PARTS

A STUDY IN ONTOLOGY

'*Parts* could easily be the standard book on mereology for the next twenty or thirty years.'

Timothy Williamson, *Grazer Philosophische Studien*

Peter Simons

Perhaps we ought to have maintained that a syllable is not the letters, but rather a single entity framed out of them, distinct from the letters, and having its own peculiar form.

Plato, *Theaetetus*, 203E

PARTS

A Study in Ontology

*

PETER SIMONS

CLARENDON PRESS · OXFORD

This book has been printed digitally and produced in a standard specification in order to ensure its continuing availability

OXFORD
UNIVERSITY PRESS

Great Clarendon Street, Oxford OX2 6DP

Oxford University Press is a department of the University of Oxford.
It furthers the University's objective of excellence in research, scholarship,
and education by publishing worldwide in

Oxford New York

Auckland Bangkok Buenos Aires Cape Town Chennai
Dar es Salaam Delhi Hong Kong Istanbul Karachi Kolkata
Kuala Lumpur Madrid Melbourne Mexico City Mumbai Nairobi
São Paulo Shanghai Taipei Tokyo Toronto

Published in the United States
by Oxford University Press Inc., New York

Reprinted 2003

ISBN 0-19-924146-5

Preface

I have many debts. First to chance. There cannot have been many other philosophy departments housing experts on both Leśniewski and Husserl at the time when I studied in Manchester. My interest in mereology dates from an occasion on which I rashly flashed some axioms for a part-relation about the departmental office (they were about the strength of a meet semilattice), and Czesław Lejewski, who happened to see them, told me they were *far* too weak (I now think they were too strong, but it took a long time to get to this). At the time I set off to read Leonard and Goodman, and then the secondary literature on Leśniewski. At about the same time, Wolfe Mays was holding reading seminars on Husserl's *Logical Investigations*. I think we skipped over the third; though its use of mereological concepts caught my attention, it seemed odd and unfamiliar, and it was mainly Kevin Mulligan's insistence on its importance which brought myself and Barry Smith back to it over and again. To Kevin and Barry I owe most stimulation and encouragement: their passionate interest in ontology, their wide knowledge, and their unconcern for philosophical fashion have helped me in countless ways. Several parts of this book go back to our discussions in meetings of the Seminar for Austro-German Philosophy and publications arising therefrom. Many other people helped in various ways, in discussion, correspondence, sending me their work and in other ways. Apart from the four mentioned, I should like in particular to thank David Bell, Hans Burkhardt, Roderick Chisholm, the late Ted Dawson, Rick Doepke, Kit Fine, Ivor Grattan-Guinness, Guido Küng, Wolfgang Künne, Karel Lambert, Audoënus Le Blanc, the late Peter Nidditch, Gilbert Null, Herman Philipse, Michael Resnik, Richard Sharvy, Peter Van Inwagen, Paul Weingartner, and David Wiggins. Other occasional debts are mentioned in the footnotes. The reader for Oxford University Press gave very helpful constructive criticisms of an earlier version.

To my colleagues, first in Bolton, then in Salzburg, many thanks for providing support and encouragement. My thanks in particular go to Edgar Morscher for his considerable help. Annemarie Brenner was the writing power behind two earlier versions, and kept the caffeine flowing.

The book is a slightly modified version of my Salzburg *Habilitationsschrift*, and I should like to express my gratitude to the members of the *Habilitationskommission*, and in particular to the referees, for their work.

For financial assistance in enabling me to travel to colloquia I am grateful to Bolton Metropolitan Borough Council, the Austrian Federal Ministry of Science and Research (Bundesministerium für Wissenschaft und Forschung) and to the Österreichische Forschungsgemeinschaft.

My greatest debt is to my wife Susan, and I dedicate this book to her in love and gratitude.

SALZBURG,
JULY 1986

Contents

List of Symbols

Page numbers indicate first or explanatory occurrences. Not listed are letters used only locally as schematic variables, Greek letters marking argument places in predicates, and single letters used locally as proper names (for example, in figures). The few symbols used with more than one meaning are listed as often as they have different meanings.

≪	10	ex	24, 65
<	11	Bpr	24, 66
∘	12	Bsm	24, 66
⌊	13	Cm	24, 66
·	13	Sm	24, 65
+	14	Pr	24, 66
−	14	Cpl	24, 66
σ	15	atm	24, 66
⌜ ⌝	15, 47	ǂ	27
π	15	τ	32
∪	15	$+'$	32
−	16	σ'	35
At	16	$\circ_*$	36
⋀	19, 63	$s, t, u, s_1, \ldots$	47
⋁	20, 63	$x, y, z, x_1, \ldots$	47
∃	20	$a, b, c, a_1, \ldots$	47
∀	20	$F, G, H, F_1, \ldots$	47
⊢	20	$P, Q, R, P_1, \ldots$	47
Σ	21	$\alpha, \beta, \gamma, \alpha_1, \ldots$	47
Π	21	$A, B, C, A_1, \ldots$	47
ı	21	~	47
●	21	∧	47
▬	22, 63	∨	47
≈	22, 63	⊃	47
≃	22, 62	≡	47
E	22, 62	.(dot)	47
!	22, 62	℩	47
E!	22, 62	$A[y//x]$	49
⊂	22, 63	Su	53
⊏	22, 63	Pr	53
Δ	22, 63	$\{x, y\}$	53
ppt	24, 65	$\{x : Fx\}$	53
pt	24	$A[s/t]$	57
ov	24, 65	$A[x_1 \ldots x_n/t_1 \ldots t_n]$	57

Introduction

This book has two major aims. The first is to give a connected account of the various kinds of mereology, or formal theory of part, whole, and related concepts, which exist, widely scattered, in the literature. This aim is fulfilled mostly in Part I. The second and more important aim is to expose the philosophical defects of most of this tradition, and to suggest why, where, and how it should be put right.

The standardly accepted formal theory of part–whole is *classical extensional mereology*, which is known in two logical guises, the Calculus of Individuals of Leonard and Goodman, and the Mereology of Leśniewski. Despite the discrepancies between the underlying logics of these two approaches, there is a precise sense in which both say the same things about parts and wholes. Classical extensional mereology (CEM) is subject to two major criticisms. The first is that, either out of conviction or for reasons of algebraic neatness, it asserts the existence of certain individuals, mereological *sums*, for whose existence in general we have no evidence outside the theory itself. The second and more fundamental criticism is that the theory is not applicable to most of the objects around us, and is accordingly of little use as a formal reconstruction of the concepts of part and whole which we actually employ.

There are two reasons for this. The first is that the underlying logic of CEM does not have the resources to deal with temporal and modal notions in connection with mereology, such as *temporary part*, *temporal part*, *essential part*, or *essential permanent part*. This is not an internal criticism of CEM, since one could envisage suitably extending it to cope with temporal and modal concepts. However there is an internal reason why CEM is not suitable for such extension, and this concerns *mereological extensionality*. This is the thesis that objects with the same parts are identical (by analogy with the extensionality of sets, whereby sets with the same members are identical). If mereological extensionality is accepted, then two prima facie facts need to be accounted for. The first is that certain things, like human beings, have different parts at different times: they are mereologically *variable* or in flux. An object with different parts at different times cannot be identical with the sum of its parts at any time, for then it would be different from itself. The second problem is that some objects (again, like human beings) might have had some parts other than those they in fact have, and yet still have been the same objects. In other words, they are not modally *rigid* in their parts. If we accept mereological extensionality in a modally strengthened form, to the effect that objects with the same parts *must* be identical, then no object could have had parts other than those it actually has, a thesis called *mereological essentialism*, and associated with Chisholm.

In the face of these two problems, a number of strategies may be adopted to preserve extensionality, some of them extreme. One may try revising the logic of identity, or denying that objects have undetached parts. The modal problem may be ignored by refusing to take modality seriously. If it is taken seriously, then it seems that mereological essentialism is the best option. One then faces the problem of explaining why it appears that ordinary objects are not modally mereologically rigid. Chisholm accounts for appearances by construing such objects as logical constructions out of objects for which mereological essentialism indeed holds.

Since it is more difficult to ignore time than modality, the problem of flux has been faced more often. The analogous solution to mereological essentialism is to deny that objects do vary their parts: they are mereologically *constant*. Chisholm holds that objects properly so called are indeed mereologically constant, and the appearance of variability among objects is to be explained by construing these, once again, as logical constructions. I argue that the price Chisholm pays is in each case too high to warrant upholding mereological extensionality.

A second and more popular solution to the flux problem is to propose replacing the things (continuants) of our usual ontology by processes, which have temporal parts. I argue that the difficulties involved in such a revision have been greatly underestimated, and that in any case the move fails to save mereological extensionality because such four-dimensional objects fall prey to the modal argument.

There are nevertheless places where extensional mereology is appropriate, in particular among events and among non-singular objects, that is, classes and masses: in the last two cases the full classical theory can apply. It emerges that 'part', like other formal concepts, is not univocal, but has analogous meanings according to whether we talk of individuals, classes, or masses.

It remains to be considered how mereology looks when extensionality is rejected. Part II considers the mereology of continuants, which may be in flux. The rejection of extensionality has as a consequence that more than one object may have exactly the same parts at the same time, and hence occupy the same position. Another somewhat controversial thesis defended is that an object may exist intermittently in special circumstances. These views are applied together to give a novel solution of the Ship of Theseus problem.

Consideration of the conditions under which distinct things may be in the same place at the same time leads to a discussion of the nature of composition, constitution, and matter in their mereological ramifications. With the rejection of extensionality, it becomes possible to distinguish different concepts of proper-or-improper-part which enrich our conceptual palette and allow disputes to be resolved as turning on equivocation.

Modal mereology has received almost no attention because the logical opinions of mereologists and modal logicians have usually been fundamentally opposed. Part III brings modality and mereology together as they are found in the work of Husserl at the beginning of the century and later in that of Chisholm. Mereological essentialism is rejected as a general doctrine, though again there are regions in which it is appropriate, essentially those where extensionality applies. For most continuants some parts are essential and others are not.

Husserl used modal mereology as a tool in developing various concepts of ontological dependence of objects on other objects, a study which quickly leads into some of the central topics of ontology, concerning substance and conditional and unconditional existence. The modal approach developed here is used to re-examine traditional problems in this light, and to reassess Husserl's achievement in this field.

The arguments have so far turned mainly on extensionality and what happens if it is rejected. In the last chapter I return to mereological sums, and ask what it is they seem to lack that other paradigmatic objects have. The leading idea here is that of a family of objects which is maximally connected under some relation; an object composed of such a family is *integrated* under the relation; such integrity—provided the relation involved is not merely formal—is what arbitrary sums and incomplete fragments lack. Among the kinds of relation constitutive of such integrity we consider forms of ontological and functional dependence, and give an account, based on work by Grelling and Oppenheim, of the characteristic structure or Gestalt of such integral wholes.

A word should be said about the use of symbolic formulae and formal systems. Since most of the existing work on mereology uses such means of expression, their use is unavoidable if we are to survey such theories and compare their relative strengths. As for the rest of the book, since the aim is to discover the most suitable mereological concepts and the formal principles governing them, though it is in principle possible to do without symbols, in practice we need them for their brevity and clarity, and to enable us to test the consequences of formulae using the established means of formal logic. Nevertheless, I want to stress that this book is about ontology, not logic. The level of logical sophistication required to understand the text is not high: a journeyman's acquaintance with first-order predicate logic with identity will suffice. Even the modal sections require no more, provided one is prepared to indulge the fiction of possible worlds. The unfamiliar language of Leśniewski is introduced in such a way that a reader without foreknowledge may follow subsequent discussion; but in general I stay closer to predicate-logical means of expression. I have tried to make the notation as unfussy as possible; a survey of the notation used generally, in particular the conventions which allow parentheses to be almost entirely dispensed with, can be found in §2.2. Apart from this, the symbolically dense Chapter 2 is not a prerequisite for understanding the main argument. It is a survey exhibiting the riches of the predominant extensional tradition, and gathers material which is otherwise widely scattered.

Finally, I should mention that the book does not deal with two areas where mereology overlaps with other important philosophical issues. The first is vagueness. Apart from some remarks in Part II, where mention of vagueness is unavoidable, this subject remains within the haven of bivalence. The second issue is whether there can be a mereology of abstract objects (if there are any). The discussion is confined to the concrete, except for a few remarks in §4.10. The reason for avoiding these two areas is that to discuss them with the required throughness would involve bringing in a good deal of material which is not in itself mereological, and which is also not relevant to the main argument, which is that extensional mereology is inadequate even for its primary intended sphere of application—concrete individuals.

Part I

Extensional Part–Whole Theory

Seek simplicity and distrust it.
Whitehead, *The Concept of Nature*, p. 163

The theories of part and whole discussed in Part I are distinguished being the only ones for which fully worked out formalizations exist. In fact, the formal systems have tended to be discussed for their own sake, and the question of the extent to which they capture intuitive notions of part and whole has less often been put. The principal theme of this book is that despite their formal elegance, these theories leave much to be desired as general theories of part and whole. Nevertheless, it is appropriate from a logical point of view to regard them as the classical formal theories. They set the standard for future formal developments and form the background to our later criticisms, amendments, and extensions. For this reason, the concepts and notations which are here introduced will be employed and modified throughout the rest of the book. However, as there is no connected text containing even a modestly thorough survey of extant modern literature on part and whole, it seems appropriate in this part to draw together information which is otherwise widely scattered. This information on formal systems is of interest to the scholar of mereology, but not so important for the main argument of this book, and is therefore collected together in Chapter 2, while the material essential for the further arguments of the book is found in the Chapter 1.

Terminological Note 1: 'mereology'

The term 'mereology' will be used generally for any formal theory of part–whole and associated concepts. So our study in the first part is of extensional mereologies. We shall speak similarly of mereological concepts, theories, principles, etc. Leśniewski called his own particular formal theory 'mereology', literally 'science or theory of parts', from the Greek μερος, 'part'. The term is often therefore applied to Leśniewski's theories alone, or to theories formulated in his fashion.

In this usage it excludes formal theories formulated in a different logical style, in particular the calculi of individuals of Leonard, Goodman, and others. However the word 'mereology' and its cognates are so convenient that it is worth using them more broadly. To distinguish Leśniewski's formal mereology from others, we write 'Mereology' with a capital letter, or else mention Leśniewski by name. This convention is similar to one employed to distinguish the general philosophical theory of being or what there is, ontology, from Leśniewski's formal logic of being, Ontology. So a Mereology is a particular kind of (extensional) mereology.

Terminological Note 2: 'extensional'

The kinds of system considered in Part I are similar either to those of Leśniewski, whose book of 1916 is the first rigorous, albeit still informal, treatment, or to the calculi of individuals of Leonard and Goodman. The latter come in two sorts, those employing some set theory and those employing none. Calculi of individuals using sets were also developed independently of Leonard and Goodman by Tarski. The logical language used in calculi of individuals is that of first-order predicate logic with identity, sometimes supplemented with a modicum of set theory, whereas that used in Mereology is the language of Leśniewski's Ontology. The two languages differ fairly considerably, as do the metalogical opinions usually associated with them. Nevertheless, as far as the principles of part and whole go, there is enough community of substance and attitude between them to warrant regarding them as variants on a single theme. This affinity is more precisely delineated in Chapter 2. Systems similar either to a calculus of individuals or to Mereology are here called *extensional* mereologies (part–whole theories). The term is chosen for its general aptness rather than its immediate lucidity, for the following reasons:

(1) Extensional mereologies bear an intended algebraic similarity to the theory of classes, which is the inner haven of extensionality. The reasons for the similarity are both historical and systematic, and will be examined in Chapter 3.
(2) The theories are by design free from modal or other intensional operators.
(3) The logicians most closely involved in their development have tended to evince a thoroughgoing extensionalist attitude to logical matters.

Finally, we may note a pun on 'extend': the most appropriate interpretation for extensional mereologies, one which renders all their axioms plausible, is one in which the singular terms of the theory stand for spatial, temporal, or spatio-temporal extents, or for extended matter.

Note on Notation

A standard notation is employed throughout Part I and, as far as possible, throughout the book. It is introduced gradually with the concepts and principles in Chapter 1, but a summary of conventions employed may be found at the beginning of Chapter 2. This should be consulted in case of doubt as to what any particular sign in this part signifies. Tables comparing our usage with others in Ontology and mereology may be found in §2.11, and there is a list of first and elucidated occurrences of symbols at the beginning of the book.

1. Concepts and Principles of Extensional Mereology

Although Leśniewski's Mereology was the first extensional part–whole theory to be rigorously developed, it is expressed in the formal language of the general logic Leśniewski called 'Ontology', and which is sometimes called his calculus of names.[1] This language is, by comparison with those of predicate logic and set theory, still relatively little known, so we first introduce mereological concepts and principles in more familiar guise. Leśniewski's notation can then be introduced without any great difficulty in its intended meaning. As far as the narrower issues of extensional mereology are concerned, there is no important difference between Leśniewski and other writers. They all in a clarifiable sense say the same. In particular, the questionable aspects of extensional mereology remain questionable no matter which language is used to express them. However, there are a number of reasons for not confining discussion and presentation to the more familiar logical basis of calculi of individuals. For one thing, much of the most detailed discussion of extensional mereologies takes place in the Leśniewskian tradition, so the interested reader should acquire a smattering of the different logical dialect. For another thing, one of the most promising interpretations for Mereology is one which is internal to Leśniewski's Ontology, knowledge of which must therefore be presupposed. Last, but not least, Leśniewski's logic and the language it is expressed in possess a number of philosophically attractive features, and for that reason alone wider acquaintance with them is desirable. A more detailed account of the relationship between predicate logic and Ontology may be found in the following chapter.

1.1 Basic Concepts

1.1.1 Proper Part

The most basic and most intuitive mereological concept, which gives the subject its name, is that of the relation of part to whole. Examples of this relation are so legion, and it is so basic to our conceptual

[1] Cf. SŁUPECKI 1955.

scheme, that it seems almost superfluous to offer examples, but those in Table 1.1 are illustrative of many others:

TABLE 1.1. *Some Typical Wholes and Parts*

Whole	Part
a (certain) man	his head
a (certain) tree	its trunk
a house	its roof
a mountain	its summit
a battle	its opening shot
an insect's life	its larval stage
a novel	its first chapter

To express that the object x is (a) part of the object y we write

$$x \ll y$$

and say 'x is (a) part of y' or, more often, 'x is a *proper* part of y'. The reason for the second (preferred) reading will become apparent presently.

Here 'x' and 'y' are (singular) variables, the entities they ambiguously signify being *individuals*. Leonard and Goodman called their mereologies 'the calculus of individuals' to emphasize that the terms of the part-relation are all of the lowest logical type, as distinct from entities of higher type such as classes, functions, or attributes. The choice of name suggests also that the principles developed apply to *all* individuals, irrespective of kind, in a way in which other concepts and principles, such as those of genetics, do not. We adopt this usage of 'individual'.[2]

The most obvious formal properties of the part-relation are its transitivity and asymmetry, from which follow its irreflexivity. That is, the following are true for any individuals:

Nothing is a (proper) part of itself.

If one thing is a proper part of another, then the second is not a proper part of the first.

If one thing is a proper part of another, and the second is a proper part of a third, then the first is a proper part of the third.

[2] Where we use 'individual', Leśniewski used equivalents of 'object' (*przedmiot*, *Gegenstand*). We use 'object' more widely, to cover also masses and classes (Ch. 4). Cf. RUSSELL 1903: 55 n.; Russell used (confusingly) 'term' for individuals.

These principles are partly constitutive of the meaning of 'part', which means that anyone who seriously disagrees with them has failed to understand the word. The relation of proper part to whole is thus a strict partial ordering, but not all strict partial orderings are part-relations; these principles will need to be supplemented.

1.1.2 Part (Proper or Improper)

Just as in arithmetic, the relation *less-than* ($<$) is often less convenient to use than the relation *less-than-or-equal* ($\leq$); so in the theory of partial orderings in general, and mereology in particular, it is often algebraically more convenient to take as the primitive of a formal theory the corresponding non-strict relation *part-of-or-equal*. It is no minor question what this 'or-equal' means. An essential characteristic of *extensional* mereology is that it mean 'or-*identical*'. If x and y are identical, then we say that x is an *improper* part of y (and vice versa). That x is a proper or improper part of y is expressed by

$$x < y$$

We read it, for simplicity, as 'x is (a) part of y'. This relatively minor deviation from normal usage is warranted by its convenience.[3]

It is trivially true that in this sense of 'part', two individuals are identical if and only if their parts are the same, and in certain calculi of individuals this is used to define the relation of identity. The question whether the part-relation is in some absolute conceptual sense prior or posterior to the identity relation is perhaps not as obvious as it may at first appear,[4] but we shall always take the identity relation as given, for reasons to be given below. With identity at our disposal, it is possible to define either 'part' or 'proper part' in terms of the other, so the choice as to which to take as primitive in a mereological system is a matter of convenience. While 'proper part' is the more natural concept, 'part' is algebraically more convenient. In any case, extensional mereology may be based on a good few more than just these two possible primitives.

1.1.3 Overlapping

Two individuals overlap mereologically if and only if they have a part in common. This includes the case where one is part of the other, and

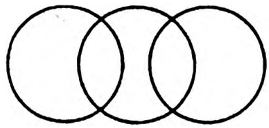
[3] The readings are mirrored in the symbols: '$<$' and 'pt' are simpler, while '$\ll$' and 'ppt' stutter. Both symbolic conventions have precedents in the literature: cf. LEONARD and GOODMAN 1940, HENRY 1972.

[4] Cf. SHARVY 1983*b*: 234, where it is contended that the part-predicate is more fundamental than identity.

also the case of identity. For '*x* overlaps *y*' we write

$x \circ y$

It is easy to see that overlapping is reflexive and symmetric, but not transitive, as shown by the diagram of overlapping discs below. In



general, overlapping individuals are such that neither is part of the other. Two intersecting roads overlap at their junction, but neither is part of the other. Overlapping sometimes occurs abnormally among human beings and other animals, in the case of Siamese twins. Where individuals overlap, but neither is part of the other, we may speak of *proper* overlapping; in mereology this concept plays hardly any role, though there is reason to think it is of importance in our conceptual scheme, if only in a negative way. There appears to be a certain conceptual uneasiness about proper overlapping. This might be connected with its abnormality for human beings (as a permanent state—the mother–foetus case is quite normal), but may be more general. We readily accept stateless tracts of sea between nations, but the idea of two nations' having overlapping territories is most uncomfortable for more than one reason. There appears to be a general tendency to draw conceptual boundaries, cast concepts of physical things and events in such a way that for most practical purposes proper overlapping is avoided. In many cases, where the objects classified are in any case marked by discontinuities at their borders with the environment, this tendency follows the natural grain of the world; but in dealing with inherently undivided continua, we have a freer hand. Yet in the case, for example, of colours we predominantly order primary colour terms so that there are stretches of no man's land between the foci of colour terms rather than allow such terms to clearly overlap.

Whatever the anthropological significance of this, the simpler idea of overlapping defined first above is mereologically and conceptually quite unproblematic, though it may interact with other mereological concepts in a questionable way. It can be and has been used as the sole primitive concept in calculi of individuals, and its counterpart likewise in Mereology.

1.1.4 Disjointness

Individuals are *disjoint* iff (if and only if) they do not overlap, i.e. they are disjoint iff they have no part in common. We write

$$x \wr y$$

for '*x* is disjoint from *y*'. Examples of disjointness abound. Most human beings are disjoint from one another. Disjointness is symmetric. Like overlapping, it is a straightforward concept, and can be used as sole primitive in a calculus of individuals.

We come now to mereological operations.

1.1.5 Binary Product

If two individuals overlap, they have, by definition, at least one part in common. In terms of the part-ordering, any such common part is a lower bound for the two individuals. Most mereological theories assume that overlapping individuals have a greatest lower bound or infimum, which is the lattice-theoretic meet: the traditional term is 'product'. The product of *x* and *y* is that individual which is part of both, and which is such that any common part of both *x* and *y* is part of it. For this product we write

$$x \cdot y$$

It is the mereological counterpart of the intersection of two sets, with the difference that in normal set theory even two disjoint sets have an intersection, namely the null set, whereas disjoint individuals precisely lack any common part. Most mereological theories have no truck with the fiction of a null individual which is part of all individuals, although it neatens up the algebra somewhat.[5] So the operational notation for product, if understood to denote a function, denotes only a partial function.[6] Rather than appealing to partial functions, we take operational notation as equivalent to definite descriptions, so that if *x* and *y* do not overlap, '$x \cdot y$' is equivalent to an improper description. Improper descriptions may be dealt with in a number of different

[5] The chief culprit in propounding this absurdity is R. M. Martin: see MARTIN 1943; 1965; 1978: 13 ff. See also CARNAP 1956: 36 f. BUNGE 1966 and 1977: 51 even proposes more than one null individual. But the idea is neatly dispatched by GEACH 1949, and we need not take it seriously. The theory of substance in BUNGE 1977 appears to be similar in strength to a classical extensional mereology plus a null individual, though Bunge claims (50) that the part–whole relation is adequately captured by a partial ordering! Bunge's views are too far off our track for it to be worth our while trying to unravel the parallels.

[6] This is the method chosen in PÖHRINGER 1982.

ways, and in this chapter no particular method is preferred. The following chapter suggests basing a calculus of individuals on a free logic with descriptions, which is philosophically the most acceptable alternative. A further advantage of this choice is that it brings calculi of individuals a step closer to Leśniewski's approach and so facilitates comparison.

Having defined binary products, it is clear that finite products of any number of places may be defined, since the product is obviously commutative and associative. For arbitrary products, however, a stronger operation is required.

1.1.6 Binary Sum

We define the mereological *sum* of two individuals x and y

$$x + y$$

as that individual which something overlaps iff it overlaps at least one of x and y. So a broom is, roughly speaking, the sum of its handle and its head.[7] We shall see below that this definition does not always coincide with that of their mereological least upper bound or lattice-theoretic join as the 'smallest' individual containing them both.

It is a central thesis of classical extensional mereologies that any two individuals possess a sum. Since individuals may be disjoint, spatio-temporally widely separated, and of quite different kinds, this assumption is very implausible, and it will be part of our task in later chapters to account for and set limits to this implausibility.

As in the case of products, the binary sum operator can be used to define sums of arbitrary finite numbers of individuals.

1.1.7 Difference

If x and y are two individuals, then their mereological difference,

$$x - y$$

is the largest individual contained in x which has no part in common with y. It exists only if x is not part of y. If x and y overlap and x is not part of y, then $x - y$ is a proper part of x. In some non-classical mereologies a unique difference does not always exist.[8]

[7] Elucidating the force of this 'roughly speaking' will occupy us a good deal. The broom example is not without its prehistory: cf. WITTGENSTEIN 1953: §60, HENRY 1972: 118 ff. Perhaps it is chosen because brooms have only two (major) components.

[8] Sharvy (1980: 621, 1983*b*: 234) calls partial orderings which are closed under least upper bounds *quasi-mereologies*, and for them differences may fail to exist where they exist in classical mereology. This is because the overlapping principle SA15 fails. For further discussion see the next chapter.

1.1.8 General Sum and Product

The existence of binary sums and products is no guarantee that every class of individuals has a sum, or that every class of individuals with a common part has a product, since such classes may be infinite. To cover such cases, a new concept and a new notation is needed. We take the *sum* (sometimes called the *fusion*) of all objects satisfying a certain predicate $F\xi$ to be denoted by a variable-binding operator σ and write

$$\sigma x \ulcorner Fx \urcorner$$

for this sum. The corner brackets mark the scope of the operator and can be omitted in the case of simple predicates, where no scope ambiguity threatens.

Similarly, the general product or *nucleus* of all the objects satisfying $F\xi$ is written

$$\pi x \ulcorner Fx \urcorner$$

with the same proviso on corners. Of course this product will only exist if all the objects satisfying the predicate have a common part. It turns out that the general sum operator suffices on its own to define the binary sum and product, the general product, and the difference operator. In fact, as we shall see below, slightly different ways of defining the general sum are possible, and they are not all equivalent in systems weaker than a full classical extensional mereology.

If the existence of binary sums poses a problem, then sums of arbitrary classes of individuals pose a greater problem. Nevertheless, Mereology and calculi of individuals, the classical extensional mereologies, have as a central thesis an axiom stating the existence of general sums.

1.1.9 The Universe

On the assumption that arbitrary sums exist, there then exists the sum of all objects whatever, a unique individual of which all individuals are part. This is denoted by 'U' and is called, for obvious reasons, 'the Universe'. It functions algebraically as a Boolean unit element in a mereological algebra. The existence of the Universe (in this special sense)[9] is perhaps slightly less controversial than the existence of arbitrary sums, so in a non-classical mereology in which arbitrary sums are not guaranteed, the existence of the Universe would have to be specially postulated, but *conditionally*. If we accept the possibility

[9] Much is made of this in MUNITZ 1974: 191 ff.

that no individuals at all exist as a logical possibility, then in such a case the Universe would not exist either. It is common to speak of this alternative as that of the *empty* world or empty Universe (by analogy with the empty set—which the empty world *is* in some semantic theories). But strictly speaking this terminology is wrong, for the world or the Universe is not a container which may or may not be filled—it is the whole filling. And so this limiting case is not one where the Universe is empty, but one where there is no Universe. Both Mereology and the calculus of individuals based on a free logic given below, admit this case.

