sum, it has now been shown that all instances of the axiom schemata A1–A6 and instances of the non-creative definitional schemata of PPL have valid afeps, and that the rules of inference guarantee that any wff inferred by these rules from the axioms has valid afeps. It follows that restricted PPL is consistent, since there are wffs of restricted PPL that are not theorems of it. That is, if the system were inconsistent, then any wff would be derivable in it. Consider all wffs of restricted PPL whose afeps are of the form $\mathbf{A}' \cdot \neg\mathbf{A}'$, e.g., $(\mathrm{P}_i)\mathrm{P}_i(x) \cdot \neg(\mathrm{P}_i)\mathrm{P}_i(x)$ whose afeps have the form $(\mathrm{p})\mathrm{p} \cdot \neg(\mathrm{p})\mathrm{p}$. All such afeps would be invalid, hence no wff whose afep is of the form $\mathbf{A}' \cdot \neg\mathbf{A}'$ can be a theorem of restricted PPL.

4. An Axiom of Comprehension and the Consistency of Full PPL

An axiom (schema) of comprehension (AC) of a formal language is the axiom that delineates the conditions for defining relations, including properties, in that object-language. It is through the form of this axiom that a predicate logic specifies how it diagnoses and solves the contradictions of the classic self-referential paradoxes. Also turning on this axiom is the extent to which the subsuming logic can provide a foundation for mathematics—specifically, arithmetic. The axiom to be added to restricted PPL will be a consistent modification of the instance analog of the standard formulation, i.e., a modification of

$$(1) \quad (\exists\mathbf{R}^n)(\mathbf{b}_1)(\mathbf{b}_2)\ldots(\mathbf{b}_m)[(\exists\mathbf{R}^n{}_i)\mathbf{R}^n{}_i(\mathbf{a}_1,\mathbf{a}_2,\ldots,\mathbf{a}_n) \equiv \phi].$$

Because of the device of extended binding, for (1) to be a closed wff of PPL, the list of variables $\mathbf{b}_1$, $\mathbf{b}_2$,…, $\mathbf{b}_n$ will contain the variables $\mathbf{a}_1$, $\mathbf{a}_2$,…, $\mathbf{a}_n$, as well as any full intension variables **P** that are not included in the list of **a**'s when one or more corresponding instances $\mathbf{P}_j$ are contained in the list of **a**'s.

To render an axiom of comprehension for PPL consistent what is crucial are modifications on the definiens ϕ, modifications that one would hope would be other than *ad hoc*, i.e., changes the content of which are positively motivated by accepted theory and not simply the negative goal of avoiding contradiction. It is in this regard that we have a powerful advantage in PPL. For through the refinement of instance predicates insights are possible that reveal the exact nature of *impredicative definitions*, together with the appropriateness and mechanism for applying the classical

condition on all definitions that they be *non idem per idem*, i.e., that entities not be defined in terms of themselves.

We will use the device of applying PPL to Russell's property paradox as a means of eliciting the needed refinements on (1). Russell's property, Rus, is defined in standard predicate logic as

(2) $(P)[Rus(P) \equiv \neg P(P)]$.

That is, a property P has Russell's property if and only if P is not a property of itself. Expression (2) is classified as 'impredicative' because the definiens (i.e., $\neg P(P)$) contains a universally quantified predicate position variable than ranges over the class containing the same entity being defined as the predicate in the definiendum (i.e., Rus(P)). If we instantiate P with Rus in (2), the result is the contradiction

$$Rus(Rus) \equiv \neg Rus(Rus).$$

Our intuitions are violated here because we think there ought to be a legitimate property of *Not-being-self-predicative*. Most properties do not apply to themselves; the property Red is not red, the property Triangular is not triangular, etc.

Now consider (2) translated into refined PPL:

(3) $(P)[(\exists Rus_i)Rus_i(P) \equiv (P_j)\neg P_j(P)]$.

This says that for every intension (universal) P, there exists at least one instance of Russell's property that applies to P if and only if no instance of P applies to P. As far as saving us from contradiction there is no advantage to (3) in itself—for, instantiating P with Rus there results

(4) $(\exists Rus_i)Rus_i(Rus) \equiv (Rus_j)\neg Rus_j(Rus)$.

Rather, the advantage (3) offers is a more detailed field where we can see how to apply the classic doctrine of *non idem per idem* definitions and at the same time save our intuition that there is a property of *Not-being-self-predicative*. For, it is clear that the universal instance quantifier binding predicates on the right side of (4) ranges over the same domain as the existential quantifier binding the defined predicate on the left side of (4). Because of this, the truth-value of the right side, the definiens, presupposes the truth-value of the left side, the definiendum, the exact opposite of the proper asymmetric dependence of definiendum upon definiens. The dependence here is one of truth-value, not of existence. The restriction that is necessary is to exempt the instance of Rus defined on the left side of (3) from falling within the domain over which the instance quantifier (P_j) of

the right side ranges. This can be accomplished using the device of a *bridging variable* Rus_h and rewriting (3) as

(5) $(\text{P})(\text{Rus}_h)\{(\exists\text{Rus}_i)[\text{Rus}_i(\text{P}) \cdot \text{Rus}_i = \text{Rus}_h] \equiv (\text{P}_j)[\text{P}_j \neq \text{Rus}_h \supset \neg\text{P}_j(\text{P})]\}$.

Upon instantiating P with Rus there results:

(6) $(\text{Rus}_h)\{(\exists\text{Rus}_i)[\text{Rus}_i(\text{Rus}) \cdot \text{Rus}_i = \text{Rus}_h] \equiv (\text{Rus}_j)[\text{Rus}_j \neq \text{Rus}_h \supset \neg\text{Rus}_j(\text{Rus})]\}$.

No self-contradiction results from (6), for assume first that Rus does not apply to itself, i.e.,

(a) $(\text{Rus}_i)\neg\text{Rus}_i(\text{Rus})$.

Applying inferences sanctioned by the transformation rules of PPL and which are analogs of inference rules of standard predicate logics,

(b) $(\text{Rus}_i)(\neg\text{Rus}_i(\text{Rus}) \vee \neg(\text{Rus}_i = \text{Rus}_i))$

(c) $\neg(\exists\text{Rus}_i)(\text{Rus}_i(\text{Rus}) \cdot (\text{Rus}_i = \text{Rus}_i))$

(d) $\neg(\text{Rus}_j)[\text{Rus}_j \neq \text{Rus}_h \supset \neg\text{Rus}_j(\text{Rus})]$
(by instantiation and negation on (6))

(e) $(\exists\text{Rus}_j)[\text{Rus}_j \neq \text{Rus}_i \cdot \text{Rus}_j(\text{Rus})]$

(f) $(\exists\text{Rus}_j)\text{Rus}_j(\text{Rus})$.

Line (f) contradicts assumption (a). Now assume the opposite; that Rus does apply to itself, i.e.,

(g) $(\exists\text{Rus}_i)\text{Rus}_i(\text{Rus})$.

Assume Rus_1 is some instance such that $\text{Rus}_1(\text{Rus})$, so that by (6)

(h) $(\text{Rus}_j)[\text{Rus}_j \neq \text{Rus}_1 \supset \neg\text{Rus}_j(\text{Rus})]$.

Line (h) does not contradict the assumption (g). Consequently, Russell's property as defined in (5) is not self-contradictory, but is, as just proven, predicable of itself.

The above analysis of Russell's Paradox reveals a needed modification on any axiom of comprehension adopted for PPL. However, there is a further modification needed which is a function of the refinement of instance predicates and the need to maintain the distinction between unrepeatable instances and their repeatable intensions. To observe the problem, consider two instances of Rus such that $\text{Rus}_1(\text{Tri})$ and $\text{Rus}_2(\text{Sqr})$. These

two propositions are true since the intension Triangular is not triangular nor is the intension Square itself square. Now instantiate (5) to yield

$$(\exists \mathrm{Rus}_i)[\mathrm{Rus}_i(\mathrm{Sqr}) \cdot \mathrm{Rus}_i = \mathrm{Rus}_1] \equiv (\mathrm{Sqr}_j)[\mathrm{Sqr}_j \neq \mathrm{Rus}_1 \supset \neg \mathrm{Sqr}_j(\mathrm{Sqr})].$$

Since the right bijunct is true, it follows that

$$(\exists \mathrm{Rus}_i)[\mathrm{Rus}_i(\mathrm{Sqr}) \cdot \mathrm{Rus}_i = \mathrm{Rus}_1],$$

and thus

$$\mathrm{Rus}_1(\mathrm{Sqr}).$$

By hypothesis $\mathrm{Rus}_1(\mathrm{Tri})$, yet here we also have $\mathrm{Rus}_1(\mathrm{Sqr})$. This means that instance Rus_1 in having two distinct subjects has the nature of a universal and not an unrepeatable instance as assumed by the motivating semantics.[11] The semantics of instance ontology requires that its formalization in PPL maintain the rigid distinction between nonrepeatable predicative instances and their repeatable nonpredicative intensions. However, a simple modification of (5) can solve this problem. Let the biconditional be split into two conditionals of the form:

$$(7)\ (\mathrm{P})(\mathrm{Rus}_h)\{[(\exists \mathrm{Rus}_i)[\mathrm{Rus}_i(\mathrm{P}) \cdot \mathrm{Rus}_i = \mathrm{Rus}_h] \supset (\mathrm{P}_j)[\mathrm{P}_j \neq \mathrm{Rus}_h \supset \neg \mathrm{P}_j(\mathrm{P})] \cdot [(\mathrm{P}_j)[\mathrm{P}_j \neq \mathrm{Rus}_h \supset \neg \mathrm{P}_j(\mathrm{P})] \supset (\exists \mathrm{Rus}_i)\mathrm{Rus}_i(\mathrm{P})]\}.$$

Instantiating P with Sqr in (7), together with the truth of

$$(\mathrm{Sqr}_j)[\mathrm{Sqr}_j \neq \mathrm{Rus}_1 \supset \neg \mathrm{Sqr}_j(\mathrm{Sqr})],$$

yields only

$$(\exists \mathrm{Rus}_i)\mathrm{Rus}_i(\mathrm{Sqr}),$$

but does not imply that Rus_i is specifically Rus_1.

This separation of the biconditional in (5) into the two conditionals of (7) is necessitated by the motivating semantics of attribute instances. Though this minor complication carries over into the formulation of the following axiom of comprehension, the reader should not be distracted by it, for it will have no material effect upon the subsequent analysis of impredicative reasoning and the paradoxes.