1.1.10 Complement

Assuming that differences and the Universe both exist, then for every individual there is a unique individual comprising the rest of the Universe outside it. If x is the individual, this *complement*, $\mathrm{U}-x$, written '$\bar{x}$', exists and is unique, providing U exists and x is not identical with U. The complement of an individual is its complement in the Boolean sense also; its interest is chiefly historical and algebraic. In weaker mereologies a complement does not always exist, even if there is a Universe.

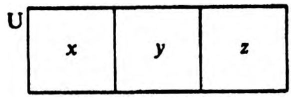
1.1.11 Atom

Finally we consider the predicate 'ξ is an atom'. An atom is an individual with no proper parts; it is accordingly indivisible either in fact or in theory, as befits the etymology of its name. Atoms in this strict mereological sense are not to be confused with atoms in the sense of physics and chemistry, which may have numerous proper parts and are far from indivisible, even in fact. Here the etymology of the name has lost touch with progress in physics; 'atom' is no longer a functional but a rigid term for certain natural kinds. Whether there really are mereological atoms is an unresolved question, and it is at present difficult to see what could resolve it. The problem of atomism is discussed in greater detail below. We shall use the expression 'At x' to mean 'x is an atom'.

There are a great many other mereological concepts which can be defined with the means we have outlined, but we have covered the most basic and important ones. To illustrate those we have listed, it is helpful to have

1.2 An Example

Consider the plane figure below: a (filled) planar rectangle U



exhaustively divided into three squares x, y, z. The letter 'U' has been chosen because we shall indulge the false supposition that U and its parts are all there is, so all the concepts mentioned above can be illustrated. Further, we confine consideration to just those individuals which can be compounded of x, y, and z, ignoring all others. That is, we are treating the squares as if they were atoms. Of course, in reality they are not so, but atoms cannot be drawn. Of the individuals compounded just of x, y, and z one has already been mentioned: U itself. We have

$$\mathrm{U} = x + y + z$$

The remaining sums may be given names as follows

$$u = x + y$$

$$v = x + z$$

$$w = y + z$$

and note that whereas u and w are connected, v is not, so the assumption that there is such an individual as v is less secure than the assumption that there are such individuals as u and w, other considerations apart.

First of all we list some representative mereological facts concerning U and these parts: $x \wr y$, $y \wr z$, $z \wr x$, $x \ll u$, $x < v$, $x < \mathrm{U}$, $u \ll \mathrm{U}$, $u \circ v$, $u \cdot v = x$, $x + w = u + v = \mathrm{U}$, $u - w = x$, $\bar{v} = y$. The general sum and product operators have not been illustrated in this finite model.

It is no accident that there are only seven individuals compounded out of the three squares. If they really were atoms, then these seven would be *all* the parts of U. In a classical mereology, there is a fixed relationship between the number of atoms and the number of objects, assuming that everything is made of atoms. If there are c atoms, where c is any cardinal number, then there are $2^c - 1$ individuals. Thus the cardinality of intended models of classical mereology is restricted. There are models with 1, 3, 7, 15, 31, $2^{\aleph_0}$, and numerous other cardinalities, but none with cardinality 2, 12, or $\aleph_0$, for example.

We have illustrated an atomistic case. In this case there is a marked algebraic affinity between the calculus of individuals and the calculus of classes, which will be pursued in greater detail in Chapter 4 below. In the figure there are of course more than just seven parts to U. If the plane figure U were a continuum, then there would be at least continuum-many parts.

Simple geometric figures like this are useful for illustrating concepts and finding ways to prove suspected theorems.[10] However, their usefulness is limited because they possess features which do not necessarily hold for all mereologies. For instance, sums of disjoint areas do not appear especially problematic, for in each case the entities summed are alike in species. There are no *conceptual* difficulties about states whose territories are not connected.

1.3 The Leśniewskian Approach

The differences between Leśniewski's Mereology and calculi of individuals spring not from the mereology as such, but from differences in the underlying logic. The languages are different, and the conception of logic entertained by Leśniewski and his disciples deviates markedly from the conception entertained by the average practitioner of predicate logic. Leśniewski's nominalism was so thorough that he applied it to his own logical systems, construing these not as abstract sets of propositions or whatever, but as concrete assemblages of physical marks. The Platonic view of logical systems envisages these as timeless constellations, whereas Leśniewski is forced to see logical systems as dynamic, growing in time by the addition of new marks called theses, according to well-defined rules. However, this thoroughgoing nominalism can be applied as much to predicate logic as to Leśniewski's own systems, and is peripheral to our concerns here. The essential differences between Ontology, Leśniewski's general logic, and standard set theories and predicate logics, concern in particular three areas: terms, quantification, and definitions. We take these in order of importance.

1.3.1 Terms

In orthodox predicate logic, all proper names and other genuine singular terms are essentially singular—that is, designate precisely one

[10] They are obviously related to Euler diagrams, whose appropriateness for mereology was noted by FREGE 1895.

individual. In free logic this restriction is relaxed to the extent of allowing empty terms. These are inter-substitutible *salva congruitate* with genuine (i.e. referential) singular terms, but do not designate anything. In other words, designation, from being a function, becomes a partial function. Leśniewski's logic likewise allows empty terms, but it goes an important step further, widening the notion of designation still further so that it becomes a general *relation*. While in orthodox logic a term designates exactly one individual, and in free logic it designates not more than one, in Ontology a term may designate more than one individual. This is not ambiguity, but straightforwardly plural designation. In this respect Ontology follows natural language, which possesses grammatically connected expressions like 'Russell and Whitehead', 'the brothers Grimm', and 'Benelux', which designate several individuals. It is a prejudice to suppose that designation must always be singular, or in default of this not occur at all.[11] Interpreters of Leśniewski frequently express this fact about his logical language by saying that the names in it are common nouns, or that the language makes no distinction between proper and common nouns. This view is debatable,[12] but the issue is not crucial to our concerns here. If we consider again our example from the previous section, then to the seven singular terms which we already have, namely 'x', 'y', 'z', 'u', 'v', 'w', and 'U', we add first of all the empty name '$\wedge$', which does not designate at all. Then we add plural names. The semantic Table 1.2 offers a fragment of the possibilities, where it is to be noted that the symbols are mentioned on the left but used on the right.

TABLE 1.2. *Some Plural Terms for the Squares Example*

Token of term	Individuals designated
a	x, y, z
b	x, y
c	x, z
d	y, z
e	u, v, w
f	u, v
$\vee$	x, y, z, u, v, w, U

Since there are seven individuals under consideration, there are $2^7 = 128$ ways of designating a selection of them, including the null

[11] Against this prejudice an increasing number of voices are being raised. Cf. the references to the discussion in Ch. 4.

[12] It is argued against in SIMONS 1982*d*.

case, so we have left 113 combinations without a term. The expression 'V' is a standard one in Ontology: dual to 'Λ', it is a *universal* term; here it designates all seven individuals of the putative universe. The distinction between 'V' and 'U' should be clearly noted: the former is a term which designates all individuals, and in any case where there is more than one individual, this means it is plural. The latter is a singular term (except in the case where nothing exists, when it is perforce empty), and designates that individual which is the mereological sum of all individuals.

The introduction of plural terms gives Ontology, and with it Mereology, an expressive power beyond that available to predicate logic of first order without the introduction of sets; for variables may be quantified which may be interpreted plurally, and predicates may be introduced which have plural subjects, allowing the direct representation of notions such as 'outnumber' or 'surround' which require paraphrase into second-order logic or set theory when translated into predicate-logical idiom. These possibilities are here simply stated; we are not here concerned with their general philosophical signifiance.

1.3.2 Quantification

Leśniewski understood quantification in a non-referential manner. In classical predicate calculus '$\exists x Fx$' may be read roughly as 'something Fs' or 'there is (exists) an F-er', and '$\forall x Fx$' as 'all (existing) things are F-ers'. This referential interpretation may be extended to accommodate plurals: '$\exists a Fa$' then means 'some thing Fs or some things F' or, slightly more accurately, 'there is (exists) some thing or things a such that it or they F'. If we use the term 'manifold' to cover both the singular and plural case, it means 'there is a manifold such that F(it)'. Such quantification still carries existential import, and if empty terms are admitted, inferences such as $Fb \vdash \exists a Fa$ are invalid.[13] For Leśniewski on the other hand such an inference is valid, so whatever else his 'for some a: Fa' means, it does not mean 'there exists a such that Fa'. It can be proved in Ontology that for some a, a does not exist, so this reading is wholly alien to Leśniewski's conception of quantification. We mark this difference in meaning with a difference in symbols: for Leśniewski's universal quantifier we write 'Π' and for its dual, which must be called the *particular* not the existential quantifier,

[13] Classical quantifiers and plural terms are mixed in this way in the manifold theory of SIMONS 1982*c*. Note that '$\vdash$', as usual, marks an inference.

we write 'Σ'. Whether Leśniewskian quantifiers are to be understood substitutionally or in some other way is a controversial matter, which we pass over here but mention in more detail in the following chapter. For our purposes it suffices to note that Leśniewskian quantification is ontologically neutral or uncommitting: the fact of quantifying variables of a certain category is not sufficient to commit one to quantifying *over* corresponding entities.

1.3.3 Definitions

For reasons which we again deal with at greater length elsewhere, Leśniewski regarded all definitions as object-language equivalences introducing a new expression, subject to detailed restrictions designed to exclude circularity, paradox, and other unpleasantnesses. We shall follow him in this regard, styling definitions with a 'D' in their nomenclature, but treating them otherwise as any other axiom, and not as a metalinguistic abbreviation.

1.3.4 Concepts of Ontology

A number of predicates may be taken as primitive in Ontology, but the most usual and useful is the predicate 'ε' of singular inclusion. This bears only a partial similarity to the membership relation '$\in$' of set theory (we have written them both so the different symbols can be noted). Whereas both expressions flanking '$\in$' are singular terms, neither one flanking '$\in$' need be singular. Whereas in most set theories self-membership is ruled out, because sentences of the form $x \in x$ are either always false or else not well formed, a formula '$a\,\varepsilon\,a$' may be true (namely in the case where 'a' denotes exactly one individual). The truth-conditions of a formula '$a\,\varepsilon\,b$' are easily given: it is true iff 'a' designates exactly one individual and it is one of the one or more individuals designated by 'b'. It is usually easiest to read the sign 'ε' as the English copula 'is a', but a more accurate reading is 'is one of' or 'is among'. The natural language reading offers in any case at best an approximation to the intended meaning.

Other predicates and further kinds of constant may be defined using 'ε' and the available logical resources of propositional connectives and quantification. The definitions are given in Chapter 2, but Table 1.3 gives a list of useful constants, together with suggested readings and truth-conditions.

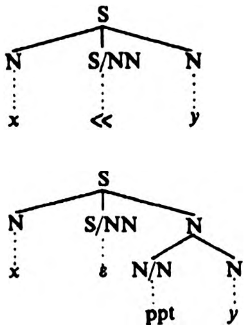
In addition to these predicates we may define the constant terms '$\vee$' and '$\wedge$' which were mentioned above.

TABLE 1.3. *Important Concepts of Ontology*

Concept	Formula	Reading	Truth-conditions
Singular Identity	$a = b$	*a* is identical with *b*; *a* is the same individual as *b*	'*a*' and '*b*' designate the same individual
Non-plural Identity	$a \approx b$	*a* is the same as *b*	'*a*' and '*b*' designate the same individual or are both empty
Identity	$a \simeq b$	*a* and *b* are the same	'*a*' and '*b*' designate the same individuals or are both empty
Existence	E*a*	*a* exist(s); there is at least one of *a*	'*a*' designates at least one individual
Uniqueness	!*a*	there is at most one of *a*	'*a*' designates at most one individual
Singular Existence	E!*a*	there is exactly one of *a*; *a* is an individual	'*a*' designates exactly one individual
Inclusion	$a \subset b$	any *a* is a *b*	'*b*' designates whatever '*a*' designates (if anything)
Strong Inclusion	$a \sqsubset b$	every *a* is a *b*	'*b*' designates whatever '*a*' designates (at least one thing)
Intersection	$a \triangle b$	some *a* is a *b*	some individual is designated by both '*a*' and '*b*'

1.3.5 Concepts of Mereology

Whatever is or has a proper or improper part, whatever overlaps or is disjoint from something, whatever is a sum or product of individuals is an individual. So, as far as the relations and operations of mereology go, there is no systematic reason why Mereology should differ greatly from calculi of individuals. In fact the difference is slightly more marked than it might be because of Leśniewski's way of expressing mereological relations. Take the sentence '*x* is a proper part of *y*'. In predicate logic this is given a tripartite structure, the transitive verb phrase 'is-a-proper-part-of' being treated as a logical unit. Leśniewski, on the other hand, uses the availability of plural terms and singular inclusion to express the same thing in a quadripartite structure, '*x* is-a proper-part-of (*y*)', breaking the predicate into the copulative predicate 'is-a' (expressed by 'ε') and a functor taking one nominal argument to form a complex term, 'proper-part-of ( )'. The whole sentence thus has the Ontological form '*x* ε *a*', where the second term is complex. This difference may be illustrated by comparing the two structure diagrams below.



The semantic work which is done in predicate logic by the predicate is here shared by the formal predicate 'ε' and the specifically mereological functor 'ppt'. In principle there would be nothing to stop Leśniewski using the same predicate as the calculus of individuals. The use of the functor 'proper-part-of' inserts an additional layer of structure which allows the term 'proper part of *y*' to emerge, and this will usually be plural. The restriction of predicate calculus to singular terms does not allow this extra level of structure to be represented. So in Mereology the sentence 'any proper part of *x* is a part of *x*', which requires the rather complex formulation '$\forall y \ulcorner y \ll x \supset y < x \urcorner$' in calculus of individuals, may be given the simpler formulation '$\text{ppt}(x) \subset \text{pt}(x)$'.

In similar manner, instead of predicates 'is-a-part-of', 'overlaps', 'is-outside', and so on, Mereology has term-forming functors 'part-of' (pt), 'overlapper-of' (ov), 'outsider-of' (ex). Whereas calculi of individuals then use operational notation for binary sum and product, in Mereology expressions of category N/NN are used, which take two nominal arguments and yield a complex term, 'binary-product-of ( ) and ( )' (Bpr(,)) and 'binary-sum-of ( ) and ( )' (Bsm(,)). This preference for terms rather than predicates is a matter of taste and expediency rather than principle; it has its advantages and disadvantages. The latter can be circumvented at any time by defining a suitable equivalent predicate, which is always possible in Mereology.

When it comes to expressing the general sum and product, Mereology looks more straightforward than calculi of individuals. Instead of a variable-binding operator, we have two functors of the same category as 'ppt', 'sum-of' (Sm) and 'product-of' (Pr). This is possible because the argument-expressions may of course be *plural*:

instead of a complex expression 'the-sum-of x such that x is-a-square' we have simply 'the-sum-of squares'. Hence a complex nested expression like 'the sum of all the overlappers of parts of x' comes out as 'Sm(ov(pt(x)))' where in predicate-logical terms we need '$\sigma z^{\ulcorner} \exists y^{\ulcorner} z \circ y \wedge y < x^{\urcorner\urcorner}$'. Of course, the variable-binding notation has advantages when it comes to expressing sums and products of individuals satisfying particularly complex predicates, where Mereology must first compress the predicative condition into a suitable complex term, if necessary via definition. In practice, each form of expression is as good as the other and the differences between them are not logically important, except for the important point that in Mereology all nominal variables may be quantified without incurring ontological commitment. Table 1.4 shows the equivalents or near-equivalents of mereological constants which we have discussed.

TABLE 1.4. *Equivalent Concepts in Calculus of Individuals and Mereology*

Concept	Calculus of individuals	Mereology
Proper part	$x \ll y$	$x \varepsilon \mathrm{ppt}(y)$
Part	$x < y$	$x \varepsilon \mathrm{pt}(y)$
Overlapping	$x \circ y$	$x \varepsilon \mathrm{ov}(y)$
Disjointness	$x \wr y$	$x \varepsilon \mathrm{ex}(y)$
Binary product	$x \cdot y$	$\mathrm{Bpr}(x, y)$
Binary sum	$x + y$	$\mathrm{Bsm}(x, y)$
Difference	$x - y$	$\mathrm{Cm}(x, y)$
General sum	$\sigma x^{\ulcorner} Fx^{\urcorner}$	$\mathrm{Sm}(a)$
General product	$\pi x^{\ulcorner} Fx^{\urcorner}$	$\mathrm{Pr}(a)$
Universe	U	U
Complement	$\bar{x}$	$\mathrm{Cpl}(x)$
Atom	At x	$x \varepsilon \mathrm{atm}$

The difference in form of expression for the binary product is not quite as minimal as it looks, since the Mereological expression is not syntactically confined to the role of a singular term. However, wherever the resulting term satisfies a uniqueness condition, we have given the corresponding Leśniewskian functor a capital letter.

This should suffice to prepare the reader for confrontation with Leśniewskian texts. As a final warning, it should be mentioned that there is no standard notation in Leśniewskian logic, and it will be possible to find numerous equivalents for the notation we have given here. As a guide to orientation, there is a table of equivalents in Chapter 2. Our notation has been chosen partly to conform with our informal terminology, partly to illuminate the parallels with the notation in calculus of individuals.

1.4 Mereological Principles

Since the first presentation of classical mereology by Leśniewski in 1916, many writers on the subject have assumed that, since the theory avoids both Platonism and paradox, it is to all extents and purposes philosophically harmless and uncontroversial. Thus many of the writings on mereology are concerned with bringing logical refinement to the existing system, by suggesting new primitives, shorter axioms, and the like. While they are interesting in themselves, these contributions take the classical theory for granted and do not question its presuppositions. For someone who questions the cogency of some of these presuppositions it is remarkable how little work has been done on systems of mereology weaker than the classical ones of Leśniewski and Leonard and Goodman.[14] So it is important to lay out the principles of extensional mereology not in the most economical and axiomatically most elegant way, where the power of the axioms is not immediately evident, but rather by starting from obviously acceptable principles and adding to them gradually in steps which are as small as possible. Only in this way are we likely to establish the boundary of what is uncontroversially acceptable.

Algebraically, a mereology is a partial ordering, but of a kind which has attracted little attention among mathematicians.[15] The algebraic structure of a full classical mereology is that of a complete Boolean algebra with zero deleted,[16] which is a much richer structure. In between these two extremes there is much room for discussion and dissent.

The family of systems S introduced below constitutes an attempt to fill in some of the gaps between the weakest and strongest systems. It uses the signs introduced in §1.1. We do not stop to explain the

[14] There is a survey of such systems in the next chapter.

[15] Among lattices, by contrast, mathematicians have worked out a detailed taxonomy. See, for example, the variety of approximations to Boolean algebra in RASIOWA 1974, which arise through considering the algebras of non-classical propositional calculi. Non-classical mereologies have been unable to command such attention.

[16] Cf. TARSKI 1956c: 333 n.

symbolism, which is straightforward: details can be found at the beginning of Chapter 2.

Systems S: *no sets; '$\ll$' primitive; '$=$' assumed*

Like the most cautious calculi of individuals, we eschew use of sets; the logical constant of identity is presupposed and not defined mereologically. The chosen primitive is the most intuitive predicate, 'is a proper part of'. The logical basis is

SA0 Any axiom set sufficient for first-order predicate calculus with identity

The relation '$\ll$' is at least a strict partial ordering; this is reflected in

SA1 $x \ll y \supset \sim y \ll x$

SA2 $x \ll y \wedge y \ll z \supset x \ll z$

We can define 'part' then as

SD1 $x < y \equiv x \ll y \vee x = y$

It is possible to take '$<$' as primitive instead, defining a proper part as a part which is different from its whole. The above axioms fall well short of characterizing the part-relation; there are many partial orderings which we should never call part–whole systems. The simplest is the partial ordering on two objects having the Hasse diagram shown below. How could an individual have a *single* proper

part? That goes against what we mean by 'part'. An individual which has a proper part needs other parts in addition to *supplement* this one to obtain the whole.[17] Principles which tell us something about how this takes place will therefore be called supplementation principles.

[17] One philosopher who apparently did not think so is Brentano (cf. BRENTANO 1933, 1981), for whom a substance (e.g. a man) is a proper part of an accident (e.g. a sitting man), without there being any extra part added to the man to get the sitting man. Cf. CHISHOLM 1978: 201 ff. A rather similar position is to be found in Fine's theory of *qua* objects in FINE 1982, which we discuss in a later chapter. However it seems unlikely that Mrs Thatcher is a *proper* part of Mrs Thatcher *qua* Prime Minister, so Fine's theory is not quite the same as Brentano's. In any case, the theory of Brentano is hardly plausible enough to shake our conviction that *some* supplementation is necessary. If there *is* (as Brentano and Fine think) a difference between a man and the same man sitting, it is not a mereological difference.

We first consider two which are clearly too weak, and then two others. The first simply suggests that any individual with a proper part must have at least two proper parts (where '$\neq$' as usual means 'not identical'):

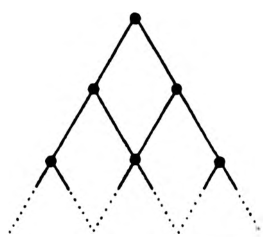
SF1 $x \ll y \supset \exists z \ulcorner z \ll y \wedge z \neq x \urcorner$

(The corners here are scope-marking brackets, a symbolization used by Leśniewski; they have the advantage of not crossing the line and so aid legibility. They have nothing to do with Quine's quasi-quotation.) This principle is far too weak to do the required job, since it is satisfied by an infinite descending chain (see below), and this no more deserves

to be called a model of part–whole relations than does the previous one. A stronger principle is

SF2 $x \ll y \supset \exists z \ulcorner z \ll y \wedge \sim z < x \urcorner$

While this blocks the chain example, it still lets in an inadmissible model, shown below.



A moment's thought shows that this too cannot be a model of part and whole. What we have here is a universe all of whose parts overlap each other. But surely if a universe is complex (i.e. has proper parts at all), then at least two of these parts will be disjoint. To express this we first define overlapping and disjointness:

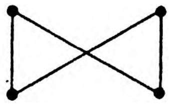
SD2 $x \circ y \equiv \exists z \ulcorner z < x \wedge z < y \urcorner$

SD3 $x \lfloor y \equiv \sim x \circ y$

We then have

SA3 $x \ll y \supset \exists z \ulcorner z \ll y \wedge z \lfloor x \urcorner$ (WSP)

This axiom will be known as the *Weak Supplementation Principle.*[18] It says that if an individual has a proper part, it has a proper part disjoint from the first. This eliminates the questionable lattice model. However the model below satisfies the axioms SA0–3. Here we have a case where two distinct individuals are made up of exactly the same proper



parts. This seems at any rate odd—why are the two not one? We shall have occasion in Part II to look with greater favour on the idea that distinct individuals can indeed share all their proper parts, but in a more complex setting, where issues of time are introduced. If we think of a complex whole as in any way like a class of its parts—albeit a *collective* rather than *distributive* class,[19] then it seems implausible that the same parts can form two distinct wholes, just as distinct classes cannot be formed from the same elements. This suggests adding, by analogy with the extensionality principle of class theory, a principle which says that if individuals have the same parts, they are identical. If 'part' meant '$<$' this would be trivial, so it must here mean 'proper part'. But as it stands this formulation will not do, because any two atoms have the same proper parts, viz. none, and so would be identified. Hence we need to add that the individuals in question are not atoms. We express a one-sided form of this *Proper Parts Principle*:

SA4 $\exists z \ulcorner z \ll x \urcorner \wedge \forall z \ulcorner z \ll x \supset z \ll y \urcorner \supset x < y$ (PPP)

Adding this to SA0–3 eliminates the four-element model.

The extensional aspect of extensional mereology appears to be closely connected with PPP, as its motivation shows. However, we

[18] A number of principles whose importance extends beyond a chapter are referred to not by their local numbers but by descriptive names or their mnemonic abbreviations given on the right of the line.

[19] The terminology is due to Leśniewski, but the distinction goes back (at least) to FREGE 1895. Cf. SINISI 1969.

shall see that in a slightly stronger setting PPP can be made, with help, to follow from WSP. To prepare the way for this, consider the following *Strong Supplementation Principle*:

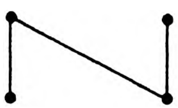
SA5 $\sim x < y \supset \exists z \ulcorner z < x \wedge z \wr y \urcorner$ (SSP)

It can easily be checked that WSP follows from SSP. In addition, PPP also follows from SSP. To show this, we take PPP in the partly contraposed equivalent form $\exists z \ulcorner z \ll x \urcorner \wedge \sim x < y \supset \exists z \ulcorner z \ll x \wedge \sim z \ll y \urcorner$. The proof is presented in a slightly abbreviated natural deduction form:

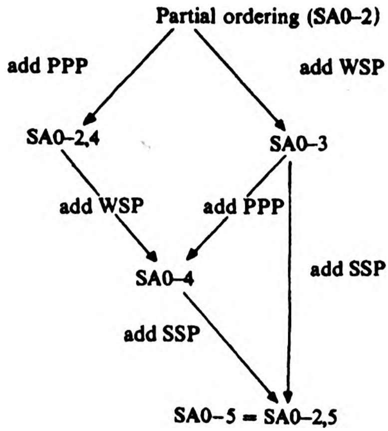
(1)	$\exists z \ulcorner z \ll x \urcorner \wedge \sim x < y$	Assumption
(2)	$\exists z \ulcorner z < x \wedge z \wr y \urcorner$	1, SSP (SA5)
(3)	$x \wr y \wedge z \ll x \supset \sim z \ll y$	SD3
(4)	$x \wr y \supset \exists z \ulcorner z \ll x \wedge \sim z \ll y \urcorner$	1, 3
(5)	$x \circ y \wedge z \wr y \supset z \neq x$	SD3
(6)	$x \circ y \wedge z < x \wedge z \wr y \supset z \ll x \wedge \sim z \ll y$	5, SD1, SD3
(7)	$x \circ y \supset \exists z \ulcorner z \ll x \wedge \sim z \ll y \urcorner$	2, 6
(8)	$x \circ y \vee x \wr y$	SD3, excluded middle
(9)	$\exists z \ulcorner z \ll x \wedge \sim z \ll y \urcorner$	8, 4, 7

It can be directly checked that SSP also rules out the four-element model, as this result implies. Hence SSP is stronger than WSP, as their names tell. In fact, however, SSP cannot be derived from SA0–4, as the following counter-model shows. If we take all half-open, half-closed intervals on the real line, and interpret '$<$' as set-theoretic inclusion, then it can be checked that all of SA0–4 are satisfied, but SA5 not. For the interval [0, 1) is not part of the interval (0, 1] (or vice versa), but no part of [0, 1) is disjoint from (0, 1]. It is characteristic of this model that overlapping intervals do not always have a unique product: this applies in particular if the overlap is either purely open or purely closed. Hence by adding SA5 directly to SA0–2, we obtain a system which is strictly stronger than that obtained by adding SA3–4.

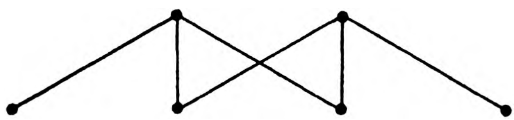
Since adding PPP directly to the partial ordering axioms SA0–2 yields something stronger than a mere partial ordering—the model



is thereby excluded—the lattice of structures shown below is thereby established:



The strongest resulting system still does not rule out all models which may appear suspicious. In particular, it still admits the model below.



This has in common with the previous interval model that overlapping individuals do not always have a unique product. In the context of an extensional mereology, the assumption of unique products appears plausible—if two objects overlap, why should there not be a maximal common part? This suggests adding

SA6 $x \circ y \supset \exists z \forall w \ulcorner w < z \equiv w < x \wedge w < y \urcorner$

Defining

SD4 $x \cdot y \approx \iota z \forall w \ulcorner w < z \equiv w < x \wedge w < y \urcorner$

this amounts to

SF3 $x \circ y \supset E!(x \cdot y)$

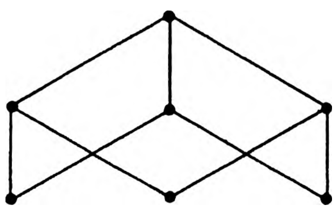
In this stronger context of SA0–3 + SA6, the Strong Supplementation Principle SA5 is derivable. We set out again a shortened natural deduction:

(1)	$\sim (x < y)$	Assumption
(2)	$x \wr y \vee x \circ y$	SD3, excluded middle
(3)	$x < x$	SD1
(4)	$x \wr y \supset \exists z \ulcorner z < x \wedge z \wr y \urcorner$	3, EG
(5)	$x \circ y \supset \mathrm{E!}(x \cdot y)$	SF3
(6)	$\mathrm{E!}(x \cdot y) \supset . \sim (x < y) \supset x \cdot y \ll x$	Simple Theorem
(7)	$x \circ y \supset \exists z \ulcorner z \ll x \wedge z \wr x \cdot y \urcorner$	1, 5, 6, WSP (SA3)
(8)	$\forall x \ulcorner z < x \wedge z \wr x \cdot y \supset z \wr y \urcorner$	Simple Theorem
(9)	$x \circ y \supset \exists z \ulcorner z < x \wedge z \wr y \urcorner$	7, 8
(10)	$\exists z \ulcorner z < x \wedge z \wr y \urcorner$	2, 4, 9

Hence, when products are guaranteed, PPP follows from WSP, which means that the extensional aspect of this system is shared between SA3 and SA6.

The system derivable from the axioms SA0–3 + SA6 may be considered as a system of *Minimal Extensional Mereology*. It excludes obviously unacceptable models but—apart from the extensional aspect—does not include anything we can take as being too strong an assumption. When we are considering the requisite formal properties of a part–whole relation in such abstractness, it is not easy to draw the line between the acceptable and the unacceptable, since intuitions are weak at this level of generality.

That this minimal mereology falls well short of the strength of the full classical systems can be seen not only by the absence of infinitary operators, but even among small finite models. For full classical extensional mereology there is only one seven-element model (up to isomorphism), shown below, whereas for this minimal mereology



there are, again up to isomorphism, twenty-eight models, essentially because there are no principles guaranteeing, either conditionally or unconditionally, the existence of sums or even upper bounds. So the most natural way to extend the system is to consider how bounds and sums may be added.

The individuals satisfying a predicate $\phi\xi$ are *bounded above* if there is an individual of which all these are part. To express this we define a constant $\zeta\, \tau x^{\ulcorner}\phi x^{\urcorner}$ which is half-predicate, half-operator, and read '$z\ \tau x^{\ulcorner}Fx^{\urcorner}$' as '$z$ is an upper bound for F-ers':

SD5 $z\ \tau x^{\ulcorner}Fx^{\urcorner} \equiv \exists x^{\ulcorner}Fx^{\urcorner} \wedge \forall x^{\ulcorner}Fx \supset x < z^{\urcorner}$

In particular, if $z\ \tau x^{\ulcorner}x = y \vee x = w^{\urcorner}$ then z is a common upper bound for w and y: both are part of it. If the universe U exists, then it is an upper bound for everything. The existence of upper bounds does not imply the existence of sums or of least upper bounds. To see this, consider the set of subsets of the natural numbers which are either non-empty and finite or are infinite and have a finite complement (possibly empty, i.e. including the set of all natural numbers). Then it can be checked that, interpreting '$<$' as the subset relation, all the axioms of minimal mereology are satisfied. Further, every collection of sets is bounded above by the whole set of natural numbers (which is here the universe), but there are collections of sets of natural numbers without a least upper bound, e.g. the collection of all finite sets of even numbers.

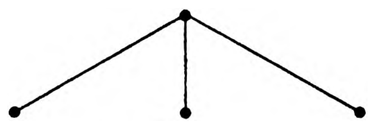
The conditions of being a least upper bound and being a sum are different. The least upper bound of two individuals may be defined as

SD6 $x +' y \approx \iota z \forall w^{\ulcorner}x < w \wedge y < w \equiv z < w^{\urcorner}$

whereas the sum of two individuals is defined as follows:

SD7 $x + y \approx \iota z \forall w^{\ulcorner}w \circ z \equiv w \circ x \vee w \circ y^{\urcorner}$

It may be checked that if a sum exists, then so does a least upper bound, and the two are identical, but the converse is not true. In the model below, the two outermost atoms have a least upper bound, the



universe, but no sum, because the only individual which overlaps both is U, but this overlaps the central atom. Similarly, if we consider open intervals on the real line, then any two open intervals have a least upper bound, namely the interval whose end points are their outermost extrema, but disjoint intervals with a gap between them have no sum.

Conditions ensuring the existence of upper bounds, least upper bounds, and sums are therefore many. We shall not explore all the possibilities, but consider the simpler and more intuitive ones.

The most plausible condition, and a weak one, is the principle that any two overlapping individuals have an upper bound. The rationale behind this is somewhat as follows. Suppose that for an individual to exist, there must be some form of connection (direct or indirect) among its parts. This vague condition will be made more precise in the final chapter, where we come to discuss the matter more fully. If then x and y overlap, then each is connected to their common part, and so they are connected to each other, connection being transitive. If being connected is then also a *sufficient* condition for belonging to a common individual, x and y are both parts of some individual, which is an upper bound for them. This principle is

SA7 $x \circ y \supset \exists z \ulcorner x < z \wedge y < z \urcorner$

and the simplest model of minimal mereology it rules out is shown below. Somewhat stronger is the principle that any two overlapping

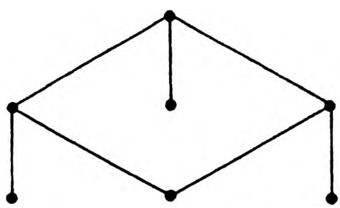
individuals have a least upper bound, which we may formulate as

SA8 $x \circ y \supset \mathrm{E!}(x +' y)$

and still stronger the principle that overlapping individuals have a sum:

SA9 $x \circ y \supset \mathrm{E!}(x + y)$

Conditions SA7 and SA8 cannot be distinguished among finite models, but SA7–8 and SA9 can: the model below satisfies SA7–8 but not SA9, the outer two non-atoms having no sum.



Somewhat different are principles guaranteeing the existence of least upper bounds or sums for pairs of individuals bounded above:

SA10 $x < z \wedge y < z \supset \mathrm{E!}(x +' y)$

SA11 $x < z \wedge y < z \supset \mathrm{E!}(x + y)$

These are both in a sense symmetric to SA6, depending on whether the product is considered as a greatest lower bound or as dual to the sum. The second implies the first, but not vice versa: the four-element model considered in connection with SD6–7 and the seven-element one above both satisfy SA10 but not SA11. Of course adding SA7 to SA10 (SA11) yields SA8 (SA9).

All these are conditional principles. Of a different sort are unconditional ones, statements to the effect that *any* two individuals have an upper bound, least upper bound, or sum:

SA12 $\exists z \ulcorner x < z \wedge y < z \urcorner$

SA13 $\mathrm{E!}(x +' y)$

SA14 $\mathrm{E!}(x + y)$ (BSP)

Note that the existence of upper bounds for two or in general finitely many individuals does not guarantee the existence of a universe. The lattice of square-free natural numbers under the partial ordering of divisibility has upper bounds only for finite sets. These unconditional principles can of course be used together with conditional ones to yield stronger conditions: for example, from SA12 and SA11 we obtain SA14. The leeway between SA8 and SA9, SA10 and SA11, or SA13 and SA14 can be made up with

SA15 $z \circ x +' y \supset z \circ x \vee z \circ y$

It is with the strongest principle SA14, which we may call the *Binary Sum Principle* (BSP), that we finally attain the strength, among finite models, of a Boolean algebra minus a zero element. This narrows, for example, the seven-element models down to just one. The square-free numbers model is one which satisfies all of SA0–6 + SA14, but which, through lacking a unit element, fails to be a Boolean algebra minus zero. The existence of a universe has to be guaranteed separately at this stage:

SA16 $\exists x \forall y \ulcorner y < x \urcorner$

As we remarked above, it is difficult in the absence of more concrete examples to set firm limits to the intuitive concept *part*; indeed, it may be doubted whether this concept has firm limits. Within the terms of reference imposed by extensionality of parts, i.e. PPP, my own feeling is that SA7 marks a point beyond which intuitions begin to waver. Of all the principles of classical mereology, it is the Sum Principle, whether in its weak binary or in its strong general form (which will be given below), which has come in for most criticism, and which has offended most intuitions concerning the meaning of 'part'. Thus a number of authors have used mereological systems in which the universal existence of even binary sums is not guaranteed. A case in point is Whitehead, whose non-classical extensional mereology is examined along with others in Chapter 2. Whitehead defines a concept of *joining* among individuals (Whitehead speaks rather of 'events') which amounts roughly to their having a sum: 'Two events which are joined have that relation to each other necessary for the existence of one event which extends over them and over no extraneous events.'[20] The concept *joining* is a first step towards analysing the idea of a common *boundary* of events (and represents therefore a step on the bridge from mereology to topology); but in classical or any other mereology, where any two individuals have a sum, each individual is trivially joined with every other. It is worth noting also that Whitehead rejects U, since he has an axiom to the effect that every individual is a proper part of some individual.

When it comes to adding principles governing general products and sums, matters are similarly complicated. However, in this area firm intuitions are if anything still rarer, so we mention only the main steps on the way to full classical mereology.

A general least upper bound operator may be defined as follows:

SD8 $\sigma' x \ulcorner Fx \urcorner \approx \iota x \forall y \ulcorner x < y \equiv \forall z \ulcorner Fz \supset z < y \urcorner \urcorner$

while a corresponding general sum operator is given by

SD9 $\sigma x \ulcorner Fx \urcorner \approx \iota x \forall y \ulcorner x \circ y \equiv \exists z \ulcorner Fz \wedge z \circ y \urcorner \urcorner$

A general product may be defined in the following way due to Breitkopf:[21]

SD10 $\pi x \ulcorner Fx \urcorner \approx \iota x \forall y \ulcorner y < x \equiv \forall z \ulcorner Fz \supset y < z \urcorner \urcorner$

The symmetry of SD8 and SD10 is pleasing, but, as we saw above, least upper bounds and sums differ in many systems.

We saw that it is safer to give conditional existence axioms for bounds and sums than unconditional. In the case of the general product, all the objects satisfying the predicate $F\zeta$ having a common

[20] WHITEHEAD 1919: 103. [21] BREITKOPF 1978: 230.

part means they all overlap each other, and in this special case it can be seen that a least upper bound and a sum are the same. This suggests the following conditional existence axiom:

SA17 $\forall xy \ulcorner Fx \wedge Fy \supset x \circ y \urcorner \supset \mathrm{E}!\sigma' x \ulcorner Fx \urcorner$

The next stronger condition, SA18, replaces 'σ'' by 'σ' in this definition. But on the motivation behind SA7, this condition appears too strong. For a number of individuals to be connected, it is not necessary that each overlap each other, but it is sufficient if each is connected to the other by a chain of overlappings. This suggests replacing the antecedent of SA17 by one where, instead of the overlapping relation, we take its ancestral $\circ_*$:

SA19 $\forall xy \ulcorner Fx \wedge Fy \supset x \circ_* y \urcorner \supset \mathrm{E}!\sigma' x \ulcorner Fx \urcorner$

and *mutatis mutandis* for SA20, conditional existence of sums. A similar rationale concerning common upper bounds rather than overlapping suggests

SA21 $\exists x \ulcorner x \, \tau y \ulcorner Fy \urcorner \urcorner \supset \mathrm{E}!\sigma' x \ulcorner Fx \urcorner$

and its σ-counterpart SA22.

There are no strictly unconditional guarantees of existence for the general operators: there have to be individuals satisfying the predicate in question. We write out the axioms fully:

SA23 $\exists x Fx \supset \exists x \forall y \ulcorner x < y \equiv \forall z \ulcorner Fz \supset z < y \urcorner \urcorner$

SA24 $\exists x Fx \supset \exists x \forall y \ulcorner y \circ x \equiv \exists z \ulcorner Fz \wedge y \circ z \urcorner \urcorner$ (GSP)

In view of SD8–9 these amount to '$\exists x Fx \supset \mathrm{E}!\sigma' x \ulcorner Fx \urcorner$' and '$\exists x Fx \supset \mathrm{E}!\sigma x \ulcorner Fx \urcorner$' respectively.

There is no need for a special axiom for products, since if something is part of all F-ers, then the product may be defined as the sum of all such common parts.[22] In fact, in this case it may equally well be defined as the least upper bound of all such common parts. The proof that these two definitions of product are equivalent uses the Strong Supplementation Principle and the existence of *binary* products to derive a contradiction from the assumption that the least upper bound of all common parts is *not* itself a common part. Since it therefore *is* a common part, it is then straightforward to show that this least upper bound is also the sum of all common parts.

The leeway between SA23 and SA24 may be made up by adding this

[22] Thus the third axiom of BREITKOPF 1978 is not independent.

strong overlapping principle to SA23:

SA25 $x \circ \sigma' z Fz \supset \exists z^{\ulcorner} Fz \wedge x \circ z^{\urcorner}$

With SA24, the *General Sum Principle* (GSP) we finally attain the full strength of classical extensional mereology.

1.5 Classical Extensional Mereology

It is an indication of the economy with which the classical theory may be presented that the axioms SA0–3 alone need be added to SA24 to secure the full system. The intermediate stages between minimal and classical extensional mereology are simply swallowed up. The strength of SA24 allows definitions to be unified, using sums. For reference, we present the system resulting from this simplification all together, retaining the numeration from the previous section, even for definitions which look different but are (in this system) equivalent to those previously given.

System SC *of Classical Mereology: no sets;* '≪' *primitive;* '=' *assumed*

Definitions

SD1 $x < y \equiv x \ll y \vee x = y$

SD2 $x \circ y \equiv \exists z^{\ulcorner} z < x \wedge z < y^{\urcorner}$

SD3 $x \wr y \equiv \sim x \circ y$

SD9 $\sigma x^{\ulcorner} Fx^{\urcorner} \approx \iota x \forall y^{\ulcorner} x \circ y \equiv \exists z^{\ulcorner} Fz \wedge z \circ y^{\urcorner\urcorner}$

SD4 $x \cdot y \approx \sigma z^{\ulcorner} z < x \wedge z < y^{\urcorner}$

SD7 $x + y \approx \sigma z^{\ulcorner} z < x \vee z < y^{\urcorner}$

SD10 $\pi x^{\ulcorner} Fx^{\urcorner} \approx \sigma x^{\ulcorner} \forall y^{\ulcorner} Fy \supset x < y^{\urcorner\urcorner}$

SD11 $x - y \approx \sigma z^{\ulcorner} z < x \wedge z \wr y^{\urcorner}$

SD12 $\mathrm{U} \approx \sigma x^{\ulcorner} x = x^{\urcorner}$

SD13 $\overline{x} \approx \mathrm{U} - x$

Axioms

SA0 Any axiom set sufficient for first-order predicate calculus with identity.

SA1 $x \ll y \supset \sim y \ll x$

SA2 $x \ll y \wedge y \ll z \supset x \ll z$

SA3 $x \ll y \supset \exists z^{\ulcorner} z \ll y \wedge z \wr x^{\urcorner}$

SAS24 $\exists x Fx \supset \exists x \forall y^{\ulcorner} y \circ x \equiv \exists z^{\ulcorner} Fz \wedge y \circ z^{\urcorner\urcorner}$

The slight difference in nomenclature in the final axiom picks up a point which in fact affects all of SA17–25, namely that these are strictly speaking not axioms but axiom *schemata*, in view of the presence of the unbound predicate variable 'F' in all of them. It is by this means that calculi of individuals may be axiomatized as first-order theories. The price is that they are not finitely axiomatized thus; SAS24 then stands for an infinite *bundle* of axioms of the same general form.

Apart from the other axioms mentioned in the previous section, a large number of intuitively fairly straightforward theorems can be derived from these axioms and definitions. The following list is inspired by a similar one to be found in Breitkopf.[23]

Theorems

Proper Part

SCT1 $\sim x \ll x$

Part

SCT2 $x < x$

SCT3 $x = y \equiv x < y \wedge y < x$

SCT4 $x < y \wedge y < z \supset x < z$

SCT5 $x < y \wedge y \ll z \supset x \ll z$

SCT6 $x \ll y \wedge y < z \supset x \ll z$

SCT7 $x = y \equiv \forall z \ulcorner z < x \equiv z < y \urcorner$

SCT8 $x = y \equiv \forall z \ulcorner x < z \equiv y < z \urcorner$

Overlap

SCT9 $x \circ x$

SCT10 $x \circ y \supset y \circ x$

SCT11 $x < y \supset x \circ y$

SCT12 $x < y \wedge x \circ z \supset y \circ z$

SCT13 $x < y \equiv \forall z \ulcorner z < x \supset z \circ y \urcorner$

SCT14 $\forall z \ulcorner z < x \supset z \circ y \urcorner \equiv \forall z \ulcorner z \circ x \supset z \circ y \urcorner$

SCT15 $x < y \equiv \forall z \ulcorner z \circ x \supset z \circ y \urcorner$

SCT16 $x = y \equiv \forall z \ulcorner z \circ x \equiv z \circ y \urcorner$

[23] BREITKOPF 1978: 231ff.

Disjoint

SCT17 $\sim x \lfloor x$

SCT18 $x \lfloor y \supset y \lfloor x$

SCT19 $x < y \wedge y \lfloor z \supset x \lfloor z$

SCT20 $x < y \equiv \forall z \ulcorner z \lfloor y \supset z \lfloor x \urcorner$

SCT21 $x < y \equiv \forall z \ulcorner z \lfloor y \supset \sim z < x \urcorner$

SCT22 $x = y \equiv \forall z \ulcorner z \lfloor y \equiv z \lfloor x \urcorner$

Product

SCT23 $x = x \cdot x$

SCT24 $x \circ y \supset x \cdot y = y \cdot x$

SCT25 $x \circ y \supset x \cdot y < x \wedge x \cdot y < y$

SCT26 $x \circ y \wedge z < x \cdot y \supset z < x \wedge z < y$

SCT27 $x < y \supset x \cdot y = x$

SCT28 $x \circ y \wedge x = x \cdot y \supset x < y$

SCT29 $x \mathrel{\text{ǂ}} y \wedge x \circ y \supset x \cdot y \ll x \vee x \cdot y \ll y$

SCT30 $\exists w \ulcorner w < x \wedge w < y \wedge w < z \urcorner \supset x \cdot (y \cdot z) = (x \cdot y) \cdot z$

Sum

SCT31 $x = x + x$

SCT32 $x + y = y + x$

SCT33 $x + (y + z) = (x + y) + z$

SCT34 $x < x + y$

SCT35 $y < x + y$

SCT36 $x + y < z \equiv x < z \wedge y < z$

SCT37 $x \lfloor y \supset x \ll x + y \wedge y \ll x + y$

SCT38 $\sim x < y \supset y \ll x + y$

SCT39 $x \mathrel{\text{ǂ}} y \equiv x \ll x + y \vee y \ll x + y$

SCT40 $x = x + y \equiv y < x$

SCT41 $x < y + z \wedge x \lfloor z \supset x < y$

SCT42 $x \ll y \wedge y \lfloor z \supset x + z \ll y + z$

Difference

SCT43 $x \circ y \wedge \sim x < y \supset x - y \ll x$

SCT44 $x \ll y \supset y - x \ll y$

SCT45 $x \lfloor y \supset y - x = y$

SCT46 $\sim y < x \wedge y - x = y \supset x \lfloor y$

SCT47 $x \lfloor y \supset (x + y) - x = y$

SCT48 $x \ll y \supset x + (y - x) = y$

SCT49 $x \circ y \wedge \sim x < y \supset x - x \cdot y = x - y$

SCT50 $\sim x < z \supset . x < y \supset x - z < y - z$

Universe

SCT51 $\mathrm{U} < x \supset x = \mathrm{U}$

Complement

SCT52 $\mathrm{E!}\overline{x} \equiv x \neq \mathrm{U}$

SCT53 $x \neq \mathrm{U} \supset x \lfloor \overline{x}$

SCT54 $x \neq \mathrm{U} \supset x + \overline{x} = \mathrm{U}$

SCT55 $y \neq \mathrm{U} \supset . x < \overline{y} \equiv x \lfloor y$

SCT56 $x \neq \mathrm{U} \supset x = \overline{\overline{x}}$

SCT57 $y \neq \mathrm{U} \wedge x \circ y \wedge \sim x < y \supset x = (x \cdot y + x \cdot \overline{y})$

General Product

SCT58 $x = \pi z \ulcorner z = x \urcorner$

SCT59 $x = \pi z \ulcorner x < z \urcorner$

SCT60 $\exists y \ulcorner x \ll y \urcorner \supset x = \pi z \ulcorner x \ll z \urcorner$

SCT61 $\exists x \forall y \ulcorner Fy \supset x < y \urcorner \supset \pi x \ulcorner Fx \urcorner = \iota x \forall y \ulcorner x < y \equiv \forall z \ulcorner Fz \supset y < z \urcorner \urcorner$

General Sum

SCT62 $x = \sigma z \ulcorner z = x \urcorner$

SCT63 $x = \sigma z \ulcorner z < x \urcorner$

SCT64 $\exists y \ulcorner y \ll x \urcorner \supset x = \sigma z \ulcorner z \ll x \urcorner$

SCT65 $\exists x \ulcorner Fx \urcorner \supset \sigma x \ulcorner Fx \urcorner = \iota x \forall y \ulcorner x < y \equiv \forall z \ulcorner Fz \supset z < y \urcorner \urcorner$

SCT66 $\exists x \ulcorner Fx \urcorner \supset \sigma x \ulcorner Fx \urcorner = \iota x \ulcorner \forall y \ulcorner Fy \supset y < x \urcorner \wedge \forall y \ulcorner y \circ x \equiv \exists z \ulcorner Fz \wedge y \circ z \urcorner \urcorner \urcorner$[24]

[24] In TARSKI 1937: 161, and TARSKI 1956*b*: 25, the sum is defined (though using sets) in a way which in this context is tantamount to

$$\sigma'' x \ulcorner Fx \urcorner \approx \iota x \ulcorner \forall y \ulcorner Fy \supset y < x \urcorner \wedge \forall y \ulcorner y < x \supset \exists z \ulcorner Fz \wedge z \circ y \urcorner \urcorner \urcorner$$

which is a cross between σ and σ': SCT66 shows that this definition is here equivalent to σ.

SCT67 $x \circ \sigma y\ulcorner Fy\urcorner \equiv \exists z\ulcorner Fz \wedge z \circ x\urcorner$

SCT68 $x \lfloor \sigma y\ulcorner Fy\urcorner \supset \forall z\ulcorner Fz \supset z \lfloor x\urcorner$

SCT69 $x \neq \mathrm{U} \supset \bar{x} = \sigma z\ulcorner z \lfloor x\urcorner$

SCT70 $\sigma x\ulcorner Fx\urcorner < \sigma y\ulcorner Gy\urcorner \supset \forall x\ulcorner Fx \supset \exists y\ulcorner Gy \wedge y \circ x\urcorner\urcorner$

It should be mentioned that the formulae containing a free '*F*' are strictly speaking not theorems but metatheorems or theorem schemata. The expression of a number of these results is slightly complicated by the need to ensure the existence of individuals satisfying a certain condition, for example in SCT24–6, SCT28–30, SCT53–7, SCT65–6, SCT69. These extra conditions are a nuisance, and are one of the main reasons for recommending the adoption of a free logic as logical basis, a step which is considered in more detail in the next chapter.

1.6 The Question of Atomism

The system SC, like other classical mereologies, does not pronounce one way or the other on the question whether or not there are atoms, and whether everything is composed of atoms. To recapitulate, an atom is here understood in the specific mereological sense as an individual which has no proper parts, or equivalently, as one whose only part is improper, i.e. itself. The simplest model of a mereology in which there are atoms is the one-element trivial model; any finite model of SC is perforce atomistic, and has cardinality $2^n - 1$, where n is the number of atoms. A model of atomless mereology is provided by the regular open sets of a Euclidean space, the part-relation being set-inclusion confined to these sets.[25] The disjoint union of such a model with an atomistic one yields a model which is neither atomistic nor atomless: there are some atoms, but not everything is composed of them.

There are a number of equivalent ways of defining an atom: we choose

SD14 At $x \equiv \sim \exists z\ulcorner z \ll x\urcorner$[26]

Theses of atomicity, atomlessness and the curious hybrid between

[25] Cf. TARSKI 1956c: 341, n. 2., also SIKORSKI 1969: 66. A regular open set is the interior of a closed set.

[26] TARSKI 1956c: 334 mentions seven different possible definitions of 'atom', and SOBOCIŃSKI 1971*a* proves the equivalence of their respective transpositions into Leśniewskian terms. There are no doubt many more equivalent definitions.

them may then be formulated as follows:

SF3 $\forall x \exists y \ulcorner \mathrm{At}\, y \wedge y < x \urcorner$

SF4 $\forall x \exists y \ulcorner y \ll x \urcorner$

SF5 $\exists x \ulcorner \mathrm{At}\, x \urcorner \wedge \exists x \forall y \ulcorner y < x \supset \exists z \ulcorner z \ll y \urcorner \urcorner$

These three formulae may be added on to SC, but only one at a time, for they are mutually incompatible.

It is fair to say that the hybrid position resulting from adding SF5 to a mereology has rarely been seriously entertained. Since the issue of atomism has been one which has been aired predominantly in metaphysics and the philosophy of nature, the compulsive tendency to a uniform picture of the world has left it out in the cold, where we too shall leave it.

The issues involved in the question whether the atomistic or the atomless mereology is in some sense the 'correct' one in application to the physical world are among the deepest that mereology has to face, and, despite some work which has been done on the alternative axiomatic bases of atomistic and atomless mereology (which will be reviewed in Chapter 2), the question has not yet been squarely faced among mereologists. The opposition comes out most clearly in the consideration of putative physical continua such as spatial extents, temporal stretches, or spatio-temporal regions. The normal mathematical tool with which such continua are worked is some kind of set theory, where the approach is essentially atomistic, from points to sets of points. This mode of working is now so deeply ingrained in pure and applied mathematics that it would take a revolution to replace it. On the other hand, no one has ever perceived a point, or will ever do so, whereas people have perceived individuals of finite extent. So the natural *philosophical* approach is to treat points and other boundaries as in some sense ideal abstractions or limits arrived at by approximation from individuals alike in kind with those which are experienced. In dealing with continua, this enjoins an atomless mereology: points, lines, etc. are not *parts* of the concrete individuals (solids, 'events', or whatever) in which they lie. This way of thinking goes back to Aristotle, but has found such notable modern representatives as Russell, Whitehead, de Laguna, and Nicod.[27] Leibniz, as he often does, faces both ways: the phenomenal sphere is atomless, but the monads are 'the true atoms of nature'.[28]

[27] RUSSELL 1914, WHITEHEAD 1919, 1920, 1929, DE LAGUNA 1922, NICOD 1970*b*.
[28] *Monadology*, §3.

The opposition between these two views of continua does not bear directly on mereology, since it comprises a geometric or at least a topological component, which introduces considerations essentially external to mereology. However, it appears very natural to base the second, atomless approach on mereology rather than set theory, and this means that many of the concepts involved are partly or wholly mereological in nature. While the introduction of topological concepts constitutes in many ways the natural 'next step' after mereology, and indeed it is possible to take as primitive a concept from which both mereological and topological concepts are derived,[29] the details and implications of this approach lie outside the scope of the present study. The question of what is meant by an ideal limit or abstraction also introduces considerations which would take us too far afield. The interesting problems raised by the atomless approach are nevertheless undoubtedly a fertile field for study. We shall confine our remarks in this chapter to indicating the impact of the adoption of atomism or atomlessness on mereology itself. Some more considerations will be found also in the survey in the next chapter.

Adopting atomism as a thesis enables mereology to be given a simple axiomatic basis. This will be shown in greater detail in Chapter 2 in connection with systems developed by Eberle; but the main points may be mentioned here. In particular, an individual is then part of another iff all the atoms of the first are atoms of the second (where an atom *of* an individual is simply an atom which is *part* of that individual). This may be expressed as

SF6 $\forall z \ulcorner \mathrm{At}\, z \wedge z < x \supset z < y \urcorner \supset x < y$

and this takes the place of the Strong Supplementation Principle in an axiomatic treatment. Similarly the General Sum Principle may be given the simple form

SF7 $\exists x \ulcorner Fx \urcorner \supset \exists x \forall y \ulcorner \mathrm{At}\, y \supset . y < x \equiv \exists z \ulcorner Fz \wedge y < z \urcorner \urcorner$

Together with the axioms for '<' being a partial order, these two principles suffice to axiomatize atomistic mereology. We then get a particularly straightforward identity condition for individuals: they are identical iff they have the same atomic parts:

SF8 $x = y \equiv \forall z \ulcorner \mathrm{At}\, z \supset . z < x \equiv z < y \urcorner$

The simplicity of the resulting system is comparable to the simplicity

[29] This is done in the work of CLARKE 1981, which we discuss briefly in the next chapter.

of atomic Boolean algebras, so that, by a modification of the well-known representation theorem for atomic Boolean algebras,[30] every atomistic mereology (i.e. every model of these axioms) is isomorphic to the field of subsets of the set of its atoms, excluding the empty set.