It should be noted that under the restriction of $\mathrm{P} \neq \mathrm{Rus}$ and thus $\mathrm{P}_j \neq \mathrm{Rus}_h$ for all P_j and Rus_h, (7) reduces to

$$(8)\quad (\exists \mathrm{Rus}_i)\mathrm{Rus}_i(\mathrm{P}) \equiv (\mathrm{P}_j)\neg \mathrm{P}_j(\mathrm{P}).$$

[11] The conclusion that $\mathrm{Rus}_1(\mathrm{Tri})$ and $\mathrm{Rus}_1(\mathrm{Sqr})$ violates the Principle of Subject Uniqueness, **SU**, given in note 5.

That is, the complications of (7) are due to its generality in including the 'impredicative' case where P = Rus, but with this case excluded, i.e., limited to the remaining predicative cases where P ≠ Rus, (7) reduces to the standard form of (8).

Before generalizing the above results it is worth observing that if (7) were added as an axiom to PPL then the enlarged system would be consistent. The afep of (7) is

$$[(\exists \mathrm{r})(\mathrm{r} \cdot T) \supset (\mathrm{p})(F \supset \neg \mathrm{p})] \cdot [(\mathrm{p})(F \supset \neg \mathrm{p}) \supset (\exists \mathrm{r})\mathrm{r}].$$

This afep resolves first to

$$\{[(T \cdot T) \vee (F \cdot T)] \supset [(F \supset \neg T) \cdot (F \supset \neg F)]\} \cdot \{[(F \supset \neg T) \cdot (F \supset \neg F)] \supset T\},$$

and this resolves to T.

Let us now generalize the successful results of our instance analysis of Russell's Property Paradox to formulate a consistent axiom of comprehension for PPL.

Axiom (Schema) of Comprehension (AC):

$$(\exists \mathbf{R}^n)(\mathbf{b}_1)(\mathbf{b}_2)\ldots(\mathbf{b}_m)(\mathbf{R}^n{}_h)\{[(\exists \mathbf{R}^n{}_i)[\mathbf{R}^n{}_i(\mathbf{a}_1,\mathbf{a}_2,\ldots,\mathbf{a}_n) \cdot \mathbf{R}^n{}_i = \mathbf{R}^n{}_h] \supset \phi] \cdot [\phi \supset (\exists \mathbf{R}^n{}_i)\mathbf{R}^n{}_i(\mathbf{a}_1,\mathbf{a}_2,\ldots,\mathbf{a}_n)]\},$$

where AC is a closed wff of PPL such that:

(i) neither $\mathbf{R}^n$ nor any of its instances occur in ϕ, except for the bridging variable $\mathbf{R}^n{}_h$ bound by the initial quantifier $(\mathbf{R}^n{}_h)$ and specified by (vi) below;

(ii) variables $\mathbf{a}_1$, $\mathbf{a}_2$, …, $\mathbf{a}_n$ exhaust all the free variables in ϕ and all the intension variables that are operator variables from one of $(\mathbf{b}_1)$, $(\mathbf{b}_2)$, …, $(\mathbf{b}_n)$ and that bind instance variables in ϕ by extended binding, e.g.,

$$(\mathrm{P})(\mathrm{R}_h)\{[(\exists \mathrm{R}_i)[\mathrm{R}_i(\mathrm{P}) \cdot \mathrm{R}_i = \mathrm{R}_h] \supset (\exists \mathrm{P}_j)\mathrm{P}_j(\mathrm{P}_j) \cdot \ldots;$$

(iii) there are no instance quantifiers among $(\mathbf{b}_1)(\mathbf{b}_2)\ldots(\mathbf{b}_n)$ that bind instance variables in predicate position in ϕ (this implies that all instance quantifiers binding instance variables in ϕ are themselves in ϕ);

(iv) all occurrences of negation in ϕ are within literals;

(v) all *existential* instance quantifiers (full or limited) of ϕ binding variables in predicate positions will be at the level of literals (i.e., the only wff within the scope of an existential quantifier, besides the quantifier itself, is a literal);
(vi) all *universal* instance quantifiers ($\mathbf{P}^m_j$) of ϕ (full or limited) binding variables in predicate position, with the following exceptions, will have occurrences of the form $(\mathbf{P}^m_j)(\mathbf{P}^m_j \neq \mathbf{R}^n_h \supset \psi)$, i.e., $(\mathbf{P}^m_j)[(\mathrm{Id}^2_i)\neg\mathrm{Id}^2_i(\mathbf{P}^m_j,\mathbf{R}^n_h) \supset \psi]$, where ψ is a literal. The exception to this requirement of the necessity of a restriction clause, $(\mathrm{Id}^2_i)\neg\mathrm{Id}^2_i(\mathbf{P}^m_j,\mathbf{R}^n_h) \supset$, between a universal quantifier ($\mathbf{P}^m_j$) and its literal ψ is that of the universal quantifier (Id^2_i) in the restriction clause itself—the quantifier (Id^2_i) in these clauses attaches directly to its literal with no such further intervening clauses.

In regard to (vi), a point worth observing is that the restriction clauses of the definiens, $\mathbf{P}^m_j \neq \mathbf{R}^n_h \supset$, do not require $m = n$, i.e., do not require that $\mathbf{P}^m$ and $\mathbf{R}^n$ have the same adicity (number of subjects). It is the case that under the intended interpretation intensions and instances of different adicities cannot be identical. But to enforce this within PPL would require an axiom (schema) to that effect, and the question becomes: Would this be a logical axiom? Intuitively, it would seem so, and thus we have another point at which logic and ontology appear indistinguishable. [See the discussion in Essay 7 of this volume.]

Generalizing upon what was observed for the specific case of Russell's property, it is important to note that if ϕ is such that its predicate position variables are quantified by only existential instance quantifiers, i.e., (vi) has no relevance to ϕ, then the instances of AC for such a ϕ will have the form

$$(\exists\mathbf{R}^n)(\mathbf{b}_1)(\mathbf{b}_2)\ldots(\mathbf{b}_m)[(\exists\mathbf{R}^n_i)\mathbf{R}^n_i(\mathbf{a}_1,\mathbf{a}_2,\ldots,\mathbf{a}_n) \equiv \phi].$$

This is the PPL analog of the axiom of comprehension as formulated in standard predicate logics. Hence, we see in a general way the limiting case status of unrefined classical predicate logic. PPL is to classic predicate logic as the entire electromagnetic spectrum is to the subrange of visible light.

It is not difficult to determine that all instances of AC have valid afeps, where these afeps have the general form of

$$[(\exists\mathbf{r})(\mathbf{r} \cdot T) \supset \phi'] \cdot [\phi' \supset (\exists\mathbf{r})\mathbf{r}],$$

ϕ' being the afep of ϕ. First, by the translation procedure for wffs of PPL into wffs of EPC, only *predicate* position instance constants or instance

variables and their quantifiers are retained, for which then distinct proposition constants or variables, respectively, are substituted. Under conditions (i)–(vi) of AC, an afep ϕ' will consist of only truth-functionally combined expressions of the possible forms $(\exists \mathbf{p})\mathbf{p}$, $(\exists \mathbf{p})\neg\mathbf{p}$, $(\mathbf{q})(F \supset \mathbf{q})$, $(\mathbf{q})(F \supset \neg\mathbf{q})$, $(\exists \mathbf{p})\mathbf{p}$, $(\exists \mathbf{p})\neg\mathbf{p}$, which correspond, respectively, to possible subformulas of ϕ of the forms $(\exists \mathbf{P}_j)\mathbf{P}_j(..)$, $(\exists \mathbf{P}_j)\neg\mathbf{P}_j(..)$, $(\mathbf{Q}_k)[\mathbf{Q}_k \neq \mathbf{R}_h \supset \mathbf{Q}_k(..)]$, $(\mathbf{Q}_k)[\mathbf{Q}_k \neq \mathbf{R}_h \supset \neg\mathbf{Q}_k(..)]$, $\mathbf{C}_n(..)$ or $\neg\mathbf{C}_n(..)$, where $\mathbf{C}_n$ is an instance constant (i.e., n is a natural number). Important is the fact that under (iv) negation occurs only at the level of literals in ϕ, and thus will occur in any corresponding afep only at the level of atomic proposition variables, not on compounds as in, for example, $\neg(\mathbf{p} \cdot \mathbf{q})$.

It follows that any instance of AC will have afeps with at most components of the schematic form

$$\text{(I)}\ \{(\exists \mathbf{r})[\mathbf{r} \cdot T] \supset [..(\exists \mathbf{p})\mathbf{p}..(\exists \mathbf{p})\neg\mathbf{p}..(\mathbf{q})(F \supset \mathbf{q})..(\mathbf{q})(F \supset \neg\mathbf{q})..]\} \cdot$$

$$\{[..(\exists \mathbf{p})\mathbf{p}..(\exists \mathbf{p})\neg\mathbf{p}..(\mathbf{q})(F \supset \mathbf{q})..(\mathbf{q})(F \supset \neg\mathbf{q})..] \supset T\},$$

where the expressions with the ellipses are abbreviations for an expression of EPC with one or more occurrences of one or more expressions of the indicated forms. The ellipses represent truth-functional combinations, if any, by the logical connective of $\cdot$, $\vee$, $\supset$, and $\equiv$, but with *no* occurrences of negation $\neg$. Because the truth-value resolution of the forms $(\exists \mathbf{p})\mathbf{p}$, $(\exists \mathbf{p})\neg\mathbf{p}$, $(\mathbf{q})(F \supset \mathbf{q})$, and $(\mathbf{q})(F \supset \neg\mathbf{q})$ are each T, (I) will resolves initially to

$$\{[(T \cdot T) \vee (F \cdot T)] \supset [..T..T..T..T..]\} \cdot \{[..T..T..T..T..] \supset T\}.$$

Now, because the ellipses represent groupings of truth-functional combination by means of only the binary logical connectives, $\cdot$, $\vee$, $\supset$, and $\equiv$, with *no* occurrences of negation $\neg$, the total expression resolves to T. It is the case, then, that all instances of AC have valid afeps. Thus, AC is a consistent extension of consistent restricted PPL, the enlarged system constituting full PPL.

We have then in full PPL a consistent extended predicate logic that is not subject to the self-referential paradoxes, and is so without an encumbering system of types or other crippling devices such as the elimination of global quantifiers or the rejection of classical negation.