In an atomless mereology, the identity condition SF8 is totally ineffective, since the right-hand side is vacuously satisfied. If, however, we can find a predicate '$F\xi$' satisfying

SF9 $\forall x \exists y \ulcorner Fy \wedge y < x \urcorner$

SF10 $\forall z \ulcorner Fz \supset . z < x \equiv z < y \urcorner \supset x = y$

then 'F' performs a role like that of 'At' in atomistic systems. We may call it a *basic* predicate, and the objects falling under it a *base* for the system. For example, in the atomless system consisting of all open regular subsets of the real numbers, the open intervals with rational end points form a base. Note that here the base is countable, whereas the open regular sets are not. An atomless system may have more than one base—for example, the open regular sets in the Euclidean plane may have as base the open discs with rational centres and radii, or the open squares with rational corners, and many more. Of course a trivial base may always be obtained by taking the predicate '$\xi = \xi$', but this is not usually very useful. However unless a definite application is in view, there is little that can be said *in concreto* about likely bases which are more useful. The idea of a base can however be relativized to things satisfying a certain predicate: we can then say that the F-ers *form a base for the* G-ers when the following conditions are fulfilled:

SF11 $\forall x \ulcorner Gx \supset \exists y \ulcorner Fy \wedge y < x \urcorner \urcorner$

SF12 $\forall xy \ulcorner Gx \wedge Gy \supset . \forall z \ulcorner Fz \supset . z < x \equiv z < y \urcorner \supset x = y \urcorner$

The interest of this idea is that it may be applied even if atomism is accepted, and may be a far handier criterion of identity than the universally available atomic one, SF8.[31] We do seem to apply such criteria approximately in everyday life. To a first approximation only,

[30] STONE 1936. Cf. SIKORSKI 1969: 23, 28.

[31] Cf. the similar attempts by YOES 1967, EBERLE 1968, 1969, and SCHULDENFREI 1969 to formulate a notion of atomless systems fulfilling a role similar to that of *atom* in giving the 'content' of an individual. These efforts have the disadvantage that they bring in set-theoretic notions extrinsic to part–whole theory, whereas the notion of a basic predicate is defined purely in part–whole terms. Similar remarks apply to the Leśniewskian equivalent defined in the next chapter. Both the term 'base' and the general idea are borrowed from topology. It can easily be shown, given some machinery for quantifying over subsets or submanifolds of bases, that every individual is the sum of elements of a base, which is just the topological definition.

and avoiding for the moment questions of the identity of things in flux (i.e. things which lose and gain parts), we might say that, *at any one time*, manufacturers components form the base for machines, while cells form a base for organisms. One machine can be part of another (as a carburettor is part of an automobile), but different machines differ by at least a component, as different organisms differ by at least a cell.

1.7 Summary

We have introduced the main concepts of extensional mereology in their predicate-logical and Leśniewskian guises, and indicated how they may be axiomatized in a system of predicate logic. The strongest resulting system SC is representative of systems which we call *classical* extensional mereologies. Such systems may be supplemented by theses of atomicity or atomlessness, but there appear at present to be no conclusive reasons for preferring one of these extensions at the expense of the other. Anticipating later critical remarks, we have hinted that certain of the principles embodied in classical extensional mereology may be philosophically suspect, and in order to demonstrate the (usually hidden) strength of the classical assumptions, we fanned out the principles which may be extracted so that we obtained a lattice of progressively stronger systems on the way to SC. In this way the source of philosophical suspicion may be more easily pin-pointed. Principles to watch out for in later chapters are in particular the *Supplementation* Principles, the closely associated *Proper Parts* Principle, and the *Sum* Principles. If the criticisms of any or all of these turn out to be justified, then a universally applicable mereology will have to be non-classical in at least one respect.

The material in this chapter suffices to enable the reader to follow the main argument from Chapter 3 onwards. The ensuing chapter is a survey whose detail is less relevant to the main argument, and so may be skipped over, though it should be noted that §9.3 of the final chapter presupposes material from §§2.9–10.

2 Survey of Extensional Mereology

2.1 Introduction

As far as its purely mereological content goes, this chapter consists predominantly of variations on the theme introduced in Chapter 1. It may therefore be left out if the reader is not interested in the details presented. Since the original formulation of a mereology by Leśniewski in 1916, a number of writers, working sometimes independently of one another, have formulated and reformulated principles which are clearly mereological in nature and which all conform to the principles laid down in the previous chapter for a minimal extensional mereology. Much of this work is scattered and some of it is difficult to obtain. Much of it has been developed less for its own sake than for some extraneous purpose. We attempt here to summarize the major contributions without aspiring to universal coverage. Since the two major lines of approach, those initiated by Leśniewski and Leonard respectively, differ considerably in their logical base, we also show in what respect their mereologies are equivalent. We finally survey several extensional mereologies which fall short of the classical in their strength, which prepares the way for later discussion on sum principles.

2.2 Presentation of Formal Systems

In the following portrayals of diverse formal systems, fixed conventions are observed regarding symbolization and formalization. All axioms are presented schematically using metavariables, dispensing with a rule of substitution. Definition are expressed in the metalanguage in the form of equivalences or identities, their status as definitions being marked by the use of a 'D' in their nomenclature. Quotation and quasi-quotation devices are dispensed with as far as practicable. The term 'parameter' is used for free variables, and 'variable' is restricted to bound variables. Where these are distinguished in a system, distinct runs of letters are used for them.

2.2.1 Metavariables

We have the following runs of metavariables, where we understand that any number may be constructed in each run using subscripting.

Singular terms: $s, t, u, s_1, \ldots$
Singular variables: $x, y, z, w, x_1, \ldots$
Leśniewskian nominal variables: $a, b, c, d, e, a_1, \ldots$
Predicate parameters: one-place: $F, G, H, F_1, \ldots$
two-place: $P, Q, R, P_1, \ldots$
Class parameters: $\alpha, \beta, \gamma, \alpha_1, \ldots$
Formula parameters: $A, B, C, A_1, \ldots$

2.2.2 Truth-Functional Connectives

We use the symbols $\sim \wedge \vee \supset \equiv$ for negation, conjunction, disjunction, material implication, and material equivalence respectively. To minimize the use of parentheses to mark grouping, these symbols will be assumed to bind their arguments in decreasing order of strength as listed, so that negation binds the strongest and equivalence the weakest. Where the same connective occurs twice or more within a formula, parentheses may be omitted on the understanding that they be restored by association to the left. This convention may be overridden by that of placing a dot after a connective, according to the stipulations of Church.[1] So the formula $A \supset B \supset C$ abbreviates $((A \supset B) \supset C)$, and the formula $A \supset .B \supset C$ abbreviates $(A \supset (B \supset C))$. The use of a dot to replace a left parenthesis is occasionally useful in other cases: for example, instead of $A \supset (B \equiv C)$ we may write $A \supset .B \equiv C$. Outermost parentheses are usually omitted.

2.2.3 Quantifiers and Operators

We let the symbols $\forall\ \exists\ \iota$ represent the universal quantifier, existential quantifier, and definite description operator respectively. However, in Leśniewskian systems the quantifiers do not, as in normal predicate logic and in free logic, carry existential import. We accordingly mark quantifiers without import by using the symbols $\Pi\Sigma$ for the universal and particular quantifiers respectively. We follow Leśniewski in marking the scope of variable-binding operators and quantifiers by a special kind of bracket: we shall use upper corners $\ulcorner\ \ \urcorner$. Where scope is obvious, however, corners will be omitted. Any number of variables may follow a quantifier. In universally quantified formulas, the outermost universal quantifier will be conventionally omitted. This is to be understood only as an abbreviation: variables apparently

[1] Cf. CHURCH 1956: 75. A dot replaces a left parenthesis whose right mate is inserted immediately before the next right parenthesis to the right of the dot which has no left mate to the right of the dot, or else the right mate goes at the end of the formula.

unbound in such formulas are *not* parameters and their binding quantifier can always be restored. In orthodox predicate logic and in Leśniewskian systems there is really no need for a distinction between parameters and variables; Leśniewskian systems do not have parameters at all, and orthodox predicate logic can always replace theorems containing parameters by ones where the parameters are replaced by variables, all universally bound at the beginning. However, in systems of free logic, which we shall have occasion to use at several junctures, the difference between parameters and variables is crucial in that parameters, like definite descriptions, may lack denotation, so formulae containing parameters are in general not interderivable with those obtained by replacing the parameters by variables and quantifying universally.

Further constants will be given for each system or group of systems as we come to them.

2.3 Calculi of Individuals Without Sets

By a 'calculus of individuals' we understand a classical mereological theory formulated in the language of standard predicate logic, possibly using sets. The first Calculi of Individuals were developed by Leonard and Goodman[2] and, independently, by Tarski,[3] using sets. For his book *The Structure of Appearance*,[4] Goodman, who for nominalist reasons rejected sets, preferred a version using only the language of first-order predicate calculus, and it is this which we give first. The mereological constants all have the meaning which was assigned them in the previous chapter.

2.3.1 System CI: *no sets;* '$\circ$' *primitive;* '$=$' *defined*

Definitions

CID1 $x \rfloor y \equiv \sim x \circ y$

CID2 $x < y \equiv \forall z \ulcorner z \circ x \supset z \circ y \urcorner$

CID3 $x = y \equiv \forall z \ulcorner z \circ x \equiv z \circ y \urcorner$

CID4 $x \ll y \equiv x < y \wedge \sim y < x$

CID5 $\sigma x Fx \approx \iota z \forall y \ulcorner y \circ z \equiv \exists x \ulcorner Fx \wedge y \circ x \urcorner \urcorner$

[2] LEONARD and GOODMAN 1940. The first version appeared in LEONARD 1930 (cf. GOODMAN 1977: 30 n.) but whether independently of Leśniewski is not said.
[3] TARSKI 1937: 1956*b*. [4] GOODMAN 1977.

We have used the sign '$\approx$' in the definition CID5 because, depending upon the theory of descriptions which is employed, a singular term defined using a description may either be empty, or may denote a selected individual, or may be constructed as a pseudo-term to be eliminated by the expansion of the description. We wish here to have these options open. Using the same convention we may define binary sum and product:

CID6 $x \cdot y \approx \sigma z \ulcorner z < x \wedge z < y \urcorner$

CID7 $x + y \approx \sigma z \ulcorner z < x \vee z < y \urcorner$

Axioms

CIA0 Any set sufficient for first-order predicate calculus *without* identity.

We are for the moment postponing a decision on what sort of theory of description we are to add. Leonard and Goodman use Russell's theory while Breitkopf[5] uses a Fregean theory. We shall eventually settle on a free description theory as the most useful, but only after discussing the use of free logic in the formulation of calculi of individuals.

CIA1 $x \circ y \equiv \exists z \forall w \ulcorner w \circ z \supset w \circ x \wedge w \circ y \urcorner$

CIA2 $\exists x Fx \supset \exists x \forall y \ulcorner y \circ x \equiv \exists z \ulcorner Fz \wedge y \circ z \urcorner \urcorner$

CIA3 $\forall z \ulcorner z \circ x \equiv z \circ y \urcorner \supset . A \supset A[y//x]$

where $A[y//x]$ is any formula obtained from A by replacing occurrences of x which are free in A by occurrences of y, taking care that no such occurrence becomes inadvertently bound by a quantifier in a well-formed part where y is bound, by changing the bound variable y to another, say alphabetically first which does not appear in A.

Axiom CIA3 is the usual Leibnizian axiom for identity, which is needed even though identity is defined by CID3. Axiom CIA1 says in effect that two individuals overlap iff they possess a binary product, while CIA2 ensures the existence of a sum or fusion of any non-empty monadic predicate. In a system where axioms rather than axiom schemata are used, and a rule of substitution for variables is given, CIA2 must still be expressed as a schema, since, again for nominalistic reasons, Goodman does not permit quantifiers binding predicate variables.

[5] BREITKOPF 1978.

Breitkopf lists forty-eight simple theorems derivable from these axioms, among them those showing that $=$ is an equivalence relation and $<$ a partial ordering. Important for our purposes are

CIT1 $x = y \equiv \forall z \ulcorner z \wr x \equiv z \wr y \urcorner$

CIT2 $x = y \equiv \forall z \ulcorner z < x \equiv z < y \urcorner$

CIT3 $x \circ y \equiv \exists z \ulcorner z < x \wedge z < y \urcorner$

which show that either $\wr$ or $<$ can be taken as primitive rather than $\circ$, and can yield identity conditions for individuals. Further identities define the universe, the complement of an individual, the difference of two individuals, and the product of the individuals satisfying a given predicate. This is very similar to what was done in SC.

CID8 $\mathrm{U} \approx \sigma x \ulcorner x = x \urcorner$

CID9 $\bar{x} \approx \sigma z \ulcorner z \wr x \urcorner$

CID10 $x - y \approx x \cdot \bar{y}$

CID11 $\pi x F x \approx \sigma x \forall y \ulcorner F y \supset x < y \urcorner$

It is not difficult to show that CI is equivalent to SC, so the general remarks we made above in connection with the latter apply here.

The assumption of atomism allows a simplification of the axioms of calculi of individuals. To see this we consider systems with '$<$' primitive. A number of such calculi have been investigated by Eberle,[6] who has proved their completeness with respect to certain algebraic structures with '$<$' interpreted as a partial ordering to which further conditions are added. The following systems are modelled on those of Eberle, and are named accordingly.

2.3.2 System E: *no sets; '$<$' primitive; '$=$' defined*

Definitions

ED1 $x \circ y \equiv \exists z \ulcorner z < x \wedge z < y \urcorner$

ED2 $x = y \equiv x < y \wedge y < x$

together with CID1, CID4, CID15.

Axioms

EA0 as CIA0

EA1 $x < y \equiv \forall z \ulcorner z \circ x \supset z \circ y \urcorner$

[6] EBERLE 1967, 1970: 50 ff.

EA2 as CIA2

EA3 as CIA3.

This system can be shown equivalent with CI, and so is neutral on atomism. However, Eberle builds a series of atomistic systems also based on '$<$'. The strongest of these is equivalent to E (or CI) + CIF1, but if we presuppose atomism from the start, we can build up to it from a weaker system as follows.

2.3.3 System AE: *atomistic, no sets;* '$<$' *primitive;* '$=$' *defined*

Definitions as system E.

Axioms

AEA0 as CIA0

AEA1 $x < x$

AEA2 $x < y \wedge y < z \supset x < z$

AEA3 $\forall z \ulcorner \mathrm{At}z \wedge z < x \supset z < y \urcorner \supset x < y$

AEA4 as CIA3.

These axioms, together with ED2, tell us that $<$ is a partial ordering, and that if all atoms of x are atoms of y then x is part of y (the converse of AEA3 follows easily from AEA2).

System AE, unlike those going before, does not guarantee the existence of sums and products. It can be extended to do this. If we add as an axiom

AEF1 $\exists z \ulcorner z < x \wedge z < y \urcorner \supset \exists w \forall z \ulcorner \mathrm{At}z \supset . z < w \equiv z < x \wedge z < y \urcorner$

which tells us that, in effect, any two overlapping individuals have a product, we guarantee binary (and thence finite) products. In the presence of the other axioms and axioms for handling descriptions, AEF1 is equivalent to

AEF2 $\exists z \ulcorner z < x \wedge z < y \urcorner \supset \mathrm{E!}x \cdot y$

Sum-closure is assured by the addition of the formula

AEF3 $\exists w \forall z \ulcorner \mathrm{At}z \supset . z < w \equiv z < x \vee z < y \urcorner$

which again, under suitable axioms for handling descriptions, is equivalent to

AEF4 $\mathrm{E!}x + y$

Eberle presents three systems based on AE; the first adds AEF1, the second adds AEF3, and the third adds both. Eberle obtains completeness with respect to his models by adding suitable closure conditions to the algebraic structures. Systems including the sum-closure axiom AEF3 ensure the existence of finite sums. The general existence of sums (finite or infinite) can be ensured by adjoining as axiom the formula

AEF5 $\exists xFx \supset \exists x\forall y^{\ulcorner}Aty \supset . y < x \equiv \exists z^{\ulcorner}Fz \wedge y < z^{\urcorner\urcorner}$

which we may call, following Eberle, the strong sum-closure condition, since once again, given axioms for descriptions, this condition is equivalent to

AEF6 $\exists xFx \supset E!\sigma xFx$

The weaker closure conditions follow at once from AEF5, which is essentially stronger: it is independent of AE + AEF1 + AEF3, as can be seen by the model consisting of all square-free natural numbers, ordered by 'divides'. The atoms are the primes, and the axioms of AE are satisfied, as are AEF1 and AEF3, which correspond to the existence of highest common factors and lowest common multiples respectively for pairs of numbers. But AEF5 is not satisfied, since there are monadic predicates satisfied in the set with no sum in the set, e.g. 'is prime' or 'is even'. So it is AE + AEF5 which is equivalent to CI plus an axiom of atomism (cf. SF3), the atomistic version of Goodman's calculus.

Although Eberle's systems decide the question of atomism in a way which Goodman's do not, Eberle is aware both of the substantiative nature of this assumption[7] and of the hidden strength of the classical axioms.

We next turn to systems which employ sets.

2.4 Calculi of Individuals with Sets

The first calculus of individuals to be presented under this name, by Leonard and Goodman, uses sets of individuals rather than predicates of individuals. Sets of sets are not needed. In place of the sum and product operators σ, π, there are binary predicates, which we write 'Su' and 'Pr' respectively,[8] whose first arguments are individuals and whose second arguments are sets. The system presupposes as its logic

[7] Cf. the brief discussion in §§2.4 and 2.9 of EBERLE 1970.

[8] We conventionally write a functor or predicate with a capital letter when the resulting functional expression or predicate satisfies a uniqueness condition. Our reasons for avoiding terminology reminiscent of classes and class algebra emerge below.

that part of *Principia Mathematica* necessary to deal with individuals, sets of individuals, and predicates of these, including identity.

2.4.1 System LG: *sets; '$\wr$' primitive; '=' assumed*

Definitions

LGD1 $x < y \equiv \forall z \ulcorner z \wr y \supset z \wr x \urcorner$

LGD2 $x \circ y \equiv \exists z \ulcorner z < x \wedge z < y \urcorner$

LGD3 $x \operatorname{Su} \alpha \equiv \forall y \ulcorner y \wr x \equiv \forall z \ulcorner z \in \alpha \supset y \wr z \urcorner\urcorner$

LGD4 $x \operatorname{Pr} \alpha \equiv \forall y \ulcorner y < x \equiv \forall z \ulcorner z \in \alpha \supset y < z \urcorner\urcorner$

LGD5 as CID4

Axioms

LGA0 Any set (such as PM) sufficient for the logic of predicates (including identity) and sets.

LGA1 $\exists x \ulcorner x \in \alpha \urcorner \supset \exists x \ulcorner x \operatorname{Su} \alpha \urcorner$

LGA2 $x < y \wedge y < x \supset x = y$

LGA3 $x \circ y \equiv \sim x \wr y$

In these terms the following Boolean concepts may be defined, where '$\{x, y\}$' designates a pair set, and '$\{x: Fx\}$' the set of individuals which are F:

LGD6 $\mathrm{U} = \operatorname{Su}`\{x: x = x\}$

LGD7 $x + y = \operatorname{Su}`\{x, y\}$

LGD8 $x \cdot y \approx \operatorname{Pr}`\{x, y\}$

LGD9 $\overline{x} \approx \operatorname{Su}`\{y: y \wr x\}$

Leonard and Goodman's work was anticipated by Tarski, who took Leśniewski's mereology as his starting point, but preferred not to use Leśniewski's underlying logic. One of Tarski's systems is atomistic, the other leaves the atomistic question open. As with our exposition of Leonard and Goodman, we have altered the notation slightly to allow ready comparison with other systems.

2.4.2 System T: *sets; '<' primitive; '=' assumed*

Definitions

TD1 $x \wr y \equiv \sim \exists z \ulcorner z < x \wedge z < y \urcorner$

TD2 $x \operatorname{Su} \alpha \equiv \forall y \ulcorner y \in \alpha \supset y < x \urcorner \wedge \sim \exists y \ulcorner y < x \wedge \forall z \ulcorner z \in \alpha \supset z \wr y \urcorner\urcorner$

Axioms

TA0 as LGA0

TA1 $x < y \wedge y < z \supset x < z$

TA2 $\exists x \ulcorner x \in \alpha \urcorner \supset \exists x \ulcorner x \,\mathrm{Su}\, \alpha \wedge \forall y \ulcorner y \,\mathrm{Su}\, \alpha \supset x = y \urcorner \urcorner$

This tells us that '$<$' is transitive, and that every non-empty set has a sum. This is a very compact axiom set, and the beginning of its development is somewhat involved. The second axiom can be replaced by two others, as in System AT below, but in either case, the system can be proved equivalent to that of Leonard and Goodman.

2.4.3 System AT: *atomistic, sets; '$<$' primitive, '$=$' assumed.*

For this system we write '$\{x\}$' for the singleton set whose only member is x.

Definitions

ATD1 $x \,\mathrm{Su}\, \alpha \equiv \forall y \ulcorner y \in \alpha \supset y < x \urcorner \wedge \forall y \ulcorner y < x \supset \exists zw \ulcorner z \in \alpha \wedge w < z \wedge w < y \urcorner \urcorner$[9]

ATD2 as CID15

Axioms

ATA0 as LGA0

ATA1 as TA1

ATA2 $y \,\mathrm{Su}\, \{x\} \supset x = y$

ATA3 as LGA1

ATA4 $\exists y \ulcorner \mathrm{At}\, y \wedge y < x \urcorner$

The system obtained by omitting the atomistic axiom ATA4 is equivalent to system T, and hence to LG.

The notion of a base can be replaced by that of a *base set*. We say that α forms a base set for β when the following conditions obtain:

B1 $\forall x \ulcorner x \in \beta \supset \exists y \ulcorner y \in \alpha \wedge y < x \urcorner \urcorner$

B2 $\forall xy \ulcorner x \in \beta \wedge y \in \beta \supset \forall z \ulcorner z \in \alpha \supset . z < x \equiv z < y \urcorner \supset x = y \urcorner$

In an atomistic system the set $\{x\text{: At } x\}$ of atoms forms a base set for every set.

In terms of part–whole theory itself, the use or non-use of sets makes no substantial difference to expressive power. The sum axiom

[9] We here write out the expression in full, whereas Tarski uses compressed *Principia Mathematica* notation.

may be expressed as an axiom using sets, while with predicates a schema must be used if second-order quantification is to be avoided. The substantial issues of the existence of sums and of atomicity are also unaffected by the use of sets. They indeed play no very forward role in the calculus, and if we disregard the elided universal quantifiers preceding such axioms as LAG1 and TA2, treating the resulting formulae as schemata with free set variables, then we are only using what Quine calls virtual classes.[10] On the other hand, while the philosophical issues of the acceptability of sets are in themselves external to mereology, the historical and philosophical motivation behind extensional mereology is far from independent of the concept of a class or set, as we shall see.

2.5 A Free Calculus of Individuals

We have hitherto been deliberately vague on how to treat improper descriptions, though everything mentioned can be handled adequately by a Russellian theory in which descriptions are treated as improper symbols.[11] In calculi of individuals, improper descriptions, such as those for products of disjoint individuals, or for sums of unsatisfied predicates, are likely to occur frequently, so it is a help in streamlining the theory to have a satisfactory treatment of such descriptions, and therefore of the compound terms defined by means of description. There are in any case good general philosophical grounds for admitting empty terms into logic, and for admitting the empty domain as a possible domain of interpretation. These arguments are well publicized and need not be repeated here.[12] So it is worth seeing, at least for the calculi without sets, how a calculus of individuals may be based on a free logic. Free calculi with sets can be treated analogously, though to take the opportunities offered by free logic to look again at the normal assumptions of set theory[13] would involve considerations leading us too far afield.

There is no single free logic, but a range of systems upon which a calculus of individuals may be constructed. For our present purposes, a system is preferable which is as nearly extensional as possible; we are

[10] QUINE 1969*a*: §2.

[11] This is how LEONARD and GOODMAN 1940 treat descriptions. BREITKOPF 1978 prefers a Fregean theory.

[12] Cf., e.g., HINTIKKA 1969*b*, LAMBERT 1965, 1967, SCHOCK 1968.

[13] An excellent example of such a rethink is provided by BENCIVENGA 1976.

dealing with extensional part–whole theory, after all, and the choice should not be taken as endorsing the system chosen as suitable for other purposes—nor would it be taken to imply that, in a wider context, the system is philosophically unproblematic.[14]

The basic logic chosen for portraying free calculus of individuals is Lambert and Van Fraassen's FD_2.[15] Because our notation differs slightly from theirs, we describe the preliminaries in greater detail than hitherto.

System F: *no sets; '<' primitive; underlying logic for '≈' assumed*

Notation and Formation Rules

We use the notation outlined in §2.1, including among the metavariables those for singular terms, singular variables, predicate parameters, and formula parameters. Well-formed formulae (wffs) and singular terms are defined by simultaneous recursion as follows:

(1) Singular parameters (for which we have no separate metavariables) and singular variables are singular terms.
(2) If s, t are terms, then Fs, Pst, $s < t$ and $s \approx t$ are atomic wffs.[16]
(3) If A and B are wffs, then so are $\sim A$, $(A \wedge B)$, $(A \vee B)$, $(A \supset B)$, $(A \equiv B)$, $\forall x \ulcorner A \urcorner$ and $\exists x \ulcorner A \urcorner$.
(4) If A is a wff, then $\iota x \ulcorner A \urcorner$ is a term.
(5) Nothing is a wff or term except by virtue of clauses 1–4.

Conventions on omitting parentheses and scope corners are as in §2.1. In writing definitions, axiom and theorem schemas, the occurrence of singular term metavariables is to be understood so that if the metavariable denotes a singular parameter, or a definite description, that is to say, a term of the form $\iota x \ulcorner A \urcorner$, this occurs freely in the formula, whereas if it denotes a variable, this is understood to be bound initially by a universal quantifier, which we however conventionally admit. So, for example, the schema

$$\exists x \ulcorner x < s \urcorner$$

covers formulae of the form $\exists x \ulcorner x < s \urcorner$ where s is a parameter, $\exists x \ulcorner x < \iota y F y \urcorner$, and also $\forall y \exists x \ulcorner x < y \urcorner$, which we have hitherto abbrevi-

[14] For criticisms of previous free logics on this point, see BENCIVENGA 1980.

[15] VAN FRAASSEN and LAMBERT 1967. FD_2 has the advantage, as is shown in this paper, of being demonstrably sound and complete.

[16] We are using '≈' where Van Fraassen and Lambert use '='. We are not here concerned with predicates of more than two places.

ated as $\exists x \ulcorner x < y \urcorner$, and shall continue to do so unless we wish to stress the quantification.

If A is a wff, $A[s/t]$ denotes that formula obtained from A by substituting s for *all* occurrences of t, where A is to be rewritten if necessary as follows: if A contains a well-formed part of the form $\forall s \ulcorner B \urcorner$ or $\exists s \ulcorner B \urcorner$, in which t occurs free, then s is to be uniformly replaced by the alphabetically first variable not occurring in A. With the same precaution, we use '$A[s//t]$' as stated in §2.4, p. 49 above for cases where the substitution need not be uniform. If A is a formula containing distinct parameters $t_1, \ldots, t_n$, let $A[x_1 \ldots x_n/t_1 \ldots t_n]$ denote the result of replacing t_1 uniformly by x_1, t_2 by $x_2, \ldots, t_n$ by x_n, provided A is rewritten as necessary to avoid clashes, and $x_1, \ldots, x_n$ are all distinct variables.

Definitions

FD1 $E!s \equiv \exists x \ulcorner x \approx s \urcorner$

FD2 $s = t \equiv s \approx t \wedge E!s$

FD3 $s \ll t \equiv s < t \wedge \sim t < s$

FD4 $s \circ t \equiv \exists x \ulcorner x < s \wedge x < t \urcorner$

FD5 $s \wr t \equiv \sim s \circ t$

FD6 $\sigma x Fx \approx \iota z \forall y \ulcorner y \circ z \equiv \exists x \ulcorner Fx \wedge y \circ x \urcorner \urcorner$

FD7 $s \cdot t \equiv \sigma z \ulcorner z < s \wedge z < t \urcorner$

FD8 $s + t \approx \sigma z \ulcorner z < s \vee z < t \urcorner$

FD9 $\Lambda \approx \iota x \ulcorner \sim x \approx x \urcorner$

Rules

FR1 Modus ponens: If A and $A \supset B$ are theorems, B is a theorem.

FR2 If A is a theorem, and $t_1, \ldots, t_n$ occur in A, then $\forall x_1 \ldots x_n \ulcorner A[x_1 \ldots x_n/t_1 \ldots t_n] \urcorner$ is a theorem, where $x_1, \ldots, x_n$ are distinct and $t_1, \ldots, t_n$ are distinct.

Axioms

FA0 All formulae which are tautologies of propositional calculus.

FA1 $\forall x \ulcorner A \supset B \urcorner \supset . \forall x A \supset \forall x B$

FA2 $A \supset \forall x A$, where x is not free in A

FA3 $\forall x A \supset A[y/x]$, where x and y are distinct variables (note that this abbreviates $\forall y \ulcorner \forall x A \supset A[y/x] \urcorner$)

FA4 $s \approx t \supset . A \supset A[t//s]$

FA5 $\iota xA \approx s \equiv \forall y \ulcorner s \approx y \equiv A[y/x] \wedge \forall x \ulcorner A \supset y \approx x \urcorner \urcorner$, where x and y are distinct variables.

FA0–5 together with FR1–2 constitute a system differing inessentially from the system FD½ of Van Fraassen and Lambert. Among the more important theorems of this system are the following.

Theorems of the Underlying Logic

FT1 $s \approx \iota x \ulcorner x \approx s \urcorner$

FT2 $s \approx s$

FT3 $E!\, \iota xA \equiv \exists y \forall x \ulcorner A \equiv x \approx y \urcorner$

FT4 $E!\, \iota xA \supset A[\iota xA/x]$

FT5 $\sim E!\, \iota xA \supset \iota xA \approx \Lambda$

FT6 $\sim E!\, s \wedge \sim E!t \supset s \approx t$

FT7 $E!s \equiv \sim s \approx \Lambda$

Given our definition of '=' we also have

FT8 $x = x$

FT9 $s = t \supset E!t$

FT10 $E!s \equiv s = s$

Now for the specifically

Mereological Axioms

FA6 $s < t \supset E!s \wedge E!t$

FA7 $s < t \wedge t < u \supset s < u$

FA8 $s < t \wedge t < s \supset s \approx t$

FA9 $\forall z \ulcorner z \circ x \supset z \circ s \urcorner \supset x < s$

FA10 $\exists x Fx \supset \exists x \forall y \ulcorner y \circ x \equiv \exists z \ulcorner Fz \wedge y \circ z \urcorner \urcorner$

FA6 tells us that whatever is or has a part exists. The restriction of the first member of the consequent of FA9 to a variable preserves this, for since

FT11 $\sim x < \Lambda$

FT12 $\sim x \circ \Lambda$

were FA9 not so restricted, we should obtain

FF1 $\Lambda < s$

which would contradict FA6.

Because the negation of FF1 is a theorem, this free calculus of individuals does not algebraically resemble a Boolean algebra despite the presence of the term 'Λ'. And indeed, it seems absurd to imagine, in the case where anything more than a single atom exists, that everything could have—albeit in the 'improper' sense—a common part. Nor do we have to go any way towards giving into this assumption to obtain the algebraic symmetry of Boolean algebra: given the definitions

FD10 $s <\cdot t \equiv s < t \vee s \approx \Lambda$

FD11 $\mathrm{U} \approx \sigma x \ulcorner x \approx x \urcorner$

FD12 $\bar{s} \approx \sigma x \ulcorner x \wr s \urcorner$

we have as theorems

FT13 $s <\cdot s$

FT14 $s <\cdot t \wedge t <\cdot u \supset s <\cdot u$

FT15 $\Lambda <\cdot s$

FT16 $s <\cdot \mathrm{U}$

FT17 $s <\cdot t \cdot u \equiv s <\cdot t \wedge s <\cdot u$

FT18 $s + t <\cdot u \equiv s <\cdot u \wedge t <\cdot u$

FT19 $s \cdot (t + u) <\cdot (s \cdot t) + (s \cdot u)$

FT20 $s \cdot \bar{s} <\cdot \Lambda$

FT21 $\mathrm{U} <\cdot s + \bar{s}$[17]

which show that F, or a weaker system with U and closure under binary sum and product, is very close to being a Boolean algebra. The defined constant '$<\cdot$' is however *not* the part-relation, but an artificial one merely introduced here by convention for algebraic comparison. This symbolic fudging through '$<\cdot$' cannot hide the fact that F falls short of being a proper Boolean algebra, because we cannot assert the formula

FF2 $\exists x \forall y \ulcorner x <\cdot y \urcorner$

although we can assert FT15 and its quantified version

FT22 $\forall x \ulcorner \Lambda <\cdot x \urcorner$

which brings us almost there.

In this section we have considered only a free version of the full

[17] These correspond to the axioms of Boolean algebra taken from Schröder by LEJEWSKI 1960–1.

calculus, but it is clear that any of the weaker mereologies envisaged in §1.4 could be formulated in similar terms.

2.6 Leśniewski's Mereology

The earliest extensional mereology to be fully worked out was Leśniewski's Mereology. Leśniewski's intention was to use Mereology, along with its logical basis, to provide a new foundation for mathematics. For various reasons, this programme was never completed, and Leśniewski's extensive unpublished writings were destroyed in the Second World War. Mereology remained however for the scientific community to evaluate. Although Mereology was the first of Leśniewski's systems to be formulated, it presupposes two more basic theories, Protothetic and Ontology, which were developed in reverse order. The whole, consisting of Protothetic and Ontology, forms a logical system of considerable originality and power, developed with an attention to exactness without parallel before or since.

Protothetic ('first principles') is Leśniewski's counterpart to the propositional calculus, which it contains as a proper fragment. In addition it contains variables of each propositional type, and quantifiers. Allowing for the fact, mentioned in §1.3 above, that Leśniewskian systems are not fixed, but grow, Protothetic is equivalent to a system of propositional types such as those of Church or Henkin.[18] Since we do not need to employ anything more from Protothetic than the classical propositional calculus, we can refer the reader elsewhere for a fuller account.[19]

2.6.1 Ontology

Ontology is the logical system coming next after Protothetic in Leśniewski's hierarchy. In its general features it combines aspects of the logic of Schröder with a simple type theory of functions, although, because Leśniewski was nominalistic by inclination, there are strictly speaking no functions for him, only functors (the expressions which, in a Fregean logic, would denote functions.) Because of his understanding of quantification, Leśniewski feels free to quantify variables

[18] CHURCH 1940, HENKIN 1963.
[19] SŁUPECKI 1953, SOBOCIŃSKI 1960, MIEVILLE 1984: pt. 2.

of all categories without incurring ontological commitment. This has led Quine to represent Leśniewskian quantification as substitutional,[20] but it is clear that Leśniewski does not quantify *over* expressions, nor does he presuppose there must be infinitely many expressions,[21] which would surely be an implausibly strong assumption for a nominalist. As an alternative hypothesis, Küng and Canty have suggested that in quantifying after the fashion of Leśniewski, we quantify neither over expressions nor over their denotations but rather over their *extensions*.[22] While this position is more plausible than Quine's, it still lets in abstract entities, so to speak, by the back door.[23] It also seemingly employs entities which Leśniewski could not even grasp, namely sets. Küng's suggestion that we should treat quantifier-expressions as *prologues* to their matrices[24] is worth pursuing independently of the set-theoretic version of the Küng–Canty position, and my own suggestion, detailed elsewhere,[25] is to employ a *combinatorial semantics* based on the primitive idea of an (extensional) *way of meaning*, so that an expression of the form '$\Pi \ldots \ulcorner$——$\urcorner$' is true iff no matter in what way the variables following the 'Π' mean, the matrix is true, the ways of meaning of compound expressions being determined combinatorially from the ways of meaning of their parts. Be that as it may, the best *reading* for 'Π' and 'Σ' is simply 'for all' and 'for some' respectively.

We presented the basics of Ontology, in particular its use of plurally designating terms, in §1.3, so we can proceed straight to the

Base System L *of Ontology*

In this section the nominal variables *a*, *b*, *c*, etc. belong to the object language to conform with usual Leśniewskian practice. Conventions and abbreviations concerning bracketing will still be the same as in previous sections: in particular, apparently free variables are to be understood without exception as being Π-universally bound with the whole formula as scope. There are no free variables in theses of Leśniewski's logical systems. As was mentioned in §1.3, definitions are here object–language equivalences rather than metalinguistic abbreviations. This conforms to Leśniewski's treatment. Later, we shall nevertheless employ abbreviations (in fact 'Σ' is already one).

[20] QUINE 1969*b*: 63, 104 ff.
[21] As do DUNN and BELNAP 1968.
[22] KÜNG and CANTY 1970.
[23] Cf. SIMONS 1985*d*: §3.4.
[24] KÜNG 1974, 1977.
[25] SIMONS 1985*e*.

Leśniewskian definitions come in two basic kinds, which we call, following Luschei,[26] *propositive* and *nominative*, according to whether the semantic/syntactic category of the definiendum has an index in the familiar quotient notation beginning with 'S' or 'N' respectively. These definitions must conform to certain conditions; we do not need to consider these in complete generality, but give schemes for those we need as follows (where quantifiers have been included for clarity):

Propositive LDS1 $\Pi a_1 \ldots a_n \ulcorner M[a_1 \ldots a_n] \equiv A \urcorner$

Such a definition defines an n-place predicate M. The right-hand side contains only $a_1, \ldots a_n$ free, and otherwise only constants which have previously been defined.

Nominative LDS2 $\Pi a a_1 \ldots a_n \ulcorner a \varepsilon f(a_1 \ldots a_n) \equiv \Sigma b \ulcorner a \varepsilon b \wedge B \urcorner \urcorner$

Such a definition defines an n-place operator (name-forming functor) f. In the case $n = 0$, f itself is a defined name. The conjunct B contains only at most $a, b, a_1, \ldots, a_n$ free and otherwise only constants which have previously been defined.[27]

For many purposes simpler nominative definition frames are sufficient:

LDS3 $\Pi a a_1 \ldots a_n \ulcorner a \varepsilon f(a_1 \ldots a_n) \equiv a \varepsilon a \wedge B \urcorner$

with similar conditions on B. Any definition conforming to LDS3 can be replaced by one conforming to LDS2. A further possibility is

LDS4 $\Pi a a_1 \ldots a_n \ulcorner a \varepsilon f(a_1 \ldots a_n) \equiv a \varepsilon g(\ldots) \wedge B \urcorner$

where at most $a_1, \ldots, a_n$ occur free in $g(\ldots)$ (which may also be a name), and g must be previously defined; B is as before.

Definitions

The following definitions include all the constants mentioned in §1.3 and a few more besides: they conform to the schemata LDS1–4:

LD1	$\mathrm{E}a \equiv \Sigma b \ulcorner b \varepsilon a \urcorner$	Existence
LD2	$!a \equiv \Pi bc \ulcorner b \varepsilon a \wedge c \varepsilon a \supset b \varepsilon c \urcorner$	Uniqueness
LD3	$\mathrm{E}!a \equiv a \varepsilon a$	Singular existence
LD4	$a \simeq b \equiv \Pi c \ulcorner c \varepsilon a \equiv c \varepsilon b \urcorner$	Identity

[26] LUSCHEI 1962: 169. Propositive definitions are often called 'propositional' or 'Protothetical'; nominative definitions 'nominal' or 'Ontological'. Both pairs of terms are misleading.

[27] The extra clauses involving 'ε' in the various kinds of nominative definition serve to prevent paradoxes. Cf. HENRY 1972: 46.

LD5	$a = b \equiv a\varepsilon b \wedge b\varepsilon a$	Singular identity (with import)
LD6	$a \approx b \equiv !a \wedge a \simeq b$	Singular identity (without import)
LD7	$a \cong b \equiv \mathrm{E}a \wedge a \simeq b$	Identity with existential import.
LD8	$a \subset b \equiv \Pi c \ulcorner c\varepsilon a \supset c\varepsilon b \urcorner$	Inclusion
LD9	$a \sqsubset b \equiv \mathrm{E}a \wedge a \subset b$	Non-empty inclusion
LD10	$a\varepsilon \bigvee \equiv a\varepsilon a$	Constant universal name
LD11	$a\varepsilon \bigwedge \equiv a\varepsilon a \wedge \sim a\varepsilon a$	Constant empty name
LD12	$a\varepsilon b \cup c$	
	$\equiv a\varepsilon a \wedge . a\varepsilon b \vee a\varepsilon c$	Nominal union
LD13	$a\varepsilon b \cap c \equiv a\varepsilon b \wedge a\varepsilon c$	Nominal intersection
LD14	$a\varepsilon \mathcal{N} b \equiv a\varepsilon a \wedge \sim a\varepsilon b$	Nominal negation

All constants here defined which are equiform to ones from previous sections bear the same interpretation. This goes for E!, $=$, $\approx$, and $\bigwedge$. For alternative notations for some of these, see the table of equivalents at the end of the chapter. Although technically speaking, LD1–14 are all definitional *axioms*, we have separated them from the proper

Axioms of Ontology

LA0 Any set of axioms and rules sufficient for propositional calculus and the manipulation of quantifiers binding variables in any category.

The 1920 ('Long') Axiom of Ontology

LA1 $a\varepsilon b \equiv \Sigma c \ulcorner c\varepsilon a \urcorner \wedge \Pi c \ulcorner c\varepsilon a \supset c\varepsilon b \urcorner \wedge \Pi cd \ulcorner c\varepsilon a \wedge d\varepsilon a \supset c\varepsilon d \urcorner$

The form of this axiom is important. It constitutes a kind of implicit definition of singular inclusion: implicit, because 'ε' occurs on both sides of the equivalence. While not the shortest possible axiom for 'ε', it is the most perspicuous: the three clauses on the right correspond to the three conditions on a Russellian definite description: there is at least one *a*, there is at most one *a*, and whatever is *a* is *b* (cf. LT7 below). To this we add as an axiom a particular thesis of

Ontological Extensionality

LA2 $\Pi fga_1 \ldots a_n \ulcorner \Pi a \ulcorner a\varepsilon f(a_1 \ldots a_n) \equiv a\varepsilon g(a_1 \ldots a_n) \urcorner$
$\equiv \Pi F \ulcorner F\langle f \rangle \equiv F\langle g \rangle \urcorner \urcorner$

Here *f* and *g* are *n*-adic nominal operator variables, and *F* is a monadic variable taking such variables as arguments. In the special case where

$n = 0$, f and g are nominal terms and F is a monadic predicate variable. Both terms $f(a_1 \ldots a_n)$ and $g(a_1 \ldots a_n)$ may contain only $a_1, \ldots, a_n$ free, and must be previously defined. Given these axioms and definitions, a large number of theorems may be proved; but we shall list without proof only such as we shall find useful. It is worth noticing that, whereas in ordinary predicate calculus the bulk of the work in a theory is done by predicates, the availability of multiply-designating terms in Ontology has the effect that much more work is done by terms and term-forming functors. This is, as we shall see, only a contingent matter of exposition, and by defining numerous predicates we shall be able to compare Mereology more readily with foregoing systems. The following are some important theorems.

Theorems of Ontology

LT1 $\mathrm{E}!a \equiv \mathrm{E}a \wedge !a$

LT2 $\mathrm{E}!a \equiv a = a$

LT3 $\mathrm{E}!a \equiv \Sigma b \ulcorner a\varepsilon b \urcorner$

LT4 $\mathrm{E}a \equiv \sim a \simeq \Lambda$

LT5 $\mathrm{E}a \equiv a \sqsubset \mathrm{V}$

LT6 $a = b \equiv \mathrm{E}!a \wedge a \simeq b$

LT7 $a\varepsilon b \equiv \mathrm{E}!a \wedge a \subset b$

LT8 $a\varepsilon b \equiv\ !a \wedge a \sqsubset b$

LT9 $!a \equiv \Pi cd \ulcorner c\varepsilon a \wedge d\varepsilon a \supset c = d \urcorner$

LT10 $a\varepsilon b \equiv \Sigma c \ulcorner a\varepsilon c \wedge c\varepsilon b \urcorner$

LT11 $a\varepsilon b \wedge b\varepsilon c \supset a\varepsilon c$

LT12 $a \simeq b \equiv \Pi F \ulcorner F[a] \equiv F[b] \urcorner$

In the presence of extensionality, LT10 can serve instead of LA1 as axiom.[28] It should be mentioned that numerous constants besides 'ε' can serve as single primitives of Ontology. We are not interested in ringing the changes here: the interested reader is referred to papers by Lejewski on the subject.[29] We now turn to Mereology.

2.6.2 *Mereology*

Mereology is the oldest and best-researched of Leśniewski's formal systems, and for that reason a large number of primitives have been

[28] Cf. SOBOCIŃSKI 1967. [29] LEJEWSKI 1958, 1960–1, 1977.

discovered on which it can be based.[30] Briefly, if f is a primitive functor on which Mereology is built, and g is another functor for which a theorem exists of the form

$$a \varepsilon f(a_1 \ldots a_n) \equiv A$$

where A contains only a, $a_1, \ldots, a_n$ free, and g as the only Mereological functor (besides possible ontological functors), then g can be primitive. Indeed, not only nominal functors need be primitive. Besides hunting for primitives, researchers in Mereology have spent much energy in producing single-axiom versions of the systems with various primitives. Our goal is not compression or economy but clarity, so we shall only mention such possibilities in passing, and prefer a multi-axiom system based on the monadic nominal functor 'pt' which corresponds effectively to the choice of '$<$' as primitive in a calculus of individuals.

System M; 'pt' *primitive*

Definitions

These definitions conform to the stipulations laid down for Ontology. Most of the expressions defined were introduced informally in §1.3. We pass special comment only on those which were not mentioned there.

MD1 $a \varepsilon \mathrm{ppt}(b) \equiv a \varepsilon \mathrm{pt}(b) \wedge \sim a = b$

MD2 $a \varepsilon \mathrm{ov}(b) \equiv a \varepsilon a \wedge b \varepsilon b \wedge \Sigma c \ulcorner c \varepsilon \mathrm{pt}(a) \wedge c \varepsilon \mathrm{pt}(b) \urcorner$

MD3 $a \varepsilon \mathrm{ex}(b) \equiv a \varepsilon a \wedge b \varepsilon b \wedge \Pi c \ulcorner c \varepsilon \mathrm{pt}(a) \supset \sim c \varepsilon \mathrm{pt}(b) \urcorner$

MD4 $a \varepsilon \mathrm{Sm}(b) \equiv a \varepsilon a \wedge \Pi c \ulcorner c \varepsilon b \supset c \varepsilon \mathrm{pt}(a) \urcorner \wedge \Pi c \ulcorner c \varepsilon \mathrm{pt}(a) \supset \Sigma d e \ulcorner d \varepsilon b \wedge e \varepsilon \mathrm{pt}(c) \wedge e \varepsilon \mathrm{pt}(d) \urcorner \urcorner$

MD5 $a \varepsilon \mathrm{fu}(b) \equiv a \varepsilon a \wedge \Pi c \ulcorner c \varepsilon \mathrm{pt}(a) \supset \Sigma d e \ulcorner d \varepsilon b \wedge d \varepsilon \mathrm{pt}(a) \wedge e \varepsilon \mathrm{pt}(c) \wedge e \varepsilon \mathrm{pt}(d) \urcorner \urcorner$

We read '$a \varepsilon \mathrm{fu}(b)$' as '$a$ is a fusion of bs'. Here the indefinite article is necessary because not all bs need be involved ('fused'). In this respect the sum is different, since it satisfies a uniqueness condition (given by the second clause in the definiens of MD4): if there is any sum, there is exactly one, whereas if there is more than one b there is more than one fusion of bs, of which the sum is the largest. We mark the uniqueness with a capital letter. Sometimes a fusion is called a 'collection' and the

[30] Cf. WELSH 1978.

sum the 'complete collection', but we prefer to steer clear of terminology which smacks of sets or classes, for reasons which will become clear.

MD6 $a \varepsilon \mathrm{U} \equiv a \varepsilon \mathrm{Sm}(\mathrm{V})$

MD7 $a \varepsilon \mathrm{nu}(b) \equiv a \varepsilon a \wedge \mathrm{E}b \wedge \Pi c \ulcorner c \varepsilon b \supset a \varepsilon \mathrm{pt}(c) \urcorner$

We read '$a \varepsilon \mathrm{nu}(b)$ as 'a is a nucleus of *b*s'. Again the indefinite article is required, since a nucleus of *b*s is any object which is part of all the *b*s.

MD8 $a \varepsilon \mathrm{Pr}(b) \equiv a \varepsilon \mathrm{Sm}(\mathrm{nu}(b))$

The general product is just the maximal fusion of *b*s. There can be at most one, and we mark this again with a capital letter. Finitary operators may be defined analogously to the way they were defined in SC:

MD9 $a \varepsilon \mathrm{Bpr}(b, c) \equiv b \varepsilon b \wedge c \varepsilon c \wedge a \varepsilon \mathrm{Pr}(b \cup c)$

MD10 $a \varepsilon \mathrm{Bsm}(b, c) \equiv b \varepsilon b \wedge c \varepsilon c \wedge a \varepsilon \mathrm{Sm}(b \cup c)$

MD11 $a \varepsilon \mathrm{Cm}(b, c) \equiv b \varepsilon b \wedge c \varepsilon c \wedge a \varepsilon \mathrm{Sm}(\mathrm{pt}(b) \cap \mathrm{ex}(c))$

MD12 $a \varepsilon \mathrm{Cpl}(b) \equiv a \varepsilon \mathrm{Cm}(\mathrm{U}, b)$

This is one way of defining the constant term 'atom':

MD13 $a \varepsilon \mathrm{atm} \equiv a \varepsilon a \wedge \Pi b \ulcorner b \varepsilon \mathrm{pt}(a) \supset b = a \urcorner$

Various alternatives to this definition have been explored by Sobociński.[31] We can also define a nominal functor 'atom of':

MTD14 $a \varepsilon \mathrm{at}(b) \equiv a \varepsilon \mathrm{atm} \wedge a \varepsilon \mathrm{pt}(b)$

If no two *a*s overlap, we say they are *discrete*:

MTD15 $\mathrm{dscr}[a] \equiv \Pi bc \ulcorner b \varepsilon a \wedge c \varepsilon a \supset b = c \vee b \varepsilon \mathrm{ex}(c) \urcorner$

MTD16 $\text{w-dscr}[a] \equiv \Pi bc \ulcorner b \varepsilon a \wedge c \subset a \wedge b \varepsilon \mathrm{pt}(\mathrm{Sm}(c)) \supset b \varepsilon c \urcorner$

We read 'w-dscr[*a*]' as '*a*s are weakly discrete'. As its name implies, this is entailed by discreteness, but not vice versa. Weak discreteness of *a*s means that any *a* which is part of the sum of one or more *a*s is one of these *a*s; that no new *a* arises by putting several other *a*s together. This implies that no *a* is ever a proper part of another *a*. The last two definitions are propositive, and because of the abundance of name-forming functors, we temporarily signal that a constant is a predicate

[31] SOBOCINSKI 1971*a*.

by putting its argument in a different shape of bracket, a practice favoured by Leśniewski and his followers.[32]

Axioms of Mereology

The following axioms are to be added to those of L.

MA1 $a \varepsilon \mathrm{pt}(b) \supset b \varepsilon b$

MA2 $a \varepsilon \mathrm{pt}(b) \wedge b \varepsilon \mathrm{pt}(c) \supset a \varepsilon \mathrm{pt}(c)$

MA3 $a \varepsilon \mathrm{Sm}(c) \wedge b \varepsilon \mathrm{Sm}(c) \supset a = b$

MA4 $\Sigma c \ulcorner c \varepsilon a \urcorner \supset \Sigma c \ulcorner c \varepsilon \mathrm{Sm}(a) \urcorner$

These axioms tell us that only (existing) individuals can have parts, that the part relation is transitive, and that if something is one of *a*, there is a unique sum of *a*. Some of the theorems which can be developed from this set are the following.

MT1 $a \varepsilon a \supset a \varepsilon \mathrm{pt}(a)$

MT2 $a \varepsilon \mathrm{pt}(b) \wedge b \varepsilon \mathrm{pt}(a) \supset a = b$

MT3 $a \varepsilon \mathrm{pt}(b) \equiv a \varepsilon \mathrm{ppt}(b) \vee a = b$

This theorem shows that ppt may be taken as primitive; using this formula, together with

MT4 $a \varepsilon \mathrm{ppt}(b) \wedge b \varepsilon \mathrm{ppt}(c) \supset a \varepsilon \mathrm{ppt}(c)$

MT5 $a \varepsilon \mathrm{ppt}(b) \supset b \varepsilon N(\mathrm{ppt}(a))$

and MD4, MA3–4, Leśniewski first constructed a system of Mereology in 1916.

MT6 $a \varepsilon a \equiv a = \mathrm{Sm}(a)$

MT7 $a \varepsilon a \equiv a = \mathrm{Sm}(\mathrm{pt}(a))$

Using MT7, it can be proved that

MT8 $a \varepsilon \mathrm{pt}(b) \equiv \Sigma c \ulcorner a \varepsilon c \wedge b \varepsilon \mathrm{Sm}(c) \urcorner$

which shows that Sm can also be taken as primitive. This is a possibility which has not been exploited in the comparable calculi of individuals (i.e. taking σ as sole primitive), because to attain something comparable to the bound '*c*' in this possible definition of pt, a calculus of individuals would have to quantify predicate or set variables, and this would no longer befit the nominalistic intentions of most calculists of individuals.

[32] Using different shapes of brackets allows analogies among constants to be expressed by using the same symbol and disambiguating by means of the brackets—e.g., one may define εε among higher types.

Other potential primitives include ov and ex: that if one may be primitive, so may the other, is easily seen by

MT9 $a\,\varepsilon\,\mathrm{ov}(b) \equiv a\,\varepsilon\,N(\mathrm{ex}(b))$

MT10 $a\,\varepsilon\,\mathrm{ex}(b) \equiv a\,\varepsilon\,N(\mathrm{ov}(b))$

The two functors have the obvious properties

MT11 $a\,\varepsilon\,a \wedge b\,\varepsilon\,b \supset. a\,\varepsilon\,\mathrm{ov}(b) \equiv \sim a\,\varepsilon\,\mathrm{ex}(b)$

MT12 $a\,\varepsilon\,a \equiv a\,\varepsilon\,\mathrm{ov}(a)$

MT13 $a\,\varepsilon\,\mathrm{ov}(b) \equiv b\,\varepsilon\,\mathrm{ov}(a)$

MT14 $a\,\varepsilon\,\mathrm{ex}(b) \equiv b\,\varepsilon\,\mathrm{ex}(a)$

and their potential primitivity can be seen by

MT15 $a\,\varepsilon\,\mathrm{pt}(b) \equiv a\,\varepsilon\,a \wedge \Pi c \ulcorner c\,\varepsilon\,\mathrm{ov}(a) \supset c\,\varepsilon\,\mathrm{ov}(b)\urcorner$

MT16 $a\,\varepsilon\,\mathrm{pt}(b) \equiv a\,\varepsilon\,a \wedge \Pi c \ulcorner c\,\varepsilon\,\mathrm{ex}(b) \supset c\,\varepsilon\,\mathrm{ex}(a)\urcorner$

Given either of MT15 or MT16, the other follows easily by previous theorems, but the proof of either from the axiom is quite involved. Another possible primitive unexploited by calculi of individuals is fu. Some of the more recent results on alternative axiomatizations are summarized in the next section. A perhaps surprising candidate for primitivity is the *predicate* dscr. This discovery was made by Sobociński in 1934, and is encapsulated in the theorem

MT17 $a\,\varepsilon\,\mathrm{ex}(b) \equiv a\,\varepsilon\,a \wedge b\,\varepsilon\,b \wedge \sim(a = b) \wedge \mathrm{dscr}[a \cup b]$

which allows us to define the potential primitive ex in terms of dscr.

The predicate dscr is historically important also in that it played an important role in Leśniewski's attempt to produce a foundation for mathematics. For a collection of objects which are discrete, all its sub-collections are discrete. This is given by

MT18 $\mathrm{dscr}[a] \wedge b \subset a \supset \mathrm{dscr}[b]$

Also, distinct discrete collections have distinct mereological sums

MT19 $\mathrm{dscr}[a] \wedge \mathrm{dscr}[b] \supset. a \simeq b \equiv \mathrm{Sm}(a) \simeq \mathrm{Sm}(b)$

which means that mereological sums of discreta can behave for once like Cantorian sets. In particular, Leśniewski showed that an analogue of Cantor's theorem about the cardinality of power sets holds: if *a*s are discrete and there are at least two of them, there are more fusions of *a*s than *a*s. In this case the fusion functor fu works rather like the subset operator. Leśniewski's investigations have been carried further in recent years by Robert Clay, who has strengthened Leśniewski's results by showing that they hold for the weaker functor w-dscr (weakly discrete), which is precisely the functor required to ensure that distinct collections have distinct mereological sums. This is based on

MT20 $\text{w-dscr}[a] \equiv \Pi bc^{\ulcorner} b \subset a \wedge c \subset a \wedge \text{Sm}(b) \simeq \text{Sm}(c) \supset b \simeq c^{\urcorner}$

which is easily proved from the definition MD16. Even more surprisingly, Clay has been able to show that w-dscr may serve as a single primitive for Mereology.[33]

2.6.3 Atomism, or Not

Like the Leonard–Goodman calculus of individuals, Mereology entails neither atomism nor atomlessness, as can be shown by the same models used for the Calculus of Individuals. The term 'atm', definable in various possible ways, and the functor 'at( )', 'atom of . . . ', can be used to axiomatize a system of atomistic mereology rather as Eberle axiomatizes atomistic calculi of individuals. The results of these researches have been presented by Sobociński,[34] building on earlier researches of Schröder and Tarski into Boolean algebra. Apart from the numerous definitions of 'atom' or 'atom of', there are numerous ways of stating the thesis of atomism, of which one of the simplest is Sobociński's

MF1 $a \varepsilon a \supset \Sigma b^{\ulcorner} b \varepsilon \text{pt}(a) \wedge \Pi c^{\ulcorner} c \varepsilon \text{pt}(b) \supset c = b^{\urcorner\urcorner}$

Taking the functor at as primitive, Sobociński has produced a straightforward system of atomistic Mereology based on the axioms[35]

MF2 $a \varepsilon \text{at}(b) \supset b \varepsilon b$

MF3 $a \varepsilon \text{at}(b) \wedge c \varepsilon \text{at}(a) \supset c = a$

MF4 $a \varepsilon a \wedge b \varepsilon b \wedge \Pi c^{\ulcorner} c \varepsilon \text{at}(a) \equiv c \varepsilon \text{at}(b)^{\urcorner} \supset a = b$

MF5 $a \varepsilon b \supset \Sigma c^{\ulcorner} \Sigma d^{\ulcorner} d \varepsilon \text{at}(c)^{\urcorner} \wedge \Pi d^{\ulcorner} d \varepsilon \text{at}(c) \equiv \Sigma e^{\ulcorner} d \varepsilon \text{at}(e) \wedge e \varepsilon b^{\urcorner\urcorner\urcorner}$

Lejewski has managed to produce a single-axiom system based on this same functor, which we present in the following section. Both of these systems are equivalent to one obtained by adding MF1 to general Mereology.