5. PPL and a General Solution to the Self-Referential Paradoxes

PPL not only avoids the paradoxes that standard extended predicate logic is subject to, but, more significantly, PPL reveals details of the mechanism of impredicative definition and role of the principle of *non idem per idem* in distinguishing legitimate for illegitimate forms. When the definition of impredicative given previously is refined with instance predicates it becomes: A definition of an intension **R**, and through it possibly instances $\mathbf{R}_i$, is *impredicative in PPL* if and only if the definiens contains predicate position instance variables bound by means of extended binding by a universal intension quantifier. With this refinement it is now possible to distinguish legitimate from illegitimate modes by first recalling that section (vi) of AC precludes definiens from containing expressions of the forms

$(\mathbf{P}^m)[..(\mathbf{P}^m{}_j)\mathbf{P}^m{}_j(..)..]$, $(\mathbf{P}^m)[..(\mathbf{P}^m{}_j)\neg\mathbf{P}^m{}_j(..)..]$, $(\exists\mathbf{P}^m)[..(\mathbf{P}^m{}_j)\mathbf{P}^m{}_j(..)..]$,

$(\exists\mathbf{P}^m)[..(\mathbf{P}^m{}_j)\neg\mathbf{P}^m{}_j(..)..]$, $(\mathbf{C}^m{}_i)\mathbf{C}^m{}_i(..)$, $(\mathbf{C}^m{}_i)\neg\mathbf{C}^m{}_i(..)$, where $\mathbf{C}^m{}_i$ is a limited instance variable.

It is the first two forms, $(\mathbf{P}^m)[..(\mathbf{P}^m{}_j)\mathbf{P}^m{}_j(..)..]$ and $(\mathbf{P}^m)[..(\mathbf{P}^m{}_j)\neg\mathbf{P}^m{}_j(..)..]$, that are relevant here; the remaining four expressions are necessitated by the theoretical generality of the logic in allowing interpretations with domains of one intension. The distinction is then:

(A) **Illegitimate Impredicative Definitions**: A definiens that contains expressions of either of the first two forms above, $(\mathbf{P}^m)[..(\mathbf{P}^m{}_j)\mathbf{P}^m{}_j(..)..]$ or $(\mathbf{P}^m)[..(\mathbf{P}^m{}_j)\neg\mathbf{P}^m{}_j(..)..]$, where a universal intension quantifier $(\mathbf{P}^m)$ binds a predicate position instance variable $\mathbf{P}^m{}_j$ through the mediation of a *universal* instance quantifier $(\mathbf{P}^m{}_j)$, is *illegitimately impredicative*;

(B) **Legitimate Impredicative Definitions**:

(1) A definiens containing either of the forms $(\mathbf{P}^m)[..(\mathbf{P}^m{}_j)\mathbf{P}^m{}_j(..)..]$ or $(\mathbf{P}^m)[..(\mathbf{P}^m{}_j)\neg\mathbf{P}^m{}_j(..)..]$, but modified as specified by AC(vi) with the clause $\mathbf{P}^m{}_j \neq \mathbf{R}^n{}_h \supset$, enforcing the *non idem per idem* principle, is *legitimately impredicative*, or

(2) a definiens containing a universal intension quantifier $(\mathbf{P}^m)$ binding a predicate position instance variable $\mathbf{P}^m{}_j$ through the mediation of an existential instance quantifier $(\exists\mathbf{P}^m{}_j)$, is *legitimately impredicative.*

That all the classic self-referential paradoxes have forms characterized by (A), and are rendered non-contradictory when translated into (B-1) forms, will be demonstrated presently. An important and relevant example of a

(B-2) case is that of the definition of the property *Natural Number*, NN, from the foundations of mathematics. It is standardly given as

(9) $\quad \mathrm{NN(P)} \equiv (\mathrm{X})\{[\mathrm{X}(0) \cdot (\mathrm{Q})(\mathrm{X(Q)} \supset \mathrm{X}(S(\mathrm{Q})))] \supset \mathrm{X(P)}\},$

where $S(..)$ is the successor function. Definition (9) was considered impredicative, and thus a source of anxiety in light of the paradoxes (e.g., by Carnap), because the universal quantifier (X) of the definiens ranges over the domain containing the property NN of the definiendum. However, if (9) is translated into its PPL analog we have

(10) $\quad (\exists \mathrm{NN}_i)\mathrm{NN}_i(\mathrm{P}) \equiv (\mathrm{X})\{[(\exists \mathrm{X}_j)\mathrm{X}_j(0) \cdot (\mathrm{Q})((\exists \mathrm{X}_k)\mathrm{X}_k(\mathrm{Q}) \supset$

$(\exists \mathrm{X}_l)\mathrm{X}_l(S(\mathrm{Q})))] \supset (\exists \mathrm{X}_m)\mathrm{X}_m(\mathrm{P})\}.$

Here the universal intension quantifier (X) binds predicate position instance variables, X_j, X_k, X_l, through the mediation of existential quantifiers $(\exists \mathrm{X}_j)$, $(\exists \mathrm{X}_k)$, $(\exists \mathrm{X}_l)$, respectively. Hence, (10) is a (B-2) case and so is legitimately impredicative.

In regard to legitimate impredicative definitions of form (B-1), these specifications avoid the contradictions derived from (A) forms by the use of the restriction clause $\mathbf{P}^m_j \neq \mathbf{R}^n_h \supset$. Clauses in definiens ϕ of the form $(\mathbf{P}^m_j)(\mathbf{P}^m_j \neq \mathbf{R}^n_h \supset \psi)$, at a point of instantiation where $\mathbf{P}^m_j = \mathbf{R}^n_h$ (the point of otherwise *idem per idem* defining), have a truth-value of *T* independent of the truth-value of ψ, and thus the truth-value of $\mathbf{R}^n_i(..)$ of the definiendum ceases at this point to be a function of itself or its negation. The PPL/AC analysis substantiates and renders transparent Gödel's conjecture that concepts are defined everywhere except at certain 'singular points', analogous to dividing by zero.

That (A) captures the logical form of the classic impredicative paradoxes can be seen by means of the Russell–Priest characterization. Making a small modification of Russell's 1905 description, Graham Priest[12] has shown how Russell's Property, Russell's Relation, Burali–Forti, Mirimarnoff's, König's, Berry's, Richard's, the Liar, the Liar Chain, the Knower, and the Heterological paradoxes all involve reference to properties ϕ and ψ, and a function δ, where

(C1) $w = \{x|\phi(x)\}$ exists and $\psi(w)$, and

(C2) if $x \subseteq w$ such that $\psi(x)$, then (a) $\delta(x) \notin x$ and (b) $\delta(x) \in w$.

[12] Graham Priest, 'The Structure of the Paradoxes of Self-reference', *Mind* 103 (1994): 25-34.

For example, Russell's Paradox arises when $\phi(x)$ is the property $\neg\in(x,x)$, w is the set of all sets not elements of themselves, $\psi(x)$ is the identity property $(\lambda x(x = x))$, and δ is the identity function. Berry's Paradox of the least integer not nameable in fewer than nineteen syllables is derived from $\phi(x)$ being the property x-is-a-natural-number-definable-in-less-than-nineteen-words, w the set of all such numbers, $\psi(x)$ the property x-is-definable-in-less-than-fourteen-words, and $\delta(x)$ the least ordinal not in x. The reader is referred to Priest on the details of the remaining paradoxes. For our purposes here (C2) will be rewritten in the weaker form:

(C2′) if $x \subseteq w$ such that $\psi(x)$, then $\delta(x) \in w \equiv \delta(x) \notin x$.

That is, $\phi(x)$, $\psi(x)$, and $\delta(x)$ are so defined in each of the paradoxes such that $\delta(x) \in w \equiv \delta(x) \notin x$. Now, if we parse the consequent of (C2′) in PPL it becomes

(11) $(\exists\in_i)\in_i(\delta(x),w) \equiv (\in_k)\neg\in_k(\delta(x),x)$.

At the singular point where $x = w$, a contradiction results:

$(\exists\in_i)\in_i(\delta(w),w) \equiv (\in_k)\neg\in_k(\delta(w),w)$.

However, the contradiction is removed if (11) is required to conform to the strictures of AC, i.e.,

(12) $(x)(\in_h)\{[(\exists\in_i)[\in_i(\delta(x),w) \cdot \in_i = \in_h] \supset (\in_k)[\in_k \neq \in_h \supset$

$\neg\in_k(\delta(x),x)]] \cdot [(\in_k)[\in_k \neq \in_h \supset \neg\in_k(\delta(x),x)] \supset (\exists\in_i)\in_i(\delta(x),w)]\}$.

No contradiction is derivable from (12). In other words, when $\psi(x)$, and $\delta(x)$, and in particular the crucial $\phi(x)$, are defined in accordance with AC which enforces the legitimate impredicative specifications of (B) above, then paradoxes conforming to conditions (C1) and (C2′) are resolved.

How the requirements of PPL and AC alter individual paradoxes, and especially the crucial definition $\phi(x)$, is different and instructive in each case. I refer the reader to *Moderate Realism* for these details concerning Russell's Set Paradox, the Generalized Fitch–Curry Paradox, the Liar and Cyclic Liar Paradoxes, and Grim's Divine Liar Paradox. Let us here attend to a further classical paradox, the representative Grelling or 'Heterological' Paradox. A predicate expression of a specified language L is said to be heterological if and only if the property that the predicate expression designates in that language does not apply to the expression. For example, in English the word 'monosyllabic' is not itself monosyllabic and so is heterological, whereas the phrase 'is in English' is in English and therefore is not het-

erological. Remaining with English, where the predicate expression that designates the property Heterological is 'heterological', the question becomes: Is 'heterological' heterological or not? By its definition as standardly construed, if 'heterological' is heterological, then it is not, and if it is not then it is. To state the heterological paradox with formal accuracy we will restrict ourselves to languages L where the predicate expressions are univocal, i.e., each linguistic predicate of L names only one property. Hence, for variables x ranging over predicate expressions of L there will be a many-one *designation* function, d, that maps expressions to their corresponding properties, e.g., in English, d('heterological') = Heterological. Then, parsed in instance logic and using convenient abbreviations, the heterological property, Het, is defined for a univocal sublanguage of English as:

$$(13)\quad (\exists \mathrm{Het}_i)\mathrm{Het}_i(x) \equiv [d(x) = \mathrm{P} \cdot (\mathrm{P}_j)\neg \mathrm{P}_j(x)],$$

where x ranges over predicate expressions of L and P ranges over the corresponding properties. When P = Het, and letting 'Het' designate Het in the sub-language, a contradiction results:

$$(\exists \mathrm{Het}_i)\mathrm{Het}_i(\text{'Het'}) \equiv [d(\text{'Het'}) = \mathrm{P} \cdot (\mathrm{P}_j)\neg \mathrm{P}_j(\text{'Het'})],$$

which implies

$$(\exists \mathrm{Het}_i)\mathrm{Het}_i(\text{'Het'}) \equiv (\mathrm{Het}_j)\neg \mathrm{Het}_j(\text{'Het'})].$$

Now, when (13) is brought under the conditions of AC, it becomes the consistent

$$\begin{aligned}(14)\quad & (\mathrm{P})(\mathrm{Het}_h)\{[(\exists \mathrm{Het}_i)[\mathrm{Het}_i(x) \cdot \mathrm{Het}_i = \mathrm{Het}_h] \supset \\ & [d(x) = \mathrm{P} \cdot (\mathrm{P}_j)[\mathrm{P}_j \neq \mathrm{Het}_h \supset \neg \mathrm{P}_j(x)]]] \cdot \\ & [[d(x) = \mathrm{P} \cdot (\mathrm{P}_j)[\mathrm{P}_j \neq \mathrm{Het}_h \supset \neg \mathrm{P}_j(x)]] \supset \\ & (\exists \mathrm{Het}_i)\mathrm{Het}_i(x)]\}.\end{aligned}$$

To translate the paradoxical specifications of (13) into an extensional form conforming to the Russell–Priest set analysis requires, as I have been able to formulate it, two additional restrictions on language L, and an assumption concerning the existence of a property and its predicate designator in L. The restrictions to be assumed are that there be only one predicate expression in L for every property designated in L, and that no two properties designated in L have the same extension. Now, we let $\phi(x) = [d(x) = \mathrm{P} \cdot \neg \mathrm{P}(x)]$, so that $w = \{x | d(x) = \mathrm{P} \cdot \neg \mathrm{P}(x)\}$, i.e., w is the set of all predicate expressions x of L such that the corresponding property P does not apply to the designating expression x. The remaining analysis requires the assump-

tion of the existence of a property whose extension is w and that L have a designator for this property. Let $\psi(x) = x \subseteq w$ such that there is a y in L where $d(y) = \mathrm{P}$ and x is the extension of P. Finally, let $\delta(x) = y$, where y is a predicate expression of L such that $d(y) = \mathrm{P}$ and x is the extension of P. Then,

(15) $(\exists\in_i)\in_i(\delta(x),w) \equiv (\in_j)\neg\in_j(\delta(x),x)$,

which implies a contradiction when $x = w$. However, as we have seen, (15) becomes (12) when rendered legitimately impredicative under AC(vi). Though circuitous in comparison with the (13)/(14) analysis, the (15)/(12) analysis likewise shows that, when properly construed, the property of Heterological is not contradictory.

The above results are evidence of the analytic power of the instance logic PPL, and provide an indirect argument for the motivating ontology. PPL holds promise of both practical and theoretical utility. As an inference tool, the minor complications of distinct variables and quantifiers for both instance predicates and intensions is but a simple extension of standard logic, and is more than offset by the avoidance of an impractical theory of types, or the non-intuitive restrictions on logical operators or relations. Moreover, PPL holds out programmatic promise as a formalized language closer in nature to what must be the particularized state structure of a mechanical/electronic modeling within computer hardware. As a tool for theoretical analysis, we have seen PPL's power in distinguishing legitimate from illegitimate impredicative definitions and in distinguishing identity from indiscernibility. These results give a new direction to intensional logic and a powerful impetus to further research. Instance ontology and its concomitant logic are fertile fields of philosophical and logical exploration.

7

A Definition for Identity And the Implying of an Infinity of Logical Entities

1. Introduction

It has been a persistent problem in that area common to logic and ontology that we have been unable to precisely define and differentiate the logical relation of ('absolute') *identity* and the ontological relation of *indiscern-ibility*. I have proposed elsewhere a recursive definition for indiscernibility[1] and shall herein argue for a refined definition for identity, one that corrects an analysis I had previously given.[2] From the start I adopt the common thesis that, because of its universal applicability—what gives it its 'topic-neutrality'—and its fundamental role in inference, the identity relation is a logical constant (as is derivatively each of its instances on the adopted on-

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[1] I have argued for a recursive definition of indiscernibility in the form:

Entities x and y are indiscernible if and only if

a) $x = R^n_i$ and $y = R^n_j$, where R^n_i and R^n_j are instances of the same intension R^n.

b) $x = :R^n_i(a_1,a_2,...,a_n)$ and $y = :R^n_j(b_1,b_2,...,b_n)$ and a_k and b_k are indiscernible for $1 \leq k \leq n$.

c) x and y are complexes such that there is a one-to-one correspondence ϕ between their constituent facts where $\phi(:R^n_i(a_1,a_2,...,a_n)) = :R^n_j(b_1,b_2,...,b_n)$ and where $:R^n_i(a_1,a_2,...,a_n)$ and $:R^n_j(b_1,b_2,...,b_n)$ are indiscernible.

The argument is given in Mertz, 'Objects as Hierarchical Structures: A Comprehensive Ontology', in *Relations and Predicates, Philosophische Analyse*, Vol. 11, eds. H. Hochberg and K. Mulligan (Frankfurt: Ontos Verlag, 2004), pp. 113-48 [Essay 4 herein].

[2] My original analysis was given in *Moderate Realism and Its Logic* (New Haven: Yale University Press, 1996), pp. 277ff, with the results repeated uncritically elsewhere. The sought-after specification is for the general or absolute identity relation having any entity whatsoever as relata—Is-Identical-to—as distinct from identity conditions on a *kind*—Is-Identical-as-an-F-to. I propose the ontologically crucial and previously offered definition of Is-Identical-as-a-*Complex*-to remains accurate. See 'Objects as Hierarchical Structures'.

tology). On the following analysis, the proper specification of identity requires giving up the standard definitional form of a single biconditional proposition where one definiens gives both necessary and sufficient conditions for the obtaining of the definiendum. Rather, the result will be a 'contextual' definition consisting of four propositions each having one or more predicate occurrences of (instances of) the identity relation, and whose logical form is such as to assert and imply a set of *ontological* relationships had uniquely by (instances of) the identity relation. The four axioms jointly provide a definition by a kind of contextual parallax, i.e., by placement in a uniquely identifying network of relations ('definition by inter-relation'), what one would expect necessary of a primitive concept such as identity that is not susceptible of decomposition ('definition by elimination').

For both identity and indiscernibility, the ontological relationships that form the defining/delimiting network derive jointly from 1) the intuitive pre-critical concept or *intension* that is the relation being defined, and this as explicated in 2) *a prior and prerequisite ontology centered around a refined analysis of ontic predication*. The latter implies the semantics and syntax for a language/predicate-logic necessary as a medium for the proper definition of these concepts. Our intuitive concept of identity is *qualitative and numerical sameness*, and of indiscernibility is *qualitative sameness with the possibility of numerical distinctness*. The foundational ontology is a realist instance ontology that I have argued for variously, a theory that delineates a certain analysis of ontic predication outlined below.[3] It has a concomitant formal language and intensional logic that I have termed 'Particularized Predicate Logic', or 'PPL', and have axiomatized elsewhere, including giving a consistency proof for it.[4] PPL is a powerful medium of analysis and inference, being in effect higher-order logic with the distinctions of orders or types removed, it subsuming all valid inferences from these logics. The consistency of PPL is assured by an axiom of comprehension that precludes in a non-*ad-hoc* way improper self-referential or

[3] E.g., see Mertz, 'Combinatorial Predication and the Ontology of Unit Attributes', *The Modern Schoolman* LXXIX (2002): 163-97 [Essay 1 herein]; 'An Instance Ontology for Structures: Their Definition, Identity, and Indiscernibility', *Metaphysica: International Journal for Ontology & Metaphysics* 4 (2003): 127-64 [Essay 2 herein]; and 'Ontic Predicates as Substance', in *Substanz: Neue Überlegungen zu einer klassischen Kategorie des Seienden*, ed. K. Trettin (Frankfurt: Klostermann, 2005), pp. 245-71 [Essay 5 herein].

[4] The logic is detailed in my 'The Logic of Instance Ontology', *Journal of Philosophical Logic* 28 (1999): 81-111 [Essay 6 herein], this correcting some problems with the version given in *Moderate Realism*.

'impredicative' specifications. The power of this logic will be further exampled in its role as an 'organon' in developing the sought after definition, this in turn giving further warrant to the motivating instance ontology.

Our starting point for the following analysis is the observation, one made by George Boolos in defending second-order logic *as logic*[5], that the notions of *property* and *relation* are classifiable as logical notions as much as any others (e.g., the standard logical constants/operators, including *identity*, as well as the notions of *set* and *concept*). This is so because there are valid inference forms that involve properties and relations essentially but independently of their having any specific intensions/contents (inferences in this regard that are 'topic-neutral'). Now, what is unique to properties and relations in their roles as logical entities is their defining natures as 'predicates', and as such their asymmetric distinction from correlative 'subjects'—a predicate qualifies/characterizes its subject(s), but not conversely. Properly assayed, the logico-grammatical predicate/subject distinction is derivative of an ontological distinction essential to the existence of resultant facts or states of affairs, the latter the prerequisite 'truth-makers' (or 'false-makers') for corresponding propositions. Succinctly, a property or relation as an ontic predicate has the nature of an intension-delimited combinator on or among its subjects, the emergent whole being a fact. And, this nature of ontic predication, with the dual aspects of repeatable ('universal') intension and (what, profoundly, is an unrepeatable) unifying agency to, and possibly the ordering unity among, a subject *n*-tuple, is modeled in a predicate logic's formal semantics. The semantics in turn guides the logic's syntactical formation and transformation rules in such a way that expressions derived under them and involving predicate terms are logically true, i.e., true on all 'interpretations' under the semantics. That is and crucially, a predicate logic as determined through its semantics assigns truth-values via 'interpretations' according to prior *ontological assumptions* about the nature of ontic predication, this the asymmetric exemplification nexus between qualifying properties and relations and their thus qualified subjects. Hence contrary to a widely-held view, predicate logic, though topic-neutral, is in the way described *not* ontologically neutral.

Now, the aspects of ontic predication that are logically relevant are *generality*—the same entity characterizing multiple subjects ('one over many')—and the *asymmetric unification* of a predicate and its subjects. In

[5] George Boolos, 'On Second-Order Logic', *The Journal of Philosophy* 72 (1975): 509-527.

the standard extensional set-theoretic semantics for predicate logics both aspects are modeled by the same device of *set-inclusion*. Specifically, a predicate expression **P** is mapped under an interpretation to a set of domain entities *s* and the unity of the set in subsuming its elements represents the shared generality or 'universal' nature of **P** as a predicate, while conversely and with the same set, the asymmetric unity under the Element-of relation, of say, $a \in s$, represents the predicable unity expressed by **P**(**a**), where the interpretation maps subject term **a** to domain entity *a*. This models the dominant and simple 'Aristotelian' assay of predication insofar as under it intension universals themselves are taken to be predicates. Note that this extensional semantics exactly inverts the classic *inherence* assay of ontic predication where a property is held to be a constituent of its subject, what is, I argue, as much an error as is the extensional 'modeling' a mis-analogy of the nature of ontic predication. For, despite their popularity reinforced by their simplicity, the intension-as-predicate thesis is not the only, nor is it the core of every other possible analysis of ontic predication (this arguably recognized by Aristotle himself[6]), nor is the standard extensional and nominalist-friendly style of semantic modeling required of a predicate logic in order to preserve its topic-neutrality and inference power. In contrast and more complex, but proportionally strengthened in their explanatory/analytic consequences, is the realist instance assay of ontic predication and its motivated formal intensional semantics outlined below, the former informally and the latter formally guiding the syntax for concomitant logic PPL. In the ontology, and modeled in the formal semantics, we have the coordinated separation of generality, what is the nature of intension universals, from unifying predicable agency, what characterizes instances of intensions. The distinction implies syntactically a modification of the standard logical operators of universal and existential quantifiers according to the device of 'extended binding', correlative with a differentiation of variables and constants into three types or sorts. These 'new' quantifier operators, together with other standard logical operators, make for the expansive logic of PPL, what, without surrendering any of the standard forms, has new valid forms of inference and logical truths. The expressions of PPL are logical truths because they are true independent of the values assigned their non-logical terms, but truth-values are assigned to expressions involving the 'new' quantifiers by a semantics motivated by a prior and refined assay of ontic predication.