One may similarly formulate an atomless Mereology with a single axiom based on the single primitive ppt. Lejewski's result on this is given in the next section. In an atomless system, it is also possible to define the notion of a base, which may partially replace that of an atom.

[33] CLAY 1961, part of which appeared as CLAY 1965.
[34] SOBOCIŃSKI 1967, building on TARSKI 1935. [35] Ibid.

The presence of plural terms in Ontology makes this particularly easy. We define the binary predicate $\lhd$ as follows, where '$a \lhd b$' may be read '*as* form a base for *bs*':

MD17 $a \lhd b \equiv \Pi c \ulcorner c \varepsilon b \supset \Sigma d \ulcorner d \varepsilon a \wedge d \varepsilon \mathrm{pt}(c) \urcorner \wedge \Pi c d \ulcorner c \varepsilon b \wedge d \varepsilon b \supset. \; \Pi e \ulcorner e \varepsilon a \supset. e \varepsilon \mathrm{pt}(c) \equiv e \varepsilon \mathrm{pt}(d) \urcorner \supset c = d \urcorner$

which, it will be observed, is the exact Leśniewskian analogue of the conjunction of the conditions SF10–11 of §1.6 or B1–2 of §2.4.3. In atomistic Mereology it is easy to show that

MF6 $\mathrm{atm} \lhd \bigvee$

that is, that atoms form a base for everything.

In this section we have attempted to survey briefly the Leśniewskian way of doing part–whole theory. That this family of systems is still undergoing development and refinement may be gathered from the next section, in which we give a summary of alternative axiomatizations. It will by now be evident however, that Mereology and calculi of individuals are beating about the same bush as far as mereology goes, and that it is their different respective logical bases which obscure this. The sense in which the two kinds of system may 'say the same thing' is the subject of the more detailed comparison of §2.8. To prepare the way for this, we may note that it is easy in Mereology to shift the expressive weight from terms to predicates using suitable definitions. So we close with a series of propositive definitions. The choice of symbols reflects our comparative aim:

MD18 $a < b \equiv a \varepsilon \mathrm{pt}(b)$

MD19 $a \ll b \equiv a \varepsilon \mathrm{ppt}(b)$

MD20 $a \circ b \equiv a \varepsilon \mathrm{ov}(b)$

MD21 $a \wr b \equiv a \varepsilon \mathrm{ex}(b)$

MD22 $\mathrm{At}[a] \equiv a \varepsilon \mathrm{atm}$

2.7 Alternatives and Developments in Leśniewskian Mereology

The exploration of alternative primitives and axiomatic bases for Mereology was instigated by Leśniewski himself, but it has been carried a good deal further by a series of workers, including Sobociński, Lejewski, Clay, Rickey, Welsh, and Le Blanc. It is fair to say that none of these has questioned the appropriateness of Mereology as an adequate analysis of the concept *part* and its relations, but their formal work is of unquestionable value nevertheless, and demonstrates the richness of mereology, in particular the richness of the Leśniewskian approach to classical mereology. Nothing of comparable detail has been attempted for the case of calculi of individuals. The following summary is just that: a brief résumé of some results. For proofs, the reader is referred to the primary literature.

2.7.1 A Symbolic Convention

Some of the formulae displayed in this section are rather long and contain a large number of variables. To help in understanding these, we adapt a convention introduced by Leśniewski and still employed by those who do logic in his wake. When a nominal term was clearly intended to be *singular*, either categorically, or when appearing on one side of a conditional or biconditional, Leśniewski would write the term using a capital letter. This applies in particular to terms standing in front of 'ε'. Thus the definition of 'E' could be written in the following form:

LD1′ $\Pi a \ulcorner \mathrm{E}a \equiv \Sigma A \ulcorner A \, \varepsilon \, a \urcorner \urcorner$

(where both quantifiers are made explicit). This convention is a considerable help in reading the often involved formulae of Ontology and Mereology. Since we have already used singular variables in the exposition of calculi of individuals, we shall adopt the slightly different convention of using Italic lower-case letters from the end of the alphabet for singular variables (u, v, w, x, y, z, etc.), reserving those at the beginning (a, b, c, d, e, etc.) for nominal variables which, it is envisaged, can be truly understood, either categorically or hypothetically, to be plural or empty as well as singular. This convention is purely informal; I know of no canon of binding rules accepted by Leśniewskians for its use, though it is intuitively easy to grasp. Nevertheless this convention can be put on a formal footing, and we shall do so in the next section for the purposes of comparing Mereology with calculi of individuals. So LD1 may now take the form

LD1″ $\mathrm{E}a \equiv \Sigma x \ulcorner x \, \varepsilon \, a \urcorner$

(eliding the universal quantifier once more).

2.7.2 Alternative Primitive Constants

Leśniewski's first choice for primitive was the most intuitive functor ppt, but he also knew that Mereology could be based on pt, Sm, ex, ⊕, ∖ and fu (for those not previously defined, see below). An exhaustive study of possible primitives and primitive combinations has been carried out by Welsh, to which the interested reader is referred for details.[36] We bring only what the sports commentators are pleased to call edited highlights.

To show that an expression is primitive, it suffices to prove an equivalence which can be used to define a known primitive in terms of it. We shall display such equivalences without proof; it suffices for our purposes if they can be seen to be valid. So taking pt as primitive, that ppt, Sm, ov, ex, and dscr are also primitive is shown respectively by MT3, MT8, MT15–17. That fu is primitive may be shown by

MT21 $x\,\varepsilon\,\mathrm{ppt}(y) \equiv x\,\varepsilon\,x \wedge \ulcorner\Sigma z\urcorner\, z\,\varepsilon\,z \wedge y\,\varepsilon\,\mathrm{fu}(x \cup z) \wedge \sim y\,\varepsilon\,\mathrm{fu}(x \cup z)\urcorner$

Further primitives which we have already defined are the binary functors Cm, Bsm, and Bpr, and the general product Pr:

MT22 $x\,\varepsilon\,\mathrm{ppt}(y) \equiv x\,\varepsilon\,x \wedge \mathrm{Cm}(y,x)\,\varepsilon\,\mathrm{Cm}(y,x) \wedge \sim \ulcorner\Sigma z\urcorner\, z\,\varepsilon\,\mathrm{Cm}(x,y)\urcorner$

MT23 $x\,\varepsilon\,\mathrm{pt}(y) \equiv x\,\varepsilon\,x \wedge y\,\varepsilon\,\mathrm{Bsm}(x,y)$

MT24 $x\,\varepsilon\,\mathrm{pt}(y) \equiv x\,\varepsilon\,\mathrm{Bpr}(x,y)$

MT25 $x\,\varepsilon\,\mathrm{pt}(y) \equiv y\,\varepsilon\,y \wedge x\,\varepsilon\,\mathrm{Pr}(x \cup y)$

That w-dscr is primitive is not so easily seen, and we refer the reader elsewhere for details.[37]

Leśniewski defined a binary disjoint sum functor ⊕:

MD23 $x\,\varepsilon\,(y \oplus z) \equiv x\,\varepsilon\,\mathrm{Sm}(y \cup z) \wedge y\,\varepsilon\,\mathrm{ex}(z)$

and its generalization to arbitrary terms

MD24 $x\,\varepsilon\,\mathrm{Dsm}(a) \equiv x\,\varepsilon\,\mathrm{Sm}(a) \wedge \mathrm{dscr}[a]$

and either of these may be taken as primitive, by

MT26 $x\,\varepsilon\,\mathrm{ex}(y) \equiv x\,\varepsilon\,x \wedge (x \oplus y)\,\varepsilon\,(x \oplus y)$

MT27 $x\,\varepsilon\,\mathrm{ex}(y) \equiv x\,\varepsilon\,x \wedge y\,\varepsilon\,y \wedge \sim x\,\varepsilon\,y \wedge \ulcorner\Sigma z\urcorner\, z\,\varepsilon\,\mathrm{Dsm}(x \cup y)\urcorner$

The following compounded primitive is due to Lejewski, who has given a single axiom for it:

MD25 $x\,\varepsilon\,\mathrm{ptSm}(y,a) \equiv x\,\varepsilon\,\mathrm{pt}(y) \wedge y\,\varepsilon\,\mathrm{Sm}(a)$

In terms of it we may easily get back the two primitives pt and Sm:

MT28 $x\,\varepsilon\,\mathrm{pt}(y) \equiv x\,\varepsilon\,\mathrm{ptSm}(y,y)$

MT29 $x\,\varepsilon\,\mathrm{Sm}(a) \equiv x\,\varepsilon\,\mathrm{ptSm}(x,a)$

It is also possible to take several Boolean operators as primitive, as has

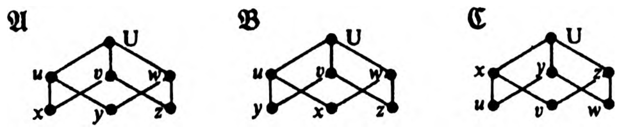
[36] WELSH 1978, building on and expanding previous work.
[37] This result took up a good part of CLAY 1961. Cf. WELSH 1978: 34.

been shown by Welsh.[38] Finally we mention another difference operator known to Leśniewski as a primitive:

MD26 $x \varepsilon y \smallsetminus z \equiv x \varepsilon \mathrm{Cm}(y, z) \wedge z \varepsilon \mathrm{pt}(y)$

MT30 $x \varepsilon \mathrm{ppt}(y) \equiv x \varepsilon x \wedge (y \smallsetminus x) \varepsilon (y \smallsetminus x)$

Primitives are so thick on the ground that Sobociński remarked, 'practically every Mereological functor of significance can be employed in this capacity.'[39] That some cannot be primitive is shown by adapting Padoa's method.[40] Let P be a functor which it is known can serve as sole primitive of Mereology and let C be another functor, and suppose we find two models in which C is given the same interpretation but the interpretation of P in each case differs. Then C cannot be a sole primitive for Mereology, since if it could, P would be definable in terms of C and would therefore have to receive the same interpretation in each model. Consider the three seven-element



models due to Clay, reproduced above. Models 𝔄 and ℭ show that neither U nor Cpl can be primitive, since in each case the interpretations of these constants remains the same, while that of ppt is different. Consider now the concept of proper overlapping mentioned in §1.1.3. The definition of its term-functor form is

MD27 $x \varepsilon \mathrm{oov}(y) \equiv x \varepsilon \mathrm{ov}(y) \wedge \Sigma z w \ulcorner z \varepsilon \mathrm{Cm}(x, y) \wedge w \varepsilon \mathrm{Cm}(y, x) \urcorner$

and models 𝔄 and 𝔅 show that it cannot be primitive, since only *u*, *v*, and *w* properly overlap each other in each model, but the proper part-relation is interpreted differently. Another non-primitive constant is that of weak exclusion:

MD28 $x \varepsilon \mathrm{wex}(y) \equiv x \varepsilon x \wedge y \varepsilon y \wedge \sim x \varepsilon \mathrm{pt}(y) \wedge \sim y \varepsilon \mathrm{pt}(x)$

That this cannot be primitive is again shown by models 𝔄 and ℭ. Interestingly, though neither proper overlapping nor weak exclusion is primitive, together they suffice as primitives for Mereology, as can

[38] WELSH 1978: 35. [39] SOBOCIŃSKI 1954: 36.
[40] PADOA 1967. Cf. TARSKI 1956*a*: ch. 10, SUPPES 1957: §8.8.

be seen by

MT31 $x \varepsilon \text{ex}(y) \equiv x \varepsilon \text{wex}(y) \wedge \sim x \varepsilon \text{oov}(y)$

Further results of this kind are given by Welsh.

2.7.3 *Alternative Axiomatic Bases*

Again we report work which was initiated by Leśniewski and carried further by his logical descendants. Following Leśniewski, Sobociński has enunciated a canon of desiderata for well-constructed axiomatic systems.[41] Among other things, a system should have as few undefined primitives as possible, preferably just one. Its axioms should not only be independent, but also as few, short, and simple as possible. Also axioms are better if they are *organic*, that is, contain no well-formed part which, if removed and universally quantified, would be a theorem. This precludes getting single axioms on the cheap by conjoining several shorter axioms. These desiderata have proved a powerful stimulus to search for ever shorter and simpler axioms for Mereology, based on its various primitives. Sometimes the various desiderata conflict: for example, one may have several short, simple axioms, or just one long one. For perspicuity and ease of handling, several short axioms are preferable to one long one, but the production and shortening of single axioms is always a logical achievement. As before, we simply list a selection of results whose proofs are to be found elsewhere.

The measure of length of a formula in Ontology or Mereology is the *Ontological unit*, which is any well-formed part of the form '$\alpha \varepsilon \beta$' in a system where singular inclusion is taken as the primitive of Ontology. So, for instance, the long axiom of Ontology, LA1, is seven units long, while the short axiom, LT10, is only three units long.

The shortest single axiom to date for Mereology is based on ov. It is due to Le Blanc, as are all the results mentioned below unless stated to the contrary.[42] It is nine units long:

$$\text{MT32} \quad x \varepsilon \text{ov}(y) \equiv \Sigma za \ulcorner z \varepsilon a \wedge \Pi wv \ulcorner \Pi u \ulcorner u \varepsilon w \equiv \Sigma t \ulcorner u \varepsilon \text{ov}(t) \equiv \Sigma s \ulcorner s \varepsilon a \wedge s \varepsilon \text{ov}(t) \urcorner\urcorner\urcorner \wedge w \varepsilon \text{ov}(v) \supset x \varepsilon \text{ov}(v) \wedge y \varepsilon \text{ov}(v) \urcorner\urcorner$$

[41] SOBOCIŃSKI 1955.

[42] Cf. LE BLANC 1983, 1985*a*, 1985*b*. The last two papers report results only up to 1980, and I must thank Audoënus Le Blanc for sending me a detailed account of his recent results, only a small selection of which have been mentioned here.

What this tells us, using only ov, is that the individuals x and y overlap iff they have a product, i.e. iff there are individuals whose sum is the largest individual such that whatever overlaps it overlaps both x and y. Here is one of two ten-unit axioms which Le Blanc has found for Mereology based on pt:

$$\text{MT33}\quad x\,\varepsilon\,\mathrm{pt}(y) \equiv \Pi z^{\ulcorner} y\,\varepsilon\,\mathrm{pt}(z) \supset x\,\varepsilon\,\mathrm{pt}(z)^{\urcorner} \wedge \Sigma a \Pi z^{\ulcorner} z\,\varepsilon\,y \equiv \Pi w^{\ulcorner} w\,\varepsilon\,a \supset w\,\varepsilon\,\mathrm{pt}(z)^{\urcorner} \wedge \Pi w^{\ulcorner} w\,\varepsilon\,\mathrm{pt}(z) \supset \Sigma v u^{\ulcorner} v\,\varepsilon\,\mathrm{pt}(w) \wedge v\,\varepsilon\,\mathrm{pt}(u) \wedge u\,\varepsilon\,a^{\urcorner\urcorner\urcorner}$$

We can work to make this understandable by defining functors of *upper bound* and *heap* respectively, where a heap is a part of a fusion:

$$\text{MD29}\quad x\,\varepsilon\,\mathrm{ub}(a) \equiv x\,\varepsilon\,x \wedge \Sigma y^{\ulcorner} y\,\varepsilon\,a^{\urcorner} \wedge \Pi z^{\ulcorner} z\,\varepsilon\,a \supset z\,\varepsilon\,\mathrm{pt}(x)^{\urcorner}$$

$$\text{MD30}\quad x\,\varepsilon\,\mathrm{hp}(a) \equiv x\,\varepsilon\,x \wedge \Pi y^{\ulcorner} y\,\varepsilon\,\mathrm{pt}(x) \supset \Sigma z^{\ulcorner} z\,\varepsilon\,a \wedge z\,\varepsilon\,\mathrm{ov}(y)^{\urcorner\urcorner}$$

It is easy then to see that

$$\text{MT34}\quad x\,\varepsilon\,\mathrm{Sm}(a) \equiv x\,\varepsilon\,\mathrm{ub}(a) \wedge x\,\varepsilon\,\mathrm{hp}(a)$$

so that MT33 has the sense that x is part of y iff whatever y is part of, x is part of, and whatever is y is a sum of one or more individuals, it being both upper bound and a heap of these.

The difficulty of these long axioms is understandable, in that a single formula having the form of an equivalence implicitly defining the primitive has not only to yield all the required properties of this primitive, but also to make possible a definition of sum, and implicitly state that any collection of individuals has a sum.

Apart from these two, Le Blanc has achieved numerous other improvements in finding shorter and simpler axioms for other functors. Before these discoveries, it was generally Lejewski who held the record on shortest axioms. He has a thirteen-unit one using ptSm,[43] and has also given single axioms for atomistic and atomless Mereology, based respectively on at and ppt:[44]

$$\text{MF7}\quad x\,\varepsilon\,\mathrm{at}(y) \equiv y\,\varepsilon\,y \wedge \Pi zwa^{\ulcorner} w\,\varepsilon\,\mathrm{at}(y) \wedge y\,\varepsilon\,a \wedge \Pi v^{\ulcorner} v\,\varepsilon\,z \equiv \Pi u^{\ulcorner} u\,\varepsilon\,\mathrm{at}(v) \equiv \Sigma t^{\ulcorner} u\,\varepsilon\,\mathrm{at}(t) \wedge t\,\varepsilon\,a^{\urcorner\urcorner\urcorner} \supset \mathrm{at}(x)\,\varepsilon\,x \wedge x\,\varepsilon\,\mathrm{at}(z)^{\urcorner}$$

(ten units), and, underlining once more the greater complexity of

[43] Cf. LEJEWSKI 1963*a*, where instead of 'ptSm' Lejewski writes 'elKl'.
[44] Cf. LEJEWSKI 1973.

atomlessness, the fourteen-unit

MF8 $x \varepsilon \mathrm{ppt}(y) \equiv y \varepsilon y \wedge \sim y \varepsilon \mathrm{ppt}(x) \wedge \Pi z w a \ulcorner w \varepsilon \mathrm{ppt}(y) \wedge y \varepsilon a \wedge \Pi v \ulcorner v \varepsilon z \equiv \Pi u \ulcorner u \varepsilon a \supset v \varepsilon u \vee u \varepsilon \mathrm{ppt}(v) \urcorner \wedge \Pi u \ulcorner u \varepsilon \mathrm{ppt}(v) \supset \Sigma t s \ulcorner t \varepsilon a \wedge s \varepsilon \mathrm{ppt}(u) \wedge s \varepsilon \mathrm{ppt}(t) \urcorner\urcorner\urcorner\urcorner$

Despite their virtuosity, these single axioms are difficult to grasp and work with; and for exposition and ease of working larger groups of short, simple axioms are better. Those given above for pt and at, both due to Sobociński, are of this type. Le Blanc has also explored such systems, variously emphasizing different kinds of simplicity, such as reducing the maximum number of units in any axiom of a set, or trying to get rid of internal universal quantifiers, as well as experimenting with systems based on more than one primitive. Our final example is of this kind: it is jointly based on pt and ov, is beautifully perspicuous, and has no axiom longer than four units:

MT35 $x \varepsilon \mathrm{pt}(y) \wedge y \varepsilon \mathrm{pt}(x) \supset x \varepsilon y$

MT36 $x \varepsilon \mathrm{ov}(y) \equiv \Sigma z \ulcorner z \varepsilon \mathrm{pt}(x) \wedge z \varepsilon \mathrm{pt}(y) \urcorner$

MT37 $x \varepsilon \mathrm{pt}(y) \equiv x \varepsilon x \wedge \Pi z \ulcorner x \varepsilon \mathrm{ov}(z) \supset y \varepsilon \mathrm{ov}(z) \urcorner$

MT38 $\Pi a \Sigma x \Pi y \ulcorner x \varepsilon \mathrm{ov}(y) \equiv \Sigma z \ulcorner z \varepsilon a \wedge z \varepsilon \mathrm{ov}(y) \urcorner\urcorner$

Here it can be seen that MT36–7 demonstrate the mutual dependence of the two primitives: they constitute a kind of internal interdefinition.

Mereology is a subject rich in logical interest, and these developments show that this interest is far from exhausted; we may expect axiomatic bases asymptotically to approach optimum forms, rather as they have done in propositional calculus.

2.8 Affinities

We have mentioned at several junctures the obvious community of content of calculi of individuals on the one hand and Mereology on the other. Those who take an interest in both kinds of system soon become adept at 'translating' concepts and principles from one idiom to the other. This informal procedure can be put on a firmer footing, however, and it is this which we attempt here. To draw the comparisons, we must overcome the differences of presentation and substance in the underlying logical systems. The alternatives of

Platonist and nominalist forms of presentation apply to either, so we may choose one or other as medium of comparison. For simplicity, not out of philosophical preference, we choose the Platonistic. Leśniewskian definitions, it was agreed above, are simply special axioms. When a Leśniewskian describes a definition as *creative*—and numerous definitions are creative, in the sense of allowing proofs of theorems not containing the defined symbol which were not possible without the definition—then all that means in more familiar terms is that the extension of a system obtained by adjoining the definitional axiom is not conservative.

We have already taken two steps towards showing how Mereology and calculi of individuals may be more directly compared, firstly by introducing a free calculus, secondly by showing how to use predicates in Mereology rather than term-forming functors. A direct comparison can be made by extending the language of Mereology so that other systems may be directly embedded in the extension, in the sense that any formula in a system under scrutiny has a synonymous equiform counterpart in the extended Mereology. Since the extensions in no way add to the expressive capacity of Mereology, it transpires that everything sayable in a normal or free calculus of individuals has a translation within Mereology.[45]

The variables *a*, *b*, *c*, . . . of Mereology are terms which may be semantically empty, singular, or plural. The two extensions of Merology we create here are obtained by adding new ranges of variables to the vocabulary. In each case, it is assumed that the formation rules, axioms, and inference rules of Mereology are extended in the obvious way to cover the new variables, so the variables of the new subcategories are inter-substitutible *salva congruitate* with the old.

2.8.1 System M′: *Mereology with Singular Variables*

The new singular variables are $x, y, z, w, x_1, \ldots$, and we use the same letters as corresponding metavariables. These variables are intended to function just like the singular variables of classical logic. We achieve this in a slightly roundabout way by adding the following axioms to M:

M′A1 $\Pi x \Pi ab \ulcorner a \varepsilon x \wedge b \varepsilon x \supset a \varepsilon b \urcorner$

M′A2 $\Pi x \ulcorner \Sigma a \mathrm{E} a \supset \mathrm{E} x \urcorner$

[45] Cf. SIMONS 1985*d*.

M′A3 $\Pi x \Sigma a \ulcorner x \simeq a \urcorner$

M′A4 $\Pi a \ulcorner \text{E!}a \supset \Sigma x \ulcorner x \simeq a \urcorner\urcorner$

M′A5 $\Sigma a \text{E}a$

The three axioms M′A1, M′A2, and M′A5 together ensure

M′T1 $\Pi x \text{E!}x$

that is, that each of the new variables is genuinely singular. We could have ensured this straight away in a single axiom, but we are planning ahead for M″. The point to notice is that M′A5 is expressed in the language of unextended Ontology, and is a way of stating the Fundamental Fact about our world—we might call it the Leibniz–Heidegger fact—that there is something rather than nothing. Since this axiom is independent of Ontology, it shows that M′ makes an ontological commitment to a non-empty universe, where M does not. But since this principle is derivable in classical logic, and our aim is to embed a classical theory, this commitment is to be expected. Axioms M′A3–4 are shuttle principles serving to allow the interchangeability of singular and neutral variables in contexts where singulars will work.

To partner the new variables we introduce two new quantifiers. In fact they are not so new after all:

M′D1 $\forall hA \equiv \Pi h \ulcorner \text{E}h \supset A \urcorner$

M′D2 $\exists hA \equiv \Sigma h \ulcorner \text{E}h \wedge A \urcorner$

We are here using $h, k, \ldots$ as metavariables ranging over both singular and neutral variables. These definitions (actually definition schemata, because A is any wff) are neither propositive nor nominative, so do not conform to Leśniewski's canons of definition. In this case they are best regarded in the conventional way as abbreviations.[46] The two new quantifiers are dual in the usual way. Their behaviour with respect to subalternation is worth noting. While the subalternation principle

$$\Pi aA \supset \Sigma aA$$

is unconditionally valid in Ontology, and is carried over to the singular variables, the analogous principle for $\forall$ and $\exists$ does not hold, since on the empty domain it is true that $\Pi a \ulcorner \text{E}a \supset \text{E}a \urcorner$, but false that $\Sigma a \ulcorner \text{E}a \wedge \text{E}a \urcorner$. On a non-empty domain, however, the subalternation

[46] In his 'official' logical language, which was developed for metalogical purposes, Leśniewski, like Frege, only used the universal quantifier. In everyday practice, he used a modified Peano–Russell language, whose '$\exists$' may be regarded as an abbreviation in the usual manner.

holds for both kinds of variables. Hence M′A5 guarantees classical subalternation.

We then embed a calculus of individuals such as CI, whose underlying logic is classical, first by eliminating all descriptions according to the Russellian schema, leaving classical quantifiers ∀ and ∃ intact, and binding any otherwise free variables (nominal or predicate) initially by 'Π'. The variables $x, y, z, \ldots$ and the signs $=$, $<$, $\circ$, $\wr$, ... all belong to M′, and the axioms and rules of CI are mapped into theorems and admissible rules of M′. When binding *singular* variables, it does not matter whether we use the restricted quantifiers ∀∃ or the unrestricted ones ΠΣ, so we can embed classical systems with or without free singular variables. But it is easy to see (by an inductive proof) that all the signs introduced with M′ are eliminable: any sentence containing such signs is equivalent to one expressed wholly in terms available in M; furthermore, this translation preserves theoremhood, provided we add the existential assumption M′A5 to M. So, for example

CIT3 $x \circ y \equiv \exists x \ulcorner z < x \wedge z < y \urcorner$

is mapped into

M′T2 $\Pi xy \ulcorner x \circ y \equiv \exists z \ulcorner z < x \wedge z < y \urcorner\urcorner$

which is equivalent in M′ (after simplification) to

M′T3 $\Pi ab \ulcorner a \varepsilon \mathrm{ov}(b) \equiv \Sigma c \ulcorner c \varepsilon \mathrm{pt}(a) \wedge c \varepsilon \mathrm{pt}(b) \urcorner\urcorner$

which is of course a theorem of M. So we can see the basis in principle of the intuitive 'translations' that researchers in the field of extensional part–whole theory have been able to carry through. Nevertheless, since M′ is not a conservative extension of M, some of the theorems of CI (such as '$x \circ x$') having counterparts in M′ which are not theorems of M, it is wrong to regard CI as equivalent to a part of M. Conversely there are many features, in particular quantifiers binding plural and predicate variables, which can be found in M and not in CI. To that extent Mereology is both more powerful and logically purer (since it makes no existence assumptions) than CI, SC, etc., but these are logical, not mereological differences. A step towards logical purity may be taken from the side of classical calculi by dropping M′A5, which leaves us with a so-called 'empty' or 'universal' logic, that is, one whose theorems are true on all domains, not just non-empty ones. In this weaker context, the subalternation principle for ∀ and ∃ and M′T1 are no longer valid. The loss of M′A5 also affects the theoremhood of

M′T4 $\Pi xA \equiv \Pi a^{\ulcorner}E!a \supset A[a/x]^{\urcorner}$

since the right–left implication fails on the empty domain (substitute ‘E! *x*’ for *A*).