[6] See *Moderate Realism*, pp. 98ff.

Relatedly, an unanticipated but significant implication of the three identified and logically valid axioms needed jointly to provide sufficient conditions for the obtaining of identity is a denumerably infinite set of predicable instances of the identity relation, i.e., a denumerably infinite set of *logical objects*. These instances are 'objects' because each is an unrepeatable individual or particular; they are 'logical' because the sole qualitative content of each is the intension of the identity relation, a logical constant; and there are infinitely many of them because as predicates they are recursively generated starting with an instance having the given identity intension, Id^2, as relata, this predicable instance then being relata for a further predicable instance, and so on. Given, then, the correctness of the following, it answers a charge made against the Fregean/logicist program for 'reducing' arithmetic to logic, viz., that logic alone cannot imply the existence of even two, let alone the needed infinitely many, distinct objects, this at least, as Boolos observed, "on any understanding of logic now available to us."[7] A claim here is that a sub-system of PPL together with the identified logically valid axioms for identity make for such a *logic*, though it is not a 'logic alone' for no predicate logic is independent of prior ontological commitments as to the nature of ontic predication.

The foundation for realist instance ontology and for the system formalizing the logical relationships that emerge from the ontology consists of a set of principles asserting aspects of the theory of *ontic predication* definitive of the ontology. These principles include: First, the *Principle of Instance Predicates* (**IP**): Ontic predicates exist only as individuated and unrepeatable relation instances: R^n_i, R^n_j, R^n_k, …, S^m_i, S^m_j, S^m_k, …, T^o_i, T^o_j, T^o_k, …, i.e., as numerically distinct attributes whose nature as individuals/particulars is indicated by subscripts on their referring terms, and the number of subjects each instance jointly characterizes indicated by superscript, *n*, *m*, *o*, … on these terms. Second, the *Principle of Immanent Instance Realism* (**IR**): Among relation instances there are those that exactly resemble, R^n_i, R^n_j, R^n_k, …, and this is so because each is composed in part by what is numerically the same intension or universal R^n, instantiated intensions being abstractions from, but, contrary to the tradition, not themselves ontic predicates. This differentiation of unrepeatable predicates and their repeatable non-predicable intensions is the key to solving the paradox of Bradley's Regress and the purported contradictory nature of all attrib-

[7] George Boolos, 'Is Hume's Principle Analytic?', in *Language, Thought, and Logic*, ed. R. Heck (Oxford: Oxford University Press, 1997), p. 246.

utes. Further and important here, **IR** is the basis of the chief and powerful technical innovation of the corresponding logic, viz., instantiation of full *intension* variables, $\mathbf{P}^n$, carries with it the instantiation of *instance* variables of the same type, $\mathbf{P}^n_i$, that are also bound via 'extended binding' by the full intension variables. What this does is build explicitly into the syntax of the logic the semantic/ontological thesis that non-predicable intensions are identically shared across their predicable instances. In technical detail, instantiating a full intension quantifier variable, $\mathbf{P}^n$, in an expression of the form $(\mathbf{P}^n)\Psi$ or $(\exists\mathbf{P}^n)\Psi$, by substituting an *intension* variable or constant $\mathbf{A}^n$ for all free occurrences of intension variables $\mathbf{P}^n$ in Ψ that are bound by leading quantifier $(\mathbf{P}^n)$ or $(\exists\mathbf{P}^n)$, carries with it also the instantiation of all *instance* variables of this same type, $\mathbf{P}^n_i$, in Ψ that are also bound via extended binding by the leading quantifier, $(\mathbf{P}^n)$ or $(\exists\mathbf{P}^n)$, including the instantiation of instance operator variables for quantifiers, $(\mathbf{P}^n_i)$ or $(\exists\mathbf{P}^n_i)$, this instance instantiation being with the appropriate instance variables or constants $\mathbf{A}^n_i$ of the type $\mathbf{A}^n$. We shall review this syntactical innovation as relevantly applied below; the reader is referred to the full exposition of PPL for details and examples.[8]

Ontologically and as argued elsewhere,[9] each predicate instance, R^n_i, consists of an individuating combinatorial act delimited as to the subjects it joins and their ordering (if any) by a single simple repeatable intension R^n. In this way relation instances of each intension-kind are, relative to each other and abstracted from their relata sets, the ontically simplest and epistemically paradigmatic indiscernible entities. For example, from facts (indicated by the colon) that :Is-Prime$_1$(3) and Is-Prime$_2$(5), the predicable instances Is-Prime$_1$ and Is-Prime$_2$ are, by the theory and intuitively, indiscernible in the primary sense of qualitatively the same but numerically distinct, each being individuated and so distinct from the other, not *solo numero*, but by its unrepeatable predicable agency, the first as qualifying 3 and the second as qualifying 5. Indeed, that instances of the same intension are indiscernible is one of the additional ontological theses needed below to formally differentiate identity from indiscernibility. Closely related to ontic principles **IP** and **IR** is a third, the *Triple-Category Principle* (**TC**): All entities whatsoever fall into one of three disjoint ontological categories: 1) individuated ontic-predicates/intensioned-combinators, i.e., predicable instances R^n_i, S^m_j, T^o_k, …; 2) non-individuated non-combinatorial inten-

[8] Given in 'The Logic of Instance Ontology'.
[9] See references in Note 2.

sions/universals, R^n, S^m, T^o, …; and 3) individuated entities that are not themselves ontic predicates, e.g., facts or states of affairs, and complexes composed of them. Category three contains what would be classified as 'individuals' in standard predicate logics, e.g., all spatio-temporal particulars, but what I have argued must and can consistently be analyzed as hierarchies of structures of structures composed of networks of facts, and this all the way down to a base level composed of only mutually supporting atomic instances from category one.[10] Instance ontic predicates are thus the fundamental ontological category: they are both the ontic atoms and the agent combinators providing unity among the yet distinct, i.e., the unifying causes of facts, and so the fundamental elements of the hierarchically organized plural reality of 'individuals' of our observation and theory. Each element instance is composed in part of an intension which, for most, is repeated and can be known in abstraction from its instances and their particularizing combinatorial agencies, intensions having a fundamental epistemic status as the means of acquiring information about the qualitative natures of their instances' relata.

Principles **IP**, **IR**, and **TC** delimit, in part, a semantics, which in turn delimits a syntax (formation and transformation rules), for the refined formal language/logic of PPL. Definitionally, this logic *qua* logic has as its core those mechanisms that identify and generate valid inferences among sets of expressions of the language, and, relatedly, those that identify and generate expressions that are *logically true*. Under a natural extension of standard model-theoretic semantics, logical truths for PPL are those expressions that are true for all interpretations as defined on it, which here means true on every non-empty domain over which the non-logical terms of the expressions are assigned values, while holding constant both assumed ontological relationships between these domain elements—those asserted by **IP**, **IR**, and **TC**—and truth-value assignments made according to the standard logical operators ('logical constants'), $\vee$, $\cdot$, $\supset$, $\neg$, $\forall$, Id (or $=$), etc. In particular and an important insight generally is the fact that *a formal semantics for a predicate logic always models prior ontological theses concerning ontic predication*, viz., theses that specify both the types of entities ('sorts') making up the partitions (if any) of a domain, and the ontic properties and relations on or among entities of these sorts that follow from the theorized nexus of predication. This is the basis for the claim that logic is 'formal ontology', and the observation that it is at predication that ontology, logic,

[10] 'An Instance Ontology for Structures', and 'Objects as Hierarchical Structures'.

and grammar meet. That is, logic, here predicate logic, does not itself make ontological commitments, rather it presupposes certain inference-relevant ones essentially, e.g., for PPL the guiding theses are those implied by the intension-determined-agency theory of predication. In the simple standard extensional semantics used to model first-order predicate logic the domain is one-sorted (unpartitioned) and predication is modeled by the Element-of relation, sets being used to model predicates. The language of PPL as descriptive of the more complex ontology and theory of ontic predication of realist instance ontology, with its non-predicable intensions shared across non-sharable instance predicates, requires a corresponding complexity in the modeling semantics, a complexity rewarded with an increased analytic and logical power.

In outline, with background ontic theses **IP**, **IR**, and **TC**, a model-theoretic semantics of PPL will have a domain consisting of three non-empty disjoint classes: individuals, instance predicates, and abstracted intensions, and on an interpretation *I* of the non-logical constants, atomic expressions of the form $\mathbf{R}^{n}_{x}(\mathbf{a}_1,\mathbf{a}_2,...,\mathbf{a}_n)$ consisting of all constant terms have assigned to each term, according to its syntactic type, an entity from the respective class or classes, and these assignments according to the following and prior functions. For, in addition to the categories themselves, the ontological relationships relevant to predication specified among their members by **IP**, **IR**, and **TC**, require that every interpretation model them by combining two functions: 1) a function *In* (for Instantiates) that maps each intension either to a set of instance predicates, $In(R^n)\rightarrow\{R^n_1,R^n_2,R^n_3,\ldots\}$ or to a set containing the empty set, $In(R^n)\rightarrow\{\varnothing\}$, the latter to accommodate uninstantiated intensions (e.g., unicorn, phlogiston).[11] And 2), a further function *Ql* (for Qualifies) that maps each *n*-adic instance predicate constant, R^n_x, to a non-empty set of *n*-tuples, $Ql(R^n_x)\rightarrow\{<a_1,a_2,...,a_n>,<b_1,b_2,..., b_n>,...\}$, each *n*-tuple having elements from any of the three categories, and that maps the empty set to the empty set, $Ql(\varnothing)\rightarrow\varnothing$. We shall immediately below modify function *Ql* from a function whose range sets each have possibly one or more *n*-tuples as elements to sets each having exactly one *n*-tuple as an element, this as part of the intensional modeling of a needed refined semantics/logic. In either language, this combination of functions is needed in order to specify recursive rules for assigning truth-values to

[11] It would seem that this empty set clause of the formal semantics solves problems concerning existential import and uninstantiated intensions, and precludes the need for the syntactical device of '$(\exists\mathbf{a})(\mathbf{a} = \mathbf{b})$' of my axiom A5III for PPL. This awaits further study.

proper expressions involving 'extended binding' by full intension variables, e.g., '$(P^1)(P^1_j)[P^1_j(\alpha) \vee \neg P^1_j(\alpha)]$' and for expressions involving limited instance variables, e.g., '$(A^1_j)[A^1_j(A) \vee \neg A^1_j(A)]$'. We need not go into these technicalities herein, with the exception of extended binding that will be explained below. An interpretation *I* on expressions of PPL is then defined as: For $\mathbf{R^n}$ an intension constant, $I(\mathbf{R^n}) \in$ set of intensions; for instance constant $\mathbf{R^n_x}$, $I(\mathbf{R^n_x}) \in In(I(\mathbf{R^n}))$; and for $\boldsymbol{a}$ an individual constant, $I(\boldsymbol{a}) \in$ set of individuals. Then truth on *I* for atomic expressions consisting only of constants and of the form $\mathbf{R^n_x(a_1,a_2,...,a_n)}$ is defined as: $\mathbf{R^n_x(a_1, a_2,...,a_n)}$ is true if and only if $<I(\mathbf{a_1}),I(\mathbf{a_2}),...,I(\mathbf{a_n})> \in Ql(I(\mathbf{R^n_x}))$. Building on this, truth on *I* for expressions containing variables, quantifiers, and logical connectives are determined in standard ways.

Except for specifying function *Ql* for instances Id^2_i, Id^2_j, ..., of the identity intension, Id^2, this needed because identity is a primitive *logical* constant of PPL, we have the key components of what would be an intensional modeling by a model-theoretic semantics of the language/logic of PPL. So interpreted, PPL's two standard rules of inference, *Modus Ponens* (R1) and Universal Instantiation (R2), preserve truth, making for expressions derived from its axioms A1-A5 (what are instance analogs of standard axioms) that are all logically true on this formal semantics. Now, despite PPL's power for which I have given examples elsewhere[12], in order to carry out a successful analysis of identity and indiscernibility it is necessary, I argue, to refine the semantics of PPL to reflect in addition to **IP**, **IR**, and **TC**, a fourth ontological thesis I have termed the *Principle of Subject Uniqueness* (**SU**): No relation instance can have two or more distinct ordered sets of subjects. **SU** is likewise a fundamental principle of instance ontology, and one implied by the agent-thesis of ontic predication. For, it is the very *raison d'être* of attribute instances that they not have multiple exemplifications, i.e., not be universals, and it is their natures as agent combinators on or among their subject *n*-tuples that individuates and renders them unique—particular act-events—for each such *n*-tuple. I will refer to this new language/logic which is just like PPL except whose semantics now enforces theses **IP**, **IR**, **TC**, *and* **SU**, but without an axiom of comprehension or axioms defining identity, by the term 'IL^-', this for 'Instance Logic' so restricted. What are to be the identity axioms is a main focus be-

[12] For example, I have argued in 'The Logic of Instance Ontology' that PPL provides a general solution to the self-referential paradoxes in its power to distinguish between legitimate and illegitimate impredicative definitions, and to avoid the latter by non-*ad-hoc* means.

low. Now, the added ontological refinement of **SU** requires of the intensional semantics modeling IL⁻ that function *Ql* map each instance R^n_i, not to a set of *one or more* ordered *n*-tuples, but rather to a set with *exactly one* *n*-tuple as an element: $Ql(R^n_i) \rightarrow \{<a_1,a_2,...,a_n>\}$. Then for identity as a logical constant of IL⁻, the intension term 'Id^2' and instance terms 'Id^2_1', 'Id^2_2', 'Id^2_3', …, will be mapped via an interpretation *I* to domain entities Id^2, and Id^2_1, Id^2_2, Id^2_3, …, of the respective sorts, where $In(Id^2) \rightarrow \{Id^2_1, Id^2_2, Id^2_3, \ldots,\}$, and $Ql(Id^2_i) \rightarrow \{<a_i,a_i>\}$, there being an instance Id^2_i mapped by *Ql* to every set $\{<a_i,a_i>\}$, for every element a_i in the domain. The latter gives what can be modeled intensionally of our intuitive concept of identity, making the truth-conditions for expressions like '$Id^2_1(\alpha,\beta)$' the facts that both the referent for 'α' and for 'β' is *a* of the domain. What is yet required is a fuller and deductively consequential 'intensional' definition of identity *in* IL⁻, what will be principles that further delineate our intuitive concept and that will be certified accurate by their being logically true on the above formal semantics.

With this as a background, then, the argument below is that IL⁻ with the above formal semantics, conforming as it does to prior principles **IP**, **IR**, **TC**, and **SU** of realist instance ontology, is the minimally sufficient analytic/inference engine for solving the otherwise intractable problem of accurately defining identity, and in this distinguishing it from indiscernibility. Four axioms will be identified as necessary to specify the identity relation, one giving a necessary condition and the other three jointly providing a sufficient condition, all having instances of 'Id^2' as their only 'constants'. These axioms will be seen to be logically true on the formal semantics. The axioms together with one of the indiscernibility axioms previously identified, viz., that two predicate instances of the same type are indiscernible, provide a means of showing deductively within IL⁻ that identity is not extensionally, and so not intensionally, equivalent to indiscernibility. Finally, it will be demonstrated how the three sufficiency axioms for identity imply a denumerable infinity of instances of the identity relation.

2. The Identity Axioms in Instance-Ontology/IL⁻

In the more ontologically and logically refined second-order predicate logic the identity relation, Id^2, is characterized by a single biconditional, as such in a single proposition giving necessary and sufficient conditions for the obtaining of the relation, viz.,

(1) $(x)(y)[\mathrm{Id}^2(x,y) \equiv (\mathrm{P}^1)[\mathrm{P}^1(x) \equiv \mathrm{P}^1(y)]]$.

And, I had offered previously as a definition for identity what is the refined analog of (1) in realist-instance-ontology/IL$^-$, indicative of this context the identity relation being 'defined' through its instances, Id^2_i, viz.,

(**Id**) $(\alpha)(\beta)[(\exists \mathrm{Id}^2_i)\mathrm{Id}^2_i(\alpha,\beta) \equiv (\mathrm{P}^1)(\mathrm{P}^1_j)[\mathrm{P}^1_j(\alpha) \equiv \mathrm{P}^1_j(\beta)]]$.

Here 'α' and 'β' are global variables ranging over all entities of the ontology/logic's three semantic categories of individuals, intensions, and instances of intensions. In words, **Id** asserts that there exists an instance of the identity relation, Id^2_i, among subject entities α and β if and only if, for every monadic intension P^1 and every instance P^1_j of P^1, instance P^1_j is an ontic predicate of α if and only if it is an ontic predicate of β. Or alternately, if entities α and β share numerically the same instances of all of their properties, then α and β are joint subjects of a predicable instance, Id^2_i, of the binary identity relation, and conversely. Now, for parallel reasons to those much discussed in the literature of why (1) remains a too crude and undifferentiated characterization of identity, the same applies to **Id**. When I first proposed **Id** I assumed that these problems could be solved with simple adjustments to the range of intension variable 'P^1'. However as we shall see, a more fundamental analysis is required. As a preliminary I will use defective **Id** as a means to illustrate the chief technical advantage of IL$^-$/PPL, what will be integral to the correcting analysis.

Under the above and intended reading, the 'definiens' of **Id**—the proposition given by '$(\mathrm{P}^1)(\mathrm{P}^1_j)[\mathrm{P}^1_j(\alpha) \equiv \mathrm{P}^1_j(\beta)]$'—implies what is IL$^-$/PPL's simple but potent syntactical refinement over standard logic, viz., instantiation under 'extending binding', here by the leading *intension* quantifier, '(P^1)'. Under extended binding the leading universal quantifier, '(P^1)', binds every occurrence of an *instance* variable within its score, i.e., within '$(\mathrm{P}^1_j)[\mathrm{P}^1_j(\alpha) \equiv \mathrm{P}^1_j(\beta)]$', that has as a part of its designator a token of the same type as the designator of the universal quantifier variable. E.g., a quantifier with symbol '(P^1)' binds every term within its scope that has symbol 'P^1'as a typographical part, including as a proper part. With analogs of the standard instantiation rules on bound variables we would have the following expansion on the definiens of **Id**:

$$(P^1)(P^1_j)[P^1_j(\alpha) \equiv P^1_j(\beta)] \equiv \{(Red^1_j)[Red^1_j(\alpha) \equiv Red^1_j(\beta)] \cdot$$
$$(Round^1_k)[Round^1_k(\alpha) \equiv Round^1_k(\beta)] \cdot$$
$$(Nutritious^1_l)[Nutritious^1_l(\alpha) \equiv Nutritious^1_l(\beta)] \cdot \ldots\}$$

Here the leading intension quantifier of '$(P^1)(P^1_j)[P^1_j(\alpha) \equiv P^1_j(\beta)]$' has been instantiated successively by names for instances of the intensions Red^1, $Round^1$, $Nutritious^1$, etc., in each case in *both* quantifier and predicate positions. As will be shown presently, the increased notational and transformational complexity of such expressions in IL^- (and PPL) are directly proportional to—are the price for—their refined power of analysis and expression.

Returning to the inadequacy of **Id**, as pointed out in the literature, there are difficulties with (1) that would carry over analogously to **Id** as stating necessary and sufficient conditions for defining identity and differentiating it from indiscernibility. Logically decomposing (1), we have first the implied conditional:

(2) $(x)(y)[Id^2(x,y) \supset (P^1)[P^1(x) \equiv P^1(y)]]$,

or what is termed the 'The Indiscernibility of Identicals'. Thesis (2) has been challenged on the ground that it does not allow for change, i.e., allow that the same entity can persist through change of properties over time. A proposed solution has been to index property predication to instances of time, and then it is intuitive that if x and y refer to numerically the same entity then this referent has, at any given instance of time, the same time-indexed properties as itself, though the properties can be different at different times.[13] Symbolically, $(x)(y)[Id^2(x,y) \supset (P^1)(t)[(\text{at } t, P^1(x)) \equiv (\text{at } t, P^1(y))]]$, where t ranges over instances of time. We can accomplish the same thing, however, utilizing the instance analog of (2), where predicate instances, P^1_j, are properly construed as existing over time intervals. I.e., using (2) refined to:

(Id-1) $(\alpha)(\beta)[(\exists Id^2_i)Id^2_i(\alpha,\beta) \supset (P^1)(P^1_j)[P^1_j(\alpha) \equiv P^1_j(\beta)]]$.