2.8.2 System M″: *Mereology with Non-Plural Variables*

Despite the increase in logical purity obtained by dropping M′A5, thereby turning *x*, *y*, *z*, . . . from singular into non-plural (singular-or-empty) variables, one of the main reasons for adopting a free logic, especially as the basis for a calculus of individuals, is that there are purportedly singular terms which are without reference even on a non-empty domain—for example, the term for product of two non-overlapping individuals. The new variables of M′ however take values on all non-empty domains (M′A2) and so cannot play the required role. So we enlarge the language of M′ by another run of non-plural variables *s*, *t*, *u*, *v*, s_1, . . . , for which

M″A1 $\Pi s^{\ulcorner}\Pi ab^{\ulcorner}a\,\varepsilon\, s \wedge b\,\varepsilon\, s \supset a\,\varepsilon\, b^{\urcorner\urcorner}$

M″A2 $\Pi s\Sigma a^{\ulcorner}s \simeq a^{\urcorner}$

M″A3 $\Pi a^{\ulcorner}!a \supset \Sigma s^{\ulcorner}s \simeq a^{\urcorner\urcorner}$

The difference between these variables and the previous non-plural ones lies in the minor alteration in the third axiom and the absence of anything corresponding to M′A2. It may seem wasteful to have two runs of non-plural variables, but in fact this is the procedure adopted by most free logics, although, since they use only ∀ and ∃ (to bind *x*, *y*, *z*, . . .), the variables *s*, *t*, *u*, . . . remain free. The identity predicate favoured by most free logics is not = but rather ≈, which is totally reflexive on non-plurals

M″T1 $\Pi m^{\ulcorner}m \approx m^{\urcorner}$

where *m*, *n* are metavariables for all non-plural variables. Another theorem is

M″T2 $\Pi m^{\ulcorner}E!m \equiv \exists x^{\ulcorner}x \approx m^{\urcorner\urcorner}$

which shows that the usual free-logic definition of singular existence is respected. It is furthermore possible, in the context of Ontology with added variables, to give an explicit nominative definition schema for singular definite descriptions:

M″DS1 $\Pi a^{\ulcorner}a\,\varepsilon\,\iota xA \equiv a\,\varepsilon\, a \wedge A[a/x] \wedge \Pi b^{\ulcorner}b\,\varepsilon\, b \wedge A[b/x] \supset b\,\varepsilon\, a^{\urcorner\urcorner}$

It can be checked that the addition of this schema enables us to embed within M″ all formulae of the free system F, binding all otherwise free variables (parameters *s*, *t*, . . . and predicate variables) initially by Π. All variables *x*, *y*, *z*, . . . are bound by ∀, ∃ or ι. In addition, the axioms and rules of F are theorems and admissible rules of M″, and each formula of F is equivalent in M″ to a formula of M, the equivalence preserving theoremhood. So, by bringing the underlying logic of the calculus of individuals nearer to that of Leśniewski, we can see how, in the sense here made precise, the calculus of individuals, when freed of existential assumptions, is equivalent to a proper fragment of Mereology. The converse project, of finding a formula in the calculus of individuals corresponding to any given formula of Mereology, cannot be carried through without resorting to a higher-order logic. In this case Leśniewskian terms would be replaced by *monadic predicates*; if the predicates F_a and F_b replace the terms *a* and *b*, then the translation for '$a \varepsilon b$' will be '$F_b(\iota x \ulcorner F_a x \urcorner)$'. The effect is to map the Leśniewskian constants and variables in Ontology onto the next higher type in a simple type theory, a suggestion which has been made by Hiż as a way of understanding Leśniewski for logicians not trained in Warsaw.[47]

2.9 Non-Classical Extensional Mereologies

In view of the criticisms which can be brought against classical extensional mereology, it is important to note that a number of authors have produced extensional mereologies which fall short of the classical in strength. While differing among themselves as to which principles to accept, all of the authors to be considered are agreed in rejecting the general sum principle. The mereologies have been developed for a variety of purposes, so it is not surprising that they diverge on some points. In each case we briefly mention the purpose for which the theory was developed, and give a compressed presentation relying on the nomenclature of Chapter 1.

2.9.1 Whitehead's Mereology of Events

The missing fourth volume of *Principia Mathematica* was to have been Whitehead's treatise on geometry. Some of its material clearly

[47] Cf. HIŻ 1977. Against this interpretation however cf. SIMONS 1982*d*. The remark about Warsaw comes from HIŻ 1984.

made its way into his masterpiece, *An Enquiry concerning the Principles of Natural Knowledge*, which we take as the definitive text for Whitehead's early mereology. The later theory of *Process and Reality* is somewhat different, and is more topological in nature. We consider it briefly in §2.10 below. Whitehead's mereology is the basis for his theory of extensive abstraction, by means of which he intended to provide certain logical constructions (in effect, classes of filters) to replace, among other things, the points, lines, and surfaces of classical geometry. Since Russell knew of this method in 1914,[48] it is possible that Whitehead's mereology was developed not only independently of Leśniewski, but maybe also earlier.

The objects to which Whitehead's mereology are meant to apply are called 'events', and his position at the time was that such events are the primary natural entities, all else being aspects of or constructions out of them. Whitehead's preference for events as ontologically basic may well stem from his preoccupation with relativity theory. Among the events he considered are certain massive ones which he called 'durations', slabs of reality cut square to the time axis, in terms of which temporal notions were to be defined. And indeed, extensional mereology is more plausible for events than for material continuants. However, we present Whitehead's mereological principles in abstraction from their intended application, leaving more general comments on events until later.

It is a pity Whitehead left behind no formal treatment of mereology, since all the accounts we have from him are informal presentations in which he does not trouble to make clear which of the stated principles are axioms and which are theorems.[49] When Leśniewski and Tarski became aware of the parallels between their work and Whitehead's, they constructed an independence proof showing that certain of Whitehead's 'theorems' did not follow from his 'axioms'.[50] The charge is unfair to the extent that in the works in question Whitehead did not set out to separate axioms from theorems. We shall follow him here,

[48] RUSSELL 1914.

[49] WHITEHEAD 1919: 75 f. and 1920: 76, drops hints concerning possible alternative axiomatic developments which indicate that he had some mathematics in the background.

[50] Cf. SINISI 1966. The writers clearly overlooked the formulation '*Some* properties of *K* [≫] essential for the method of extensive abstraction are. . .' (WHITEHEAD 1919: 101, my emphasis). It is not denied however, that Whitehead's presentation fails to come up to Warsaw standards.

calling all asserted sentences indiscriminately 'principles'.[51]

System W: The Theory of Extension: sets; '≫' primitive; '=' assumed

Whitehead's primitive is the concept of *covering* or *extending over*, which he writes '*K*' and which we might write '≫'. Since it is merely the converse of the proper part-relation ≪, we simply transpose Whitehead's theses into ones where the latter is taken as the primitive. The following definitions and principles are in the order Whitehead gives them.

WP1 $x \ll y \supset x \neq y$

WP2 $\forall x \exists yz \ulcorner y \ll x \wedge x \ll z \urcorner$

WP3 $\forall z \ulcorner z \ll x \supset z \ll y \urcorner \wedge x \neq y \supset x \ll y$

WP4 $x \ll y \wedge y \ll z \supset x \ll z$

WP5 $x \ll z \supset \exists y \ulcorner x \ll y \wedge y \ll z \urcorner$

WP6 $\forall xy \exists z \ulcorner x \ll z \wedge y \ll z \urcorner$

It is worth pausing here to take stock. WP1 and WP4 require no comment. WP3 is a version of PPP, the proper parts principle. WP2 says that every event both is and has a proper part. This entails on the one hand atomlessness and on the other hand the non-existence of a maximal object U. It follows immediately that the General Sum Principle cannot hold, for if it did, U, the sum of everything, would exist. WP6 tells us that nevertheless any two events have an upper bound (by WP2 therefore infinitely many). WP5 is a denseness principle: between any two events of which one is a proper part of the other, there is a third. In classical mereology this follows from atomlessness. That the world is 'open' both above and below seems to have been something which Whithead found self-evident, for he gives no arguments for it.

WD1 $x \circ y \equiv \exists z \ulcorner z \ll x \wedge z \ll y \urcorner$

Whitehead does not define '<' (cf. the way WP3 is stated): this definition of overlapping (which he calls 'intersection') is different from the usual one, but in an atomless context they are equivalent.

WP7 $\forall z \ulcorner z \circ x \supset z \circ y \urcorner \supset x \ll y \vee x = y$

This is the Strong Supplementation Principle. Whitehead simply states it. That it does not follow from WP1–6 can be seen by taking the

[51] Exercise: do the job properly.

rational numbers with their natural ordering as a model for WP1–6.[52]

WD2 $x \wr y \equiv \sim x \circ y$

Whitehead calls disjoint events *separated*. A *set* of events is separated iff its members are pairwise disjoint (cf. 'dscr' in Mereology):

WD3 $\mathrm{sep}(\alpha) \equiv \forall xy \ulcorner x \in \alpha \wedge y \in \alpha \wedge x \neq y \supset x \wr y \urcorner$

A *dissection* of an event is a 'non-overlapping exhaustive analysis of an event into a set of parts, and conversely the dissected event is the one and only event of which that set is a dissection'.[53]

WD4 $\alpha \text{ diss } x \equiv \mathrm{sep}(\alpha) \wedge \forall y \ulcorner y \circ x \equiv \exists z \ulcorner z \in \alpha \wedge y \circ z \urcorner\urcorner$

WP8 $\alpha \text{ diss } x \wedge \alpha \text{ diss } y \supset x = y$

(Cf. 'Dsm' in Mereology.)

WP9 $x \ll y \supset \exists \alpha \ulcorner \alpha \text{ diss } y \wedge x \in \alpha \urcorner$

WP10 $x \ll y \supset \exists z \ulcorner z \ll y \wedge z \wr x \urcorner$

So far, apart from being open above, Whitehead's mereology is still reasonably familiar. The definition of the relation of *joining* between events changes this:

WD5 $x \text{ jn } y \equiv \exists z \ulcorner z \circ x \wedge z \circ y \wedge \exists \alpha \ulcorner \alpha \text{ diss } z \wedge \forall w \ulcorner w \in \alpha \supset w \ll x \vee w \ll y \urcorner\urcorner\urcorner$

Joined events are those which are *continuous* with one another. If events overlap, they are joined

WP11 $x \circ y \supset x \text{ jn } y$

but Whitehead says that joined events may be separated:

WP12 $\exists xy \ulcorner x \text{ jn } y \wedge x \wr y \urcorner$

This shows that Whitehead does not accept the Binary Sum Principle, since in classical mereology any two individuals are joined: the sum $x + y$ can be dissected into $x \cdot y$, $x - y$, and $y - x$ (all three of which will exist only if x and y properly overlap).

Whitehead then defines *adjunction* and *injunction*:

WD6 $x \text{ adj } y \equiv x \wr y \wedge x \text{ jn } y$

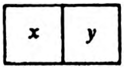
WD7 $x \text{ inj } y \equiv y \ll x \wedge \exists z \ulcorner z \wr x \wedge z \text{ adj } y \urcorner$

and states

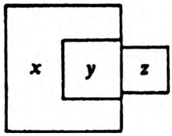
WP13 $y \ll x \wedge z \wr x \wedge z \text{ adj } y \supset z \text{ adj } x$

[52] Cf. SINISI 1966. The other independent principle is WP10, i.e. WSP.
[53] WHITEHEAD 1919: 102.

Example of adjunction



Example of injunction



Injunction is the *boundary union* of an event and its part, adjunction is the boundary union of separated events. With these concepts Whitehead can begin to give an analysis of the concept *boundary*, and from this point in his exposition the method of extensive abstraction takes over the account.

In *The Concept of Nature* Whitehead produces a slight variant of the mereology of the *Enquiry*, restricting the strict upper bound thesis WP6 to what he calls 'finite' events, i.e. excluding durations. This decision was later reversed.[54] In the later work a variant definition of joining is offered

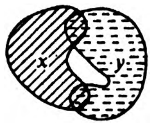
WD5′ $x \,\mathrm{jn}\, y \equiv \exists z \ulcorner z \circ x \wedge z \circ y \wedge \forall w \ulcorner w \ll z \supset w \circ x \vee w \circ y \urcorner\urcorner$

which effectively says that events are joined iff they have a binary sum. Whitehead clearly holds that all events are connected or continuous, in the sense of not consisting of two or more disconnected pieces. But he cannot say so in purely mereological vocabulary, since a definition of connectedness would presumably have to use the concept of joining. But in any mereology accepting binary sums without reservation, any two individuals are joined, so this could not be used to discriminate connected from disconnected individuals. To express the topological concept of connection, we need more than plain mereology, and when we have it, we need not stint on binary or other sums, as § 2.10 and § 9.3 show.

In any case, the assumption that all events are connected is no longer laudable caution, but downright miserliness. Continuity or connection is important to many objects, but there is no reason to think that all objects, even all individuals, should be connected. A trombone does not cease to exist when dismantled in its case, and a football match is a single event despite the break at half-time. If we refuse to accept disconnected individuals, then a purely mereological

[54] WHITEHEAD 1920: 76, reversed at 197 f. Cf. also the 2nd (1925) edn. of the *Enquiry*, 204.

oddity results: individuals may overlap and yet have no product, for example two kidney-shaped areas shown below.[55] Of course, a mere



oddity or algebraic inconvenience is no philosophical argument, but since we do in any case already accept disconnected individuals, we have good reason to think that Whitehead's informal restriction is too stringent. The ostensible reason for wanting to restrict the existence of events to connected ones, namely the interest in defining *boundary*, is not binding, since the tools required to do that are available in an extended theory whether or not we accept disconnected individuals. Accepting *some* disconnected objects, however, falls well short of accepting arbitrary sums, as the next section shows.

2.9.2 Bostock's Theory of Parts

Bostock develops a mereology in connection with providing a foundation for the theory of extensive measurement, rational and irrational numbers.[56] By contrast with Whitehead's treatment, Bostock's more recent one is explicit in its axiomatization, very clear, and gives reasons for the principles adopted, why they are so strong and no stronger. We can therefore be brief in exposition. Despite the specificity of its use, and Bostock's sceptical view that 'it is unprofitable to concentrate upon some one relation which is supposed to be *the* relation of part to whole', his system is still 'an attempt to capture the formal features shared by all relations which are reasonably regarded as part–whole relations'.[57] Both the scepticism and the aim seem to me to be just right, but Part II will offer considerations suggesting that Bostock's assumptions are still too strong to be minimal in the way he suggests.

System P: *no sets;* '$<$' *primitive*[58]

We present the system with inessential notational alterations, referring for brevity to Chapter 1. Bostock in effect presents two

[55] Cf. BOSTOCK 1979: 125. [56] Ibid: ch. 2, §4, 'Parts'.
[57] Ibid: p. 124.
[58] Bostock uses some set theory *notation*, but regards it as merely syncategorematic.

systems at once, one with several simple axioms, and an equivalent one with just two compact axioms. First defining disjointness as usual

PD1 $x \wr y \equiv \sim \exists z \ulcorner z < x \wedge z < y \urcorner$

Bostock then has

PA1 $x < y \equiv \forall z \ulcorner y \wr z \supset x \wr z \urcorner$

which is equivalent to four theses stating that < is a partial ordering satisfying SSP. The definitions of binary product, difference, and general product are equivalent to those of SD4, SD10, and SD11. As duals to the products he takes not the sums + and σ but rather the least upper bounds +′ and σ' (cf. SD6, SD8). The second axiom of the compact set is

PA2 $\exists xFx \wedge \exists y \forall x \ulcorner Fx \supset x < y \urcorner \supset \exists y \forall z \ulcorner y \wr z \equiv \forall x \ulcorner Fx \supset z \wr x \urcorner \urcorner$

This tells us that if there are any *F*-ers *and* they are bounded above, then they have a *sum* (σ, not just a least upper bound σ') (cf. SA22). The resulting system is a little weaker than classical mereology; it does not commit us to the existence of U, but either adding this (SA16) or replacing PA2 by the General Sum Principle SA24 brings us back to classical strength.

Bostock's system is thus fairly strong, certainly much stronger than the very weak minimal system envisaged in Chapter 1. It has only six non-isomorphic seven-element models. Binary least upper bounds exist if two objects have *an* upper bound (SA10), binary and general products, and differences exist under classical conditions. This relative strength is due to Bostock's express purpose 'precisely to exploit the analogy between parts and subclasses',[59] a motive lying behind classical mereology as well. Bostock's reason is that it is this analogy 'that explains why the notion of a part is at all important to us'.[60] That there is an analogy in many cases cannot be doubted; more will be said about this in Chapter 4. On the other hand, Part II will try to undermine the analogy for other cases.

On the other hand Bostock avoids the most extreme theses of classical mereology, because there are or may be classes of objects unbounded above. His argument is that a stronger postulate, such as that of classical mereology, might result in sums which were in some

[59] Ibid: 127. [60] Ibid: 125.

sense too large, or too heterogeneous in composition.[61] The system P therefore represents one fairly natural compromise between the demands of universal applicability and algebraic neatness.

Bostock also notes the possibility of interpretations of P which are not intended and not mereological; to exclude these he adds the informal demand that sums should be understood as being *made up out of* their summands. Unintended interpretations of a formal system are of course possible at any time. An example is provided by our Hasse diagram models (§1.4), since the higher dots are certainly not made up out of the lower dots. But 'make up' is itself a mereological notion—the lower dots fail to make up the higher ones because they are not *parts* of them. Bostock's informal demand reduces to the unobjectionable requirement that when doing mereology we understood 'part' to mean *part*, and not another thing, which is just to say that inintended interpretations are not wanted.

2.9.3 Sharvy's Quasi-Mereologies

In this case we outline the formal aspect first and discuss the context and applications afterwards. Both classically and in Bostock's P we have

$$\text{RP} \quad \sim x < y \supset \exists z \forall w \ulcorner w < z \equiv w < x \wedge w \wr y \urcorner$$

which amounts to saying that if x is not part of y, the difference $x - y$ exists. This is a principle even stronger than the Strong Supplementation Principle: we call it the *Remainder Principle*, since $x - y$ is the remainder of x when $x \cdot y$ is substracted; it is the *maximal* supplement to $x \cdot y$ in x (and vice versa). Its strength can be gauged by the fact that if added to the axioms for partial order and the existence of U it ensures the appropriate existence of binary sums and products, the product as $x - \overline{y}$, the sum as $\overline{\overline{x} \cdot \overline{y}}$.[62] Sharvy calls a *quasi-mereology* any partial ordering which is closed under taking general least upper bounds. Such a system can be axiomatized by SA1–3 + SA23. In general a quasi-mereology does not satisfy RP. A model of such a theory, essentially similar to examples offered by Sharvy, is the following. Take as individuals all sets of natural numbers containing

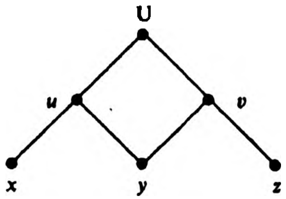
[61] Ibid: 126. Exactly what is meant here is not made clear. The reference to objects which are too large suggests Bostock has set-theoretic applications in mind, while the idea of heterogeneity suggests rather empirical applications. The heterogeneity objection is fully endorsed below.

[62] This is the general case, assuming all complements involved exist. Special cases have to be handled separately.

at least one odd and one even number, taking set inclusion as the part-relation. Then although the set {1, 2} is a proper part of the set {1, 2, 3, 4} there is no difference in the domain, since {1, 2} can be supplemented by any of {3, 4}, {1, 3, 4} and {2, 3, 4} to get {1, 2, 3, 4}. Each of these three is disjoint from {1, 2}, there being no intersection containing both an odd and an even number, but as none is a unique maximum the difference does not exist. This is a special case of a general method for constructing new quasi-mereologies from old, by taking the direct product of two or more.

Sharvy's quasi-mereology is thus non-classical in a very different way from Bostock's system. Quasi-mereologies admit nine non-isomorphic seven-element models, but the only one they have in common with the six of Bostock's system is the classical model, since quasi-mereologies have U.

The *example* of a quasi-mereology appears somewhat suspect, since after all, *qua* sets, {1, 2} and {1, 2, 3, 4} *do* have a unique difference, namely {3, 4}. It is only *qua* odd-and-even sets that {1, 2} and {1, 3, 4} are disjoint. Bostock has noted that restricting part–whole relations to certain parts may yield something which fails to be a part-relation, and that appears to apply here.[63] However, a more intuitive model for quasi-mereology can be found if we take the example of § 1.2 and drop the disconnected sum *v*: we then obtain a six-element model with the Hasse diagram shown below (note that *y* has no complement).



Sharvy's aim in noting quasi-mereologies is not that of giving a *general* mereology, but rather of giving a general account of reference and definite descriptions for all kinds of nouns: count singular, count plural, and mass. This is a topic considered in Chapter 4. Sharvy considers quasi-mereologies only for the case of domains which are the extensions of predicates corresponding to such nouns. The

[63] Cf. BOSTOCK 1979: 128. Of course this restriction is precisely what Sharvy wants. Sharvy and Bostock would presumably not agree as to what counts as a part-relation.

<-relation then represents the *part of* or *some of* relation on this extension. Thus viewed, the part-relation bears a non-accidental similarity to the Ontological functor $\sqsubset$, which can also be read as 'are some of (the)'. In Sharvy's view all predicates determine a quasi-mereology on their extensions. In certain cases—for example, for predicates corresponding to some count nouns—the part-relation is identity, for example for 'boot'—the only boot-part of a boot is the boot itself. For other predicates the part-relation is not trivial, for example, for 'water'—the water-parts of some quantity of water are the sub-quantities which are water (as against hydrogen, for example). Sharvy sees this as a sign that the part-of or some-of relation, thus relativized, is more fundamental than identity, since identity appears as a special case: in place of Quine's 'No entity without identity' he proposes 'Nothing without a quasi-mereology'.[64] However, the attempt to see identity as a special case fails, because if the partial ordering corresponding to a predicate is identity, the =-least upper bound of a subset of the predicate's extension exists only if the subset contains a single element, so the ordering is not a quasi-mereology, for which the least upper bound must exist for all subsets of the extension. Sharvy's approach works better for mass nouns and for count nouns unrestricted as to singular or plural. To discuss this now would mean anticipating Chapter 4, so we postpone further comment.

2.9.4 Needham and Van Benthem on Intervals

These two theories are taken together because of their similarity of aim and content. Both authors are interested in exploring temporal concepts using the notion of a temporal interval. Both present special-purpose mereologies for such intervals where the pure mereology is joined by principles for temporal betweenness (Needham) or order (Van Benthem). The special subject matter enjoins principles which are not valid for all mereologies, so, for example, Needham's definition of 'between' works only for a single dimension and for connected individuals. In each case we mention just the pure mereological component.

Needham[65] has a mereology similar to LG, based on $\wr$, but with weaker summation conditions, because intervals are normally taken to be finite and connected. He defines <, $\circ$, and ≪ as do Leonard and

[64] SHARVY 1983*b*: 234. The claim is already hinted at in SHARVY 1980: 621 f.
[65] NEEDHAM 1981.

Goodman (LGD1–2, and the obvious definition for $\ll$), and the binary product is also defined as one would expect. The interesting definition is that of an interval *abutting* another

$$\text{ND5} \quad x \rhd\lhd y \equiv x \lfloor y \wedge \exists z \ulcorner z \circ x \wedge z \circ y \wedge \forall w \ulcorner w \circ z \supset w \circ x \vee w \circ y \urcorner\urcorner$$

(cf. WD6 defining *adjunction* in Whitehead. If we define junction using WD5′ the two definitions are equivalent in the atomless context.) Needham's axioms are then LGA2–3, product existence (equivalent to SA6), a limited sum principle to the effect that sums exist of intervals which overlap *or abut*

$$\text{NA5} \quad x \circ y \vee x \rhd\lhd y \supset \exists z \forall w \ulcorner w \lfloor z \equiv w \lfloor x \wedge w \lfloor y \urcorner$$

and finally atomlessness (SF4.)

Van Benthem[66] is more flexible in the assigning of axioms, being prepared to ring the changes as circumstances warrant. He bases the pure inclusion fragment of his calculi on $<$, which can be interpreted as set-inclusion or as mereological part according to taste. Pure principles governing temporal precedence and mixed ones relating precedence and inclusion are also added. A suggested *minimal* theory of inclusion makes $<$ a partial ordering with product existence (SA6). As next likely candidate for a near-minimal theory, Van Benthem adds a version of SSP:[67]

$$\text{SCT13a} \quad \forall z \ulcorner z < x \supset z \circ y \urcorner \supset x < y$$

Supplementation principles Van Benthem calls *freedom* postulates, and he has reservations about their status as pure inclusion principles.[68] When considering interval models which yield structures isomorphic to the integers or the rationals he adds further SA10 + SA12 (= SA13).[69] These models also enjoin connectedness, which is expressed as a mixed principle. Connectedness is not part of the minimal set. As dual to the product Van Benthem takes not the sum + but the least upper bound +′, but in the minimal case this need not exist. It is interesting that for a minimal theory Van Benthem is prepared to do without any supplementation principles at all. This admits models of the pure inclusion fragment which are not

[66] VAN BENTHEM 1983, esp. ch. I.3. [67] Ibid: 69.

[68] Ibid: 242 f. Van Benthem notices here the tendency to view part–whole theory through Boolean-algebraic spectacles, about which he rightly has reservations.

[69] Ibid: 72. The difference between integer (discrete) and rational (dense) models is not a purely mereological one, but resides in the principles relating parts and precedence.

extensional in the sense of § 1.4, indeed not even mereological, such as the two-element model. But it would be over-hasty to conclude that Van Benthem's axioms are too weak to characterize a *part*-relation, since he, like Needham, is not developing a mereology in isolation for its own sake, but as part of a more inclusive theory, where the mixed principles have their repercussions on the mereological component, for example, entailing that intervals be finite.[70]

2.10 The Boundary with Topology

Several of the systems considered in the last section have an eye to applications outside mereology, but for which mereology is to serve as the basis. In this role mereology is often more intuitively attractive than the more usual set theory. In addition to the interval theories just mentioned, there have been several attempts to base various kinds of geometry on mereology. For instance, Tarski's system T (§2.4.2) forms the basis for a treatment of solid geometry based on the primitive *sphere*.[71] Whitehead's method of extensive abstraction uses mereology and set theory to define not just points but all other geometrical and temporal boundary entities, such as moments, point-moments, or surfaces, in terms of sets of concrete events. After the appearance of Whitehead's early work, an alternative approach to solid geometry was suggested by de Laguna,[72] using the primitive predicate 'ξ connects η to ζ'. Whitehead thereupon adopted a modified version of de Laguna's approach in *Process and Reality* with the simpler predicate 'ξ is connected with η', which may be intuitively understood to mean the sharing of a point, though of course this is to be used to define points and other kinds of boundary.[73] The relata of this relation are no longer events, but *regions*. Analogously to the earlier treatment of events, Whitehead assumes that all regions are connected internally.

It would take us beyond the bounds of our study to consider either abstraction or topology in detail. However, it seems plausible both that topology should be given a mereological rather than point-set-theoretical basis,[74] and that the introduction of topological notions is

[70] Ibid: 64 f.

[71] TARSKI 1956*b*. The mereology is atomless. Points are regarded as sets of concentric spheres. This is a special form of Whitehead's more general maximal filter construction.

[72] DE LAGUNA 1922. [73] WHITEHEAD 1929: 449 ff.

[74] For an early plea, cf. MENGER 1940.

in some sense the natural next step after mereology. Thus the attempt to graft a theory of time directly onto mereology, as exemplified either by the work just discussed, or by recent work of Lejewski, jumps right across the topological concepts common to space and time to deal with a special case.[75]

The theoretically most satisfactory way to proceed is to present a pure mereology first, then add on topological concepts and principles. Another possibility is to develop a unified mereo-topology based on a single concept straddling both disciplines. This is less clear, but more economical as a single theory. We outline one approach of each type below. For the first sort only the work of Tiles lies to hand. The second kind of theory is that developed by de Laguna and Whitehead, but we take as representative much more recent work by B. L. Clarke, which is free of both the limitations and the defects of the earlier work.[76]

2.10.1 TI: *Theory of Tiles*

Tiles's account[77] is developed not for its own sake, but as part of a discussion of the discriminatory capacities required for someone to talk about and identify events. We abstract from the context and consider just the formal theory. For his mereology, Tiles takes a full classical theory similar to LG, though understanding the sets as virtual. Taking $<$ as primitive, he defines $\circ$ and $\wr$, Su and Pr (TIDI–4) by LGD2, CIDI, LGD3–4 respectively.[78] His axioms TIA1–3 are LGA2, SCT14, and LGD1. The topological addition is based on the single intuitive notion of one individual's being wholly *within* another, or being an *internal part* of it. We write this '$<\circ$'. The following axioms are suggested for this notion:

TIA4 $x <\circ \mathrm{U}$

TIA5 $x <\circ y \wedge y < z \supset x <\circ z$

[75] Cf. LEJEWSKI 1982. Lejewski's *Chronology*, as he calls it, is based directly on Mereology, and uses *two* primitives, 'is an object wholly earlier than' and 'is an object whose duration is shorter than that of'. The second is of course metric rather than topological. Lejewski points out that TARSKI 1937 initiated this approach—of which we have mentioned only the mereological part—but that Tarski is there committed to momentary objects, on which Lejewski remains neutral.

[76] DE LAGUNA 1922 is limited to solid geometry, and WHITEHEAD 1929 is not only not formally presented, but actually inconsistent: cf. CLARKE 1981: 204 f.

[77] TILES 1981: 40 ff. and 53 ff.

[78] Both of Su and Pr are wrongly defined in terms of implications rather than equivalences, and there is a misprint in the definition of Pr. TID5 is missing: it should presumably be a definition of $\ll$ (cf. TIA7).