[13] André Gallois, *Occasions of Identity: The Metaphysics of Persistence, Change, and Sameness* (Oxford: Oxford University Press, 1998), pp. 36ff.

where the semantics for the logic guarantee under **SU** that the single instance P^1_j has a single subject. The idea here is that if α is characterized by instance P^1_j, it is so over the interval of time for which P^1_j exists (a proper or improper sub-interval over which α exists), and any entity β that is numerically the same as α will be characterized by numerically the same P^1_j over this time interval. But, α need not have P^1_j over the entire interval of α's existence. Over proper sub-intervals of the interval over which α exists α can have different and contrary properties, and any entity β numerically the same as α will have the same set of property instances. This is what is asserted in **Id-1**. Now importantly and as the reader can easily verify, on the formal semantics given above axiom **Id-1** is logically true, i.e., true on all interpretations, given the identity instances, Id^2_1, Id^2_2, Id^2_3, …, over which the logical variables, 'Id^2_i', 'Id^2_j', …, are given assignments and how the *Ql* function is defined on them.

It is appropriate at this point to add the following axiom as further characterizing the identity relation, one whose necessity for distinguishing identity from indiscernibility will be seen below. It is the instance analog of one commonly given in first-order logic:

(**Id-2**) $(\alpha)(\exists Id^2_i)Id^2_i(\alpha,\alpha)$.

As the reader can also easily verify, axiom **Id-2** is logically true on the above formal semantics.

Returning again to the deficiencies of (1), there are the much discussed problems that arise with the implied conditional from right ('definiens')-to-left, what is called the 'Identity of Indiscernibles':

(3) $(x)(y)[(P^1)[P^1(x) \equiv P^1(y)] \supset Id^2(x,y)]$.

The problems with (3) come into focus with the observation, first, that within the unrestricted range of variable 'P^1' in the proposed sufficiency-condition stated by '$(P^1)[P^1(x) \equiv P^1(y)]$', there are so-called 'identity-properties', e.g., 'identical-to-α', or 'identical-to-α-and-green', and these properties render (3) necessarily true.[14] Some have argued that such 'properties' are not proper-

[14] For an overview of the issues see Bernard Katz, 'The Identity of Indiscernibles Revisited', *Philosophical Studies* 44 (1983): 37-44. For more detail see Gonzalo Rodriguez-

ties at all and so do not count against (3). However, it seems to me, as it has to others, that attempts to rule them such have the quality of being *ad hoc*. Yet, if these properties are legitimate, then, contrary to what is the tradition's intent in proposing the antecedent of (3), we do not need the mutual possession of every one of their properties as a sufficient condition for the identity of *x* and *y*, but only the mutual possession of one of these identity-properties. In this way these properties are said to 'trivialize' the principle, rendering it true 'on the cheap'. Now, these properties do indeed preempt the antecedent of (3) from providing a non-trivial condition for the obtaining of identity, but the conclusion to be drawn here is not that the problem is with identity-properties, but rather with the antecedent of (3) as a sufficiency condition for identity. To see this consider that there are equally corresponding 'indiscernibility-properties': 'indiscernible-from-α', 'indiscernible-from-α-and-green', etc. And, the names for any one of these properties substituted for 'P^1' in the antecedent of (3) shows the latter to state a sufficient condition for the obtaining of the indiscernibility relation between *x* and *y*. So, assuming no prior means of distinguishing between identity-properties and indiscernibility-properties, which presupposes a prior means of defining identity in such a way as to distinguish it from indiscernibility, then the antecedent of (3) can be sufficient for implying only the weaker relation of indiscernibility, not identity. In short, the antecedent of (3) is in itself insufficient as a sufficiency condition for identity.

In an attempt to save something close to (3) as a sufficiency condition for identity it has been suggested that the range of 'P^1' be restricted to only 'shared properties', i.e., properties that are exemplified by more than one entity. This means we assert only the 'strong form' of the Identity of Indiscernibles:

(4) $(x)(y)[(P^1)[\text{Is-a-shared-property}(P^1) \cdot P^1(x) \equiv P^1(y)] \supset \text{Id}^2(x,y)]$.

But, here again we cannot define 'shared property' without using identity properly defined and distinguished from indiscernibility, and thus we must presuppose a prior established sufficiency condition. But setting this failure aside, it is logically possible that there could be two *numerically distinct* entities *x* and *y* that nevertheless share all their shareable properties, *x* and *y*

Pereyra, 'How not to Trivialize the Identity of Indiscernibles', in *Universals, Concepts, and Qualities: New Essays on Meaning of Predicates and Abstract Entities*, eds. P. F. Strawson and A. Chakrabarti (Burlington: Ashgate, 2005).

then being qualitatively indiscernible but not identical. This is the basis for the plausibility of the proffered classic counter-examples to (4): Max Black's exactly resembling spheres in a symmetric universe, and A. J. Ayer's sequential series of repeating indiscernible universes, as well as by the apparent fact from current micro-physics that distinct quantum entities can be absolutely without qualitative difference.[15] These objections to (4) turn on the assumption that the same properties P^1 can simultaneously characterize numerically distinct subjects, i.e., be universals, and in this way $(P^1)[P^1(x) \equiv P^1(y)]$ is sufficient for specifying only indiscernibility. This was the motivation for Peter Geach's theory of 'relative identity', Geach arguing in effect that if (1) or a logical equivalent is the best we can do to specify strict identity then the notion is incoherent, and we are left instead with an indefinite number of indiscernibility relations, one for each monadic property P^1, viz., x is the same P^1 as y.[16]

In sum, neither the antecedent of (3) nor its modification in (4) prescribe a sufficient condition for the obtaining of the identity relation. In response I shall now argue that a proper specification of identity, one that combines both necessary and sufficient conditions for the obtaining of identity and what distinguishes it from indiscernibility, consists of the joint assertions of four axioms—**Id-1** and **Id-2** above, and two further theses. Of the remaining two, the first and guiding thesis parallels assumed ontological principle **SU**. What I shall term the *Axiom (Schema) of Subject Uniqueness on Identity* (**SU-Id**), it asserts that: For any instances R^n_j and R^n_k of any n-adic intension R^n, if the instances are identical and are predicable, respectively, of subject n-tuples $<\alpha_1,\alpha_2,\ldots,\alpha_n>$ and $<\beta_1,\beta_2,\ldots,\beta_n>$, then the respective elements of the n-tuples are identical. Fully formalized in IL^-, the axiom schema is given as:

$$\begin{aligned}(\mathbf{SU\text{-}Id})\ \ &(\alpha_1)\ldots(\alpha_n)(\beta_1)\ldots(\beta_n)(R^n)(R^n_j)(R^n_k)\{[(\exists Id^2_i)Id^2_i(R^n_j,R^n_k)\cdot\\ &R^n_j(\alpha_1,\alpha_2,\ldots,\alpha_n)\cdot R^n_k(\beta_1,\beta_2,\ldots,\beta_n)]\supset\\ &[(\exists Id^2_j)Id^2_j(\alpha_1,\beta_1)\cdot(\exists Id^2_k)Id^2_k(\alpha_2,\beta_2)\cdot\ldots\cdot(\exists Id^2_l)Id^2_l(\alpha_n,\beta_n)]\}\end{aligned}$$

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[15] Max Black, 'The Identity of Indiscernibles', *Mind* 61 (1952): 153-64; A. J. Ayer, 'The Identity of Indiscernibles', *Philosophical Essays* (London: Macmillan, 1954), p. 29; Steven French, 'Identity and Individuality in Quantum Theory', *The Stanford Encyclopedia of Philosophy (Spring 2006 Edition)*, ed. E. Zalta, URL = <http://plato.stanford.edu/archives/spr2006/entries/qt-idind/>.

[16] Peter Geach, 'Identity', *Review of Metaphysics* 21 (1967): 3-12.

where again subscripted ‘α’s and ‘β’s are global variables ranging over all categories of entities in IL⁻. It is clear here as well that on the formal semantics given above, as it models the prior ontic principle **SU** and the Identity relation as a logical constant, that **SU-Id** is logically true.

It is to be noted that from **SU-Id** and **Id-2** one can derive:

(5) $(\alpha)(\beta)(P^1)(P^1_j)\{[P^1_j(\alpha) \cdot P^1_j(\beta)] \supset (\exists Id^2_i)Id^2_i(\alpha,\beta)\}$,

where **SU-Id** in IL⁻, in conformity with **SU** of the background ontology, makes any instance P^1_j a ‘non-shared’ ontic predicate. (5) can be used to derive the symmetry and transitivity properties of the identity relation.

Now, for the remaining theses necessary to establish and from which to demonstrate the distinction between identity and indiscernibility, consider the instantiation of **SU-Id** so as to produce the following [the use of the superscript ‘2’ will be temporarily suspended for ease of reading]:

(6) $[(\exists Id_i)Id_i(Id_1,Id_2) \cdot Id_1(Id,Id) \cdot Id_2(Id_1,Id_1)] \supset$
$[(\exists Id_j)Id_j(Id,Id_1) \cdot (\exists Id_k)Id_k(Id,Id_2)]$.

In the antecedent of (6), two of the three conjuncts, viz., $Id_1(Id,Id)$ and $Id_2(Id_1,Id_1)$, are derivable in IL⁻ from **Id-2**, and are true on the formal semantics. Moreover, on this formal semantics, mirroring here a fact of the motivating realist instance ontology, each conjunct of the consequent of (6), $(\exists Id_j)Id_j(Id,Id_1) \cdot (\exists Id_k)Id_k(Id,Id_2)$, is false. Yet, there is no axiom in IL⁻ from which we can demonstrate in this system the falsity of these conjuncts. What is needed then can be termed the *Axiom (Schema) of Non-Identity of Intensions and Instances*:

(**NII**) $(Q^n)(P^m)(P^m_i)\neg(\exists Id^2_j)Id^2_j(Q^n,P^m_i)$.

In words, no intension Q^n is identical to any instance P^m_i of any intension P^m. It is an instantiation instance of **NII**, viz., where ‘Q^n’ and ‘P^m’ are instantiated with ‘Id^2’, that contradicts the consequent of (6):

(**NII**-1) $(Id^2_i)\neg(\exists Id^2_j)Id^2_j(Id^2,Id^2_i)$.

Like the previous identity axioms, **NII** (and **NII**-1) is logically true—true on all interpretations in the given semantics of IL⁻ due to the value assignments given the identity terms 'Id^2', 'Id^2_i', and 'Id^2_j', and the fact that the assignments given to 'Id^2' and 'Id^2_i' are from the disjoint sub-domains of intensions and instances. Now, given these results and the fact that (6) is true, the leading conjunct of the antecedent, $(\exists Id_i)Id_i(Id_1,Id_2)$, must be false, this being an intended implication of **SU-Id**—that instances, e.g., $Id_1$and Id_2, that do not have the same relata *n*-tuples are not numerically the same.