TIA6 $x <\circ y \supset \overline{y} <\circ \overline{x}$

TIA7 $x \neq \mathrm{U} \wedge x <\circ y \supset x \ll y$

TIA8 $y <\circ x \wedge z <\circ x \supset y + z <\circ x$

TIA9 $x <\circ y \wedge x <\circ z \supset x <\circ y \cdot z$

In terms of $<\circ$ Tiles defines *separation*, *connection* and (*self*) *connectedness*:

TID6 $x][y \equiv x <\circ \bar{y}$ (*x* is separated from *y*)

TID7 $x \between y \equiv \sim (x][y)$ (*x* is connected with *y*)

TID8 $\mathrm{cn}(x) \equiv \forall yz \ulcorner x = y + z \supset y \between z \urcorner$ (*x* is (self-)connected)

The condition TIA8 on finite sums can be strengthened to cope with arbitrary sums. Rather than use indexing sets, as Tiles does, we might employ

TIA8′ $\exists x Fx \wedge \forall x \ulcorner Fx \supset x <\circ y \urcorner \supset \sigma x Fx <\circ y$

Although the basic idea is good, the formal execution is defective in several ways. A minor point concerns TIA6, which ought to have an antecedent '$x \neq \mathrm{U} \wedge y \neq \mathrm{U}$' to ensure the existence of complements. This difficulty is inherited by the definitions TID6–8. TIA7 has as consequence that the only *open* individual (one which is its own interior part) is U, but it is desirable to allow more than just this trivial case—open individuals ought after all to correspond to open sets in an orthodox topology. The definition of self-connectedness does not work. For consider the three-element mereology {x, y, U} with x and y as atoms, and everything an interior part of U; x and y without interior parts. Then x and y are by the definition connected with each other, and U is self-connected. If we alter this model to the extent of stipulating that U $<\circ$ x, we can check that the axioms of TI are all satisfied, which means that the axioms fail to entail the obvious required result that an interior part is a part:

TIFI $x <\circ y \supset x < y$

2.10.2 CL: *Theory of Clarke*

No such complaints can be raised against the nice formal development of Clarke.[79] His system is based on Whitehead's primitive of connection, the relata being informally understood as spatio-

[79] CLARKE 1981. See also the development in CLARKE 1985, which goes on to construct points. In the later paper Clarke corrects a misprint in what is our CLA5 (cf. 74, n. 4) and defines separation and self-connectedness in the same way as Tiles (69.)

temporal regions. As with Whitehead, that x is connected with y means intuitively that x and y share a point. But neither points nor any other boundaries appear as individuals; all individuals have an interior. As we shall see, this yields a mereology which is in various respects non-classical. Clarke's account being easily accessible, we refer the reader to it for details, and summarize here just the main points of interest.

In terms of connection ($\between$) we may define a number of mereotopological concepts, some familiar, some less familiar, as follows:

CLD1 $x][y \equiv \sim (x \between y)$

CLD2 $x < y \equiv \forall z \ulcorner z \between x \supset z \between y \urcorner$

CLD3 $x \ll y \equiv x < y \wedge \sim y < x$

CLD4 $x \circ y \equiv \exists z \ulcorner z < x \wedge z < y \urcorner$

CLD5 $x \lfloor y \equiv \sim x \circ y$

CLD6 $x \Join y \equiv x \between y \wedge \sim x \circ y$

CLD7 $x \leqslant y \equiv x < y \wedge \exists z \ulcorner z \Join x \wedge z \Join y \urcorner$

CLD8 $x <\circ\, y \equiv x < y \wedge \sim \exists z \ulcorner z \Join x \Join x \wedge z \Join y \urcorner$

We may read '$x][y$' as 'x is separated from y' (or, as Clarke says, 'disconnected'), '$x \Join y$' as 'x is externally connected to y' or 'x just touches y', '$x \leqslant y$' as 'x is a tangential part of y', and '$x <\circ\, y$', as before, as 'x is an interior part of y' (Clarke says 'non-tangential part'.) The axiomatization begins as follows:

CLA1 $x \between x$

CLAS $x \between y \supset y \between x$

CLA3 $\forall z \ulcorner z \between x \equiv z \between y \urcorner \supset x = y$

Here the first two axioms correspond to Clarke's first axiom, which is inorganic. The third axiom shows the theory is extensional, and Clarke indeed compares it to the extensionality principle of set theory. It will be noticed that $\circ$ and $<$ do not interact together as they do in classical mereology, but that $\between$ takes over the role of $\circ$ in this regard. The following two theorems show the relationship clearly:

CLT1 $\sim (x \Join y) \equiv . x \circ y \equiv x \between y$

CLT2 $\sim \exists z \ulcorner z \Join x \urcorner \supset . x < y \equiv \forall z \ulcorner z \circ x \supset z \circ y \urcorner$

Only when an object touches nothing tangentially (intuitively, when it is open) can we treat its parts in the way we do in classical mereology. Among the further theorems mentioned by Clarke are ones corresponding to TIA5 and TIF1. In the definition of Boolean operators, we note a similar transformation from the classical:

CLD9 $\sigma x \ulcorner Fx \urcorner \approx \iota x \forall y \ulcorner y \between x \equiv \exists z \ulcorner Fz \wedge y \between z \urcorner \urcorner$

CLD10 $x + y \approx \sigma z \ulcorner z < x \vee z < y \urcorner$

CLD11 $\bar{x} \approx \sigma y \ulcorner y][x \urcorner$

CLD12 $\mathrm{U} \approx \sigma x \ulcorner x \between x \urcorner$

CLD13 $x \cdot y \approx \sigma z \ulcorner z < x \wedge z < y \urcorner$

Clarke mentions a sizeable number of theorems using these operators, some of which again differ from those of classical mereology. An example is

CLT3 $\mathrm{E!}\bar{x} \wedge \mathrm{E!}\bar{y} \wedge \sim x >< \bar{y} \supset . \bar{x} + y = \mathrm{U} \equiv x < y$

where the deviation from the classical may be noted in the extra conjunct in the antecedent. Of more intrinsic interest are the topological operators which are now definable—because of the absence of a null element and boundary elements Clarke calls them 'quasi-topological':

CLD14 $^{\circ}x \approx \sigma y \ulcorner y <\circ x \urcorner$ (interior of x)

CLD15 $^{c}x \approx \sigma y \ulcorner \sim (y \between {}^{\circ}\bar{x}) \urcorner$ (closure of x)

CLD16 $^{x}x \approx \sigma y \ulcorner y <\circ \bar{x} \urcorner$ (exterior of x)

CLD17 $\mathrm{op}(x) \equiv x = {}^{\circ}x$ (x is open)

CLD18 $\mathrm{cl}(x) \equiv x = {}^{c}x$ (x is closed)

At this point we need additional axioms

CLA4 $\exists z \ulcorner z <\circ x \urcorner$

CLA5 $\forall z \ulcorner (z \between x \supset z \bigcirc x) \wedge (z \between y \supset z \bigcirc y) \urcorner \supset \forall z \ulcorner z \between x \cdot y$
$\supset z \bigcirc x \cdot y \urcorner$

which again correspond to a single inorganic axiom of Clarke. They tell us that every individual has an interior, and that the product of any two open individuals is again open. Among the many interesting theorems which Clarke lists are the following:

CLT4 $\forall x \exists y \ulcorner y = {}^{\circ}x \urcorner$

CLT5 $^{\circ}x < x$

CLT6 $y < {}^{\circ}x \equiv y <\circ x$

CLT7 $x < y \supset {}^{\circ}x < {}^{\circ}y$

CLT8 $x <\circ x \equiv {}^{\circ}x = x$

CLT9 $\sim(x >\!< {}^{\circ}y)$

CLT10 ${}^{\circ\circ}x = {}^{\circ}x$

CLT11 ${}^{\circ}\mathrm{U} = \mathrm{U}$

CLT12 $\mathrm{E!}(x \cdot y) \supset {}^{\circ}x \cdot {}^{\circ}y = {}^{\circ}(x \cdot y)$

The theorems CLT5, 10, 11, and 12 are particularly interesting, because they show that the interior operator is almost like that of orthodox topology, the difference lying only in the antecedent to CLT12. With similar conditions ensuring the existence of appropriate individuals, the closure operator also approximates that of topology:

CLT13 $\mathrm{E!}\,\bar{x} \supset \mathrm{E!}\,{}^{c}x$

CLT14 $\mathrm{E!}\,\bar{x} \supset {}^{c}x = \overline{({}^{\circ}\bar{x})}$

CLT15 $\mathrm{E!}\,\bar{x} \supset {}^{\circ}x = \overline{({}^{c}\bar{x})}$

CLT16 $\mathrm{E!}\,\bar{x} \supset x < {}^{c}x$

CLT17 $\mathrm{E!}\,\bar{x} \supset {}^{cc}x = {}^{c}x$

CLT18 $\mathrm{E!}\bar{x} \wedge \mathrm{E!}\bar{y} \wedge \mathrm{E!}(\bar{x} \cdot \bar{y}) \supset {}^{c}x + {}^{c}y = {}^{c}(x + y)$

CLT19 $\mathrm{E!}\bar{x} \wedge \mathrm{E!}\,\bar{y} \supset x < y \supset {}^{c}x < {}^{c}y$

CLT20 $\mathrm{E!}\,\bar{x} \supset {}^{x}x = {}^{\circ}(\bar{x})$

Clarke's estimation of the nature and significance of his system is worth quoting:[80]

> ... just as the linguistic domain of the classical calculus of individuals is a [complete] Boolean algebra with the null element removed, our theorems indicate that the domain of the present calculus is a closure algebra with the null element and the boundary elements removed. It is interesting, however, that so much topology can be reflected under these conditions and with such minimal assumptions.

It is indeed interesting, although it is unlikely that a busy mathematician will be prepared to forego the convenience of set theory to have to put up with the troublesome antecedents of the latter theorems. The significance of the quasi-topology with a mereological basis must surely be philosophical. And just here we have reservations. For the idea of the 'removal' of boundary elements can be taken in two ways. On the first interpretation, points and other boundary elements 'really' exist, so $\between$ really means $\circ$, and so on, but the domain of individuals has been artificially restricted to individuals with an

[80] Ibid: 216.

interior, which would explain why the resulting mereology is non-classical, it having been deprived of some elements. This is hardly an open-faced attitude to boundary elements, so suggesting the second interpretation, namely that there 'really' are no such elements, and that the calculus is to be taken at face value. Then we see that one of the mereological principles missing is the remainder principle, for removing the interior of a (non-open) individual leaves no difference. Indeed, it leaves nothing at all. If we take any individual, then its interior is a proper part of its closure, yet there is no proper part of the closure which is disjoint from the interior. We do not even have the weak supplementation principle. What we are being asked to believe is that there are two kinds of individuals, 'soft' (open) ones, which touch nothing, and partly or wholly 'hard' ones, which touch something. Yet we are not allowed to believe that there are any individuals which make up the difference. We can discriminate individuals which differ by as little as a point, but are unable to discriminate the point. It is hard to find satisfaction in this picture.[81] On the other hand, if we are to do something like *topology* without accepting points and other boundaries, it is hard to see how else it can be approached. This suggests that a philosophically adequate account must be more complex, involving for example successive vague *approximations* to sharp boundaries, a variant suggested in outline by Menger[82] and Tiles.[83] This would be less mathematically crisp, but would offer a more realistic basis for explaining how topological concepts might be *acquired* by someone who—like all of us—has never experienced an ideal, zero-thickness boundary.

2.11 Appendix: The Notational Jungle

Mereology is more than usually cursed with a plethora of different notations. The only safe rule when reading a text is to check exactly what the author intends by each symbol. But there are a number of more frequently recurring variants, and to aid in scanning the literature we tabulate the most important ones (see Tables 2.1–2.3 below). The plot is thickened by the presence of the relatively

[81] The view was explicitly held by BOLZANO: cf. his 1975: §66 f., according to which two bodies may only properly touch at places where one of them has, and the other one lacks, boundary points. Against this cf. BRENTANO 1976: 174. Cf. also VAN BENTHEM 1983: 68.

[82] MENGER 1940: 107.

[83] TILES 1981: 56 f.

unfamiliar functors of Leśniewski's Ontology and Mereology, which also have variant notations, and these too are tabulated as far as is practicable. We have to plead guilty to occasionally adding to the number of symbols on the market, but in a comparative study like this a large repertoire is inevitable if ambiguity is to be avoided. Our symbols have been chosen to be as far as possible mnemonic either phonetically or iconically, and we have sought to maximize the distance between mereological and set-theoretical notations without completely losing the analogies.

TABLE 2.1. *Calculi of Individuals*

Simons	Leonard–Goodman	Tarski[84]	Others
[illegible]	<	P	⊂ ⊆ ≤ ≼ ⊑
[illegible]	≪		⊂ ⊊ <
o	∘		0
(	(		|)(Z
x · y	xy	x ∩ y	x ∧ y
x + y	x + y	x ∪ y	
U	U	w	a* Un
x Su α	x Fu α	x Σ α	S F
x Pr α	x Nu α	x Π α	P N
[illegible]	−x	−x	

TABLE 2.2. *Leśniewski's Ontology*

Simons	*NDJFL**	Lejewski–Henry[85]	Others[86]
a ε *b*	*A* ε *a*	*a* ε *b*	ε{*Aa*}
x ε *a*			
E	!	ex	
!	→	sol	
E!		ob	
⊂	⊂	⊂	
≃	∘	∘	
=	=	=	
≈		≅	
≅	□	□	
⊏	⊏	⊏	

* *Notre Dame Journal of Formal Logic*

[84] TARSKI 1937. [85] LEJEWSKI 1958, 1967*b*, HENRY 1972.

[86] Cf. e.g. SOBOCIŃSKI 1967. It was Leśniewski's practice, sometimes also followed in *NDJFL*, to write functors before their argument expressions, which occur in a string enclosed by special brackets. For predicates, these had the shape of braces { }.

TABLE 2.3. *Leśniewski's Mereology*

Simons	Lejewski–Henry[87]	*NDJFL*	Le Blanc	RPT[88]	Leśniewski[89]
pt	pt	el	in	el	ingr
ppt	ppt	pr, pt	pt	pt	cz
ov		ov	ov	ov	
ex	ex	ex	ex	extr	zw
Sm	ccl	Kl	Ccl	Kl	Kl
fu	cl	cl	cl	cl	zb

[87] Lejewski's publications in *NDJFL* use that journal's symbolism; his own notation, more mnemonic for English speakers, is used in HENRY 1972. Le Blanc informs me that he disapproves, as I do, of the attempt to imitate set theory in the *NDJFL* notation. The Lejewski–Henry notation avoids this best, but the reading '(complete) collection' for '(c)cl' still carries such undertones.

[88] *Rocznik Polskiego Towarzystwa* (SOBOCIŃSKI 1954, LEJEWSKI 1954.)

[89] LEŚNIEWSKI 1927–30. (The abbreviations are mnemonic in Polish!)

3 Problems

Extensional mereologies are mostly very similar to Boolean algebras or complete Boolean algebras in their algebraic structure, and this similarity is not accidental. It can be accounted for by considering the jobs which they were and are called upon to perform. In most applications of mereology the natural rival is some kind of set theory, and mereology is usually preferred, when it is preferred, for philosophical rather than mathematical reasons. This can be seen by examining both the development of the main systems of extensional mereology and more recent applications. The resulting systems have tended to be strong, to hold their own more readily against their inherently more powerful set-theoretic rival. In our view this heritage has had detrimental effects on mereology, which will be considered in some detail in this chapter. Beforehand, it is worth briefly outlining the genesis of extensional mereology, as this helps to explain why the main systems are as they are.

3.1 Historical Remarks

We have already noted that in chronological order of appearance, the main systems of extensional mereology are those of Leśniewski, Whitehead, Tarski, and Leonard and Goodman. Apart from the separately developed systems of Whitehead, all subsequent mereologies were influenced to a greater or lesser extent by Leśniewski.

Leśniewski conceived Mereology as a replacement for the calculus of classes.[1] His original name for it was 'calculus of manifolds'. The sums or fusions of Mereology are often called 'collective classes' by Leśniewskians, and their parts 'elements'. We noted in the previous section the unfortunate tendency that many mereologists of Leśniewskian stamp have of employing terminology and notation reminiscent of set theory. This tendency has its origins in the master. There is indeed a good algebraic analogy between mereology and class theory, which is examined in greater detail in the next chapter, but we prefer to replace the notation reminiscent of classes by one without

[1] For information on Leśniewski's development, see LUSCHEI 1962, SURMA 1977, MIÉVILLE 1984.

such connotations, since in our view the notion of class is essentially distributive rather than collective (the difference will be explained below).

Leśniewski was struck forcibly by Russell's paradox of naïve set theory, and Mereology arose out of his efforts to track down what he considered to be the mistake in the reasoning leading to the paradox. In Leśniewski's view the problem lies in overlooking an ambiguity in the term 'class'. If 'class' is understood in the mereological sense as a sum, then, since every sum is a part of itself or, in Leśniewski's terms, an element of itself, there is no class which is not an element of itself, and hence no class of classes which are not elements of themselves. Hence Russell's paradox cannot arise in Mereology. One might be forgiven for thinking that this analysis is not directly relevant to Russell's paradox, since the classes Russell was considering are not concrete wholes such that every part is a 'member'. One of Russell's examples is that the class of teaspoons is not a member of itself, since it is not a teaspoon.[2] The members of such a class cannot be just any part of the world-heap of teaspoons, but only teaspoons. Leśniewski distinguished such classes from mereological wholes by calling them 'distributive classes'. However, he believed that such classes do not exist, since expressions in which apparent reference is made to such classes are replaceable without change of sense by expressions in which no such apparent reference is made; in particular '*a* is a member of the class of *b*s' may be replaced simply by '*a* is a *b*'; in Ontological terms, '$a \varepsilon b$'. If the term 'the class of teaspoons' is not so eliminable, then in Leśniewski's view it can only stand for the sum of all teaspoons. But as a sum, or collective class, it is variously describable: in particular it is the sum of all objects consisting only of teaspoons and undetached teaspoon-parts, and since it consists itself only of teaspoons and their parts (it is a fusion of teaspoons), it is, in that sense, 'self-membered'.

Some years previously, Frege had also distinguished between a 'concrete' and a 'logical' notion of class. Frege's point was directed against Schröder, whom he accused of confusing the two senses.[3]

[2] RUSSELL 1972: 119. [3] FREGE 1895.

Frege points out that a calculus of collective classes is simply a calculus of part and whole.[4]

Leśniewski knew Frege's criticisms,[5] and his development of Ontology and Mereology may be seen as making good Frege's criticisms of Schröder by developing two calculi, distinguishing Frege's two concepts of *class*, along broadly Schröderian lines.[6] It is notable that Schröder's basic sentence form is '$a \subset b$',[7] a tripartite subject–copula–predicate form, such as one finds in Leśniewski, where subject and predicate are both nominal expressions exchangeable *salva congruitate.* The tradition passes back from Schröder through Boole and Leibniz to Aristotle, whereas the modern function-argument analysis stemming from Frege, and transmitted into the modern tradition by Russell and Whitehead, while finding acknowledgement by Leśniewski,[8] does not obscure this Schröderian heritage.[9]

Schröder's major work, *Vorlesungen über die Algebra der Logik*, despite just criticism by Husserl and Frege[10] of its philosophical basis, nevertheless exercised wide influence as *the* work in symbolic logic up

[4] Frege's own 'logical' classes, the *Wertverläufe* of monadic predicates, turned out, ironically, to be far more vituperative than Schröder's rather innocent manifolds, which were cushioned against paradox by an early form of type theory (cf. CHURCH 1976). Now that the pioneering days are over and formal set theory has stabilized, it is easy to forget how fluid conceptions of class were around the turn of the century. The popular success of Zermelo–Fraenkel set theory is based on its undoubted mathematical virtues, not on its philosophical motivation. On the unnaturalness of sets, see the nice discussion in BEALER 1982: ch. 5.

[5] Cf. SINISI 1969.

[6] Leśniewski described his Ontology as 'a kind of modernized traditional logic', which is closest in content and 'power' to Schröder's 'calculus of classes', assuming this is taken to include the theory of 'individuals' (LEŚNIEWSKI 1929: 5.) He translates the title of his 1916 monograph as *Die Grundlagen der allgemeinen Mengenlehre*. In those days *Menge* was variously translated as 'set', 'multiplicity', 'manifold', or 'aggregate'.

[7] Schröder writes '$a \Subset b$'.

[8] LEŚNIEWSKI writes (1929) that Frege's *Grundgesetze* is for him the most imposing embodiment of the achievements of the deductive method in the history of the foundations of mathematics. Not many logicians preferred Frege so decisively over Russell and Whitehead at that time.

[9] In SIMONS 1983*b*: § 2, I argue that Ontology does not however rest on a two-name theory of predication, but is clearly Fregean in inspiration. Leśniewski merely preserves more of the old logic than do Frege, Whitehead, and Russell. The close relations of Ontology to Aristotelian and Boolean logic can be seen from LEJEWSKI 1960–1, 1963*b*.

[10] HUSSERL 1891, FREGE 1895.

until the publication of *Principia Mathematica*.[11] The algebra of logic which Schröder axiomatizes is Boolean algebra, and he regards this as essentially capable of various interpretations or applications, one of which is a calculus of 'manifolds' or 'domains'. Schröder's investigations, and those of Huntington,[12] were carried forward by Tarski,[13] who introduced infinitary sums and products, thus inventing complete Boolean algebra. Tarski was well aware of the close relationship between complete Boolean algebra and Mereology. So both Ontology and Mereology are closely related to Boolean algebra, in the form presented by Schröder. The algebraic similarities are probed further in the next chapter, where our conclusion is that there are different, analogous senses of 'part' even among extensional systems, and that these cannot be combined into a single overarching interpretation. Though, like Leśniewski, we seek to avoid Schröder's mistakes, this will amount to a partial vindication of his approach against the criticisms of Frege.

Both of the main previous extensional part–whole theories have been burdened with providing the foundations for ambitious philosophical programmes: Mereology was to provide, along with the logical systems it presupposes, a new foundation for mathematics, while calculi of individuals were to serve in Goodman's programme of constructive nominalism. Goodman makes clear[14] that for him the calculus of individuals is meant to supplant set theory as a theoretical tool. Its essentially weaker character is perceived by him as an advantage, since it prevents the creation of infinitely ascending chains of new entities from the same basic entities, which contravenes the principle of 'no difference of entities without difference of content'.[15] The need to provide sufficiently powerful principles to carry the weight of such programmes has tended to interfere with the more modest aim of assessing the place and characteristics of various concepts of part in our actual conceptual scheme. The two aims would

[11] DAMBSKA 1978: 123, claims that the lectures Kasimir Twardowski gave on 'The Attempts to Reform Traditional Logic' at Lemberg (Lwów), which concerned the work of Bolzano, Brentano, Boole, and Schröder, were attended by Łukasiewicz, Leśniewski, Kotarbiński, Czeżowski, and others. This claim requires substantiation, but it is certainly true that Polish logic in general did not break with the past as radically as Frege, Whitehead, and Russell (cf. Łukasiewicz's lifelong interest in Aristotle's syllogistic).

[12] HUNTINGTON 1904, 1933. [13] TARSKI 1935.

[14] GOODMAN 1956, 1977: § II. 4.

[15] The principle will be judiciously injured in Part II.

not be incompatible, were it not that extensional mereologists by and large refuse to accept that concepts of part which deviate from theirs are not in some way inferior or defective. We are pursuing here the more modest aim, as free as possible from programmatic burdens. The concepts of part which emerge are indeed richer and more varied than those we come across in extensional mereology, the unifying drive of which obscures this richness.

Extensional mereologists by and large share several other characteristics. One is mistrust or rejection of intensional notions such as necessity. Another is a preference for some form of nominalism. A third is a tendency to a form of materialism, the nature of which will become more apparent below. Finally, it will be noticed that the part-relation and all the other mereological concepts dealt with in the previous two chapters are tenseless or 'eternal' relations, like those found in pure mathematics. This is to be explained in good part by the purpose for which they were developed in the case of Leśniewski, and by the philosophical view of time held by others such as Whitehead. This cluster of opinions and manners of treatment may not have a tight internal logical structure, but regarded as a package, it exerts considerable epistemic thrust on the way in which part–whole theory is considered and applied.

3.2 Criticisms

The historical community of most extensional mereologies lent them common aspects of content which have been criticized. We deal with these criticisms and problems in ascending order of importance: they are:

(1) That there are senses of 'part' where a whole does not count as one of its own parts.
(2) That there are senses of 'part' for which the part–whole relation is not transitive.
(3) That there is no guarantee of the existence of 'sum-individuals'; equivalently, that the various axioms guaranteeing the existence of such sums for arbitrary classes of individuals, or for the individuals satisfying an arbitrary predicate or denoted by an arbitrary plural term, are false.
(4) That the identity criteria for individuals imposed by the axioms which identify individuals having all parts in common are in general false.

(5) That the ontology forced onto us by the acceptance of the principles of extensional part–whole theory is a materialistic ontology of four-dimensional objects.

The last three and most serious criticisms are all linked, but we separate them for clarity. We deal with the criticisms in turn, but first make a general point. In constructing a formal theory such as those of the previous sections, there is a certain tension between, on the one hand, the demands of consistency and technical simplicity and, on the other hand, those of being true to the features and complexities of the linguistic usages corresponding to the concept whose meaning is built on and modified by the formal theories in question. Clearly a formal theory cannot adhere slavishly to usage if this usage is conflicting and/or if its formalization would result in an inconsistent theory. Against that, a formal theory claiming to represent—even in part—or regiment a concept in general use, such as that of 'part', cannot lose all sight of the informal usage from which it is intended to derive its interpretation and plausibility. A compromise between these two unacceptable extremes has to be found, but it is no easy matter to determine where. This whole book is in part an attempt to hit on such a compromise for the small word 'part'. It is by no means obvious that there is a single, uniform sense of this term: indeed, we shall claim in the next chapter that there are a number of distinct concepts of part, some of which possess formal analogies to others, and that it is highly doubtful whether these are all restrictions of some single, overarching part concept. If this is correct, then it is not a conclusive argument against this or that theory that there are senses of 'part' which it does not reflect, nor can we thereby argue against it that it cannot therefore be *the* theory of part and whole. For perhaps there is no such theory.

3.2.1 Proper Parts

In an article criticizing extensional part–whole theory, Rescher makes points similar to (1)–(4) above.[16] Point (1) is, however, quickly dealt with. All extensional part–whole theories recognize a sense of 'part', namely 'proper part' (represented by '$\ll$' or 'ppt' above) in which it is false to say that something is part of itself. This asymmetry indeed seems to characterize our most basic conception of part. It is possible to construct the theories on this concept as primitive, as we have seen clearly in the case of system S above. The choice of '$<$' or 'pt' as

[16] RESCHER 1955. Cf. the criticism of it in LEJEWSKI 1957.

primitive is purely a matter of convenience, and the only possible objection can be to calling the predicate or functor involved a 'part'-relation. However, in describing the relation we *have* to bring in the notion of 'part' somewhere, so it is mere verbal quibbling to object to the inconsiderable adjustment involved in including the limiting case.

3.2.2 Non-transitivity

As examples of non-transitive part-relations, Rescher mentions those between military and biological units. A platoon is part of a company, a company part of a battalion, but a platoon is not part of a battalion. A nucleus is part of a cell and the cell is part of an organ, but the nucleus is not part of the organ. Or, to take an example from a more detailed linguistic investigation by D. A. Cruse,[17] a handle is part of a door, and the door is part of a house, but the handle is not part of the house.

In all these cases, and in many others like them, there seems to be a way of understanding the term 'part' so that the objection is clearly wrong, namely that sense where the one object is spatio-temporally included in the other. If the cell nucleus is not part of the organ, is it somewhere outside the organ, or perhaps merely adhering to it? But then either its cell/is not inside the organ or it is not the nucleus of a cell inside the organ. Again, if the handle is not part of the house, is it lying somewhere detached from the house? But if so, how can it be part of a door that is part of the house? If we consider merely the men concerned, the platoon *is* part of the battalion.

It may perhaps be true, however, that there are senses of 'part' in each case which are narrower than this basic spatio-temporal sense, and involve some other relation, perhaps one of being a part making a *direct functional contribution* to the functioning of the whole. In this sense, for example, the nucleus contributes directly to the functioning of the cell, the cell directly to the functioning of the organ, but the nucleus contributes thereby only indirectly to the functioning of the organ, so is not a part in the narrower sense. Similar points apply to the door-handle example. The case of military units is slightly different. Here we are not strictly dealing with an individual in the

[17] CRUSE 1979. Cruse's (albeit tentative) use of the frame '*A* has a *c*' for the part–whole relation is certainly unhelpful, as the frame is too weak to pick out such relations from others. Not even the most ardent male chauvinist would use the truth of 'John has a wife' to argue that she is part of him. But Cruse makes numerous useful remarks, e.g. concerning the difference between parts and attachments.

sense that a soldier is an individual. Nor does a mere collection or set of soldiers count as a military unit. Such units are institutions, and are in part constituted by such relatively abstract objects as Army Ordinances and lines of command. The sense of 'part' which is not transitive probably has to do here with direct lines of command and responsibility, perhaps also with function.

In each of these cases, non-transitivity arises by narrowing or specifying some basic broad part-relation, the specifications introducing concepts themselves extrinsic to part–whole theory, such as function, causal contribution, and lines of command. The existence of such examples does not in any way entail that there are not more basic senses of 'part' which are transitive: indeed, it tends on analysis to confirm that there are. Further, since the considerations *restricting* the transitive concepts are frequently themselves