The above analysis also points immediately to the final axiom that will assure the demonstrability in IL⁻ of the distinction between identity and indiscernibility. The truth of (6) requires that $(\exists Id_i)Id_i(Id_1,Id_2)$ be false—that instances Id_1 and Id_2 not be numerically the same, but as stated previously it is a fundamental principle of the motivating instance ontology that predicate instances of the same type, e.g., $Id_1$and Id_2, are indiscernible. The latter will be added to IL⁻ under the name of *Axiom (Schema) of Instance Indiscernibility*:

(II) $(P^m)(P^m_i)\,(P^m_j)(\exists Ind^2_k)Ind^2_k(P^n_i,P^m_j)$,

i.e., any instances, P^m_i and P^m_j, of the same intension P^m are indiscernible. Clearly, **II** is not a logical truth—not on the formal semantics for IL⁻, nor intuitively does the indiscernibility relation have the universality/'subject-indifference' of a logical operator.

Now, the claim is that at this point we have with axioms **Id-1**, **Id-2**, **SU-Id**, and **NII**, and the expressive/inference power of IL⁻, what is formally and explicitly sufficient to specify/define our intuitive notion of identity, and what when conjoined with **II** which is primary to specifying the notion of indiscernibility, is sufficient to formally differentiate identity from indiscernibility. The identity relation is reflexive by **Id-2**, and is demonstrably symmetric and transitive under **Id-1** and the contrapositive of (5), (5) itself implied by **SU-Id** and **Id-2**. The formal differentiation of identity from indiscernibility, specifically, that there are entities that are indiscernible but not identical, is derivable in the logic of IL⁻ from premises **Id-2**, **SU-Id**, **II**, and **NII** (specifically **NII**-1) [again the superscript '^{2}' will be left implicit]:

Proof A: First, by **Id-2**, $(\exists Id_i)Id_i(Id,Id)$. In words, the intension universal Id is identical to itself by some predicable instance of itself, Id_i. Let such an (what is the only) instance be Id_1, i.e., $Id_1(Id,Id)$. By the same reasoning applied now to Id_1, $(\exists Id_j)Id_j(Id_1,Id_1)$. In words, the individual instance of the identity intension, Id_1, is identical to itself by some (what can now be proven is a further) predicable instance of the identity relation. Let such an instance be Id_2, so $Id_2(Id_1,Id_1)$. It is easily demonstrated that Id_1 and Id_2 are indiscernible but not identical.

For, first by **II**,

$$(\exists Ind_k)Ind_k(Id_1,Id_2),$$

i.e., Id_1 and Id_2 are indiscernible. Assume further that Id_1 and Id_2 are identical, i.e.,

$$(\exists Id_k)Id_k(Id_1,Id_2).$$

This leads immediately to a contradiction:

For given $(\exists Id_k)Id_k(Id_1,Id_2)$, $Id_1(Id,Id)$, and $Id_2(Id_1,Id_1)$, then by **SU-Id**,

$$(\exists Id_j)Id_j(Id,Id_1).$$

Yet, by **NII**-1,

$$\neg(\exists Id_j)Id_j(Id,Id_1).$$

Hence, Id_1 and Id_2 are indiscernible but not identical.

3. An Implied Infinity of Instances of the Identity Relation

We are now in a position to observe the significant result that the sufficiency principles for identity, **Id-2**, **SU-Id**, and **NII**, guarantee a denumerably infinite set of instances of the identity relation—what then is *an infinity of logical entities*. As seen below, a certain infinite set of identity instances, Id^2_i, Id^2_j, Id^2_k, …, is generated recursively via **Id-2**, with their mutual distinctness—non-identity—guaranteed by **SU-Id** and **NII**. The proof that each instance Id^2_n is not identical to any previous instance in the recursion, and therefore that the non-terminating sequence consists of only mutually non-identical entities, i.e., is a denumerable infinity, requires the use of the inference rule of Strong Mathematical Induction, what is not a rule of inference of IL^-. Hence, distinct from the fact that axioms **Id-2**, **SU-Id**, and **NII**, jointly imply an infinity of identity instances, the *proof* that they do so is carried out in the metalanguage to *T*. If we were to add an axiom of infinity to *T*, AI, asserting the existence of an infinity of identity instances, AI would be logically true in that its truth would follow from the

logical truths **Id-2**, **SU-Id**, and **NII**, on the semantics of IL⁻. The significance of this result is: Given that IL⁻ with its semantics constitutes a logic and that, consequently, **Id-2**, **SU-Id**, and **NII**, are logical truths, we have a demonstration of the crucial but thus far elusive requirement of the logicist program for the reduction/modeling of arithmetic to/in logic, viz., that a logic is committed to infinitely many logical entities. I note that this argument for the recursive generation of infinitely many *a priori* logical entities resembles in form (but contrasts in its successful rigor with) an epistemic argument once given by Richard Dedekind—we have a thought, we can then have a thought of the thought, we can then have a thought of the thought of the thought, etc.[17] If, as I would argue, identity is a concept of 'second intension' with a conceptual existence only, in this way like exemplification, propositions, truth, etc., then Dedekind's argument would have a closer relationship with the argument presented here than one might initially think.

Let Theory *T* consist of IL⁻ together with **Id-2**, **SU-Id** and **NII**. We can demonstrate using Strong Mathematical Induction that *T* guarantees an infinity of distinct instances of the identity intension Id (as before I shall here suppress the superscripts '^{2}' for ease of reading). The symbol '=' will designate identity in the meta-language in which the following proof is constructed. The strategy is to show that a certain set of instances of the identity relation recursively generated from non-terminating iterated applications of axiom **Id-2** is denumerably infinite by demonstrating using **SU-Id** and **NII** that no two instances in the sequence are identical, i.e., are relata for an instance of the identity relation in *T*. This means that in the three-sorted domain of IL⁻/*T*, the axioms **Id-2**, **SU-Id**, and **NII** require the sub-domain containing predicate instances to be countable infinite in cardinality.

Proof B: The identity relation intension, Id, is a primitive constant of IL⁻/*T*. As in **Proof A**, by **Id-2** it is the case in *T* that $(\exists Id_i)Id_i(Id,Id)$, i.e., that the identity intension Id is identical to itself in the precise sense that there is at least one (indeed exactly one on the modeling semantics under **SU**) instance of intension Id relating it to itself. Let Id_1 be such an instance.

[17] Richard Dedekind, *Essays on the Theory of Numbers*, trans. W. Beham (New York: Dover, 1963), p. 64. Dedekind notes that a similar argument was given by Bolzano. According to Dedekind, the theory of numbers is a part of logic taken as "laws of thought", and in this "numbers are free creations of the human mind." Ibid., p.32.

Then, applying Axiom **Id-2** in turn to instance Id_1, we have $(\exists Id_j)Id_j(Id_1, Id_1)$. Let Id_2 be such an instance. This recursive process carries on indefinitely effecting the set of facts for: $Id_1(Id,Id)$, $Id_2(Id_1,Id_1)$, $Id_3(Id_2,Id_2)$,…, $Id_n(Id_{n-1},Id_{n-1})$, $Id_{n+1}(Id_n,Id_n)$,… What is to be shown is that in theory *T*, for any two instances Id_i and Id_j in the sequence: Id_1, Id_2, Id_3,…, Id_n, Id_{n+1},…, it is the case that $\neg(\exists Id_j)Id_j(Id_i,Id_j)$.

That is, we shall prove by Strong Mathematical Induction that, for every *n*,

$\neg(\exists Id_i)Id_i(Id_n,Id_y)$, for every $y < n$.

I.e., Theory *T* provides a denumerable infinite set of distinct instances of the identity intension, Id.

First, let $n = 1$, and $Id_0 = Id$. We must prove that $\neg(\exists Id_i)Id_i(Id_1,Id)$. But this follows immediately from **NII**, and in particular **NII**-1.

Now consider the induction step:

Assume $\neg(\exists Id_i)Id_i(Id_n,Id_y)$, for every $y < n$. We must prove that:

$\neg(\exists Id_i)Id_i(Id_{n+1},Id_y)$,, for every $y < n+1$.

Proof by Contradiction:

Assume $(\exists Id_i)Id_i(Id_{n+1},Id_y)$, for some $y < n+1$.

Now it is the case that $Id_{n+1}(Id_n,Id_n)$ and $Id_y(Id_{y-1},Id_{y-1})$. And so by **SU-Id**,

$(\exists Id_j)Id_j(Id_n,Id_{y-1})$.

But, since $y-1 < n$, $(\exists Id_j)Id_j(Id_n,Id_{y-1})$ violates the initial assumption of the induction step. Hence the induction step is established. Hence, there is a denumerable infinity of identity instances, no two of which are identical.

It is the case, then, that **Id-2**, **SU-Id,** and **NII** in IL^- imply the existence of a denumerably infinite set of individuals/'objects'—relations instances—that are logical entities. As noted above, this fact is available ('proved') *in T* by adding an axiom of infinity, AI, to *T* asserting the existence of infinitely many instances of the identity relation, what then is a logical truth. Further, if T + AI were expanded to include a proper axiom of comprehension, AC[18], then it may be possible to succeed with a Fregean/logicist program for Peano Arithmetic in, say, a realist instance modification of a construction once outlined by Charles Chihara[19]. Here natural numbers would be cardinal properties of properties, in contrast to their

[18] As, e.g., given in my 'The Logic of Instance Ontology'.

[19] Charles Chihara, *Constructibility and Mathematical Existence* (Oxford: Clarendon Press, 1990), pp. 80ff.

standard modeling as sets of sets. Given the ontology/logic of intensions and their instances, this assay would have the explanatory power of accounting for both the substantival uses of number terms where non-predicable repeatable intensions are subjects, e.g., 'Five is a prime number' or symbolically ($\exists$Is-Prime$_i$)Is-Prime$_i$(Five), and adjectival uses of number terms where unrepeatable instances are predicates, e.g., 'There are five regular solids' or symbolically ($\exists$Is-Five$_i$)Is-Five$_i$(Regular-Solid).[20] Assuming a logicist program can be carried out in T + AI + AC, with this turning on the guarantee from T of infinitely many logical entities, it would remain a remarkable result despite the fact that T implements the logical implications of a prior ontological analysis of predication—predication being a juncture of ontology, logic, and grammar/syntax. For though ontology-out only if ontology-in, it is astonishing that the ontology-out of an infinity of instances of identity follows only from logically valid axioms of logic IL$^-$ and identity, axioms that singly give no hint of, nor were formulated with the intent of providing, this implication.

[20] See my *Moderate Realism*, pp. 265ff.

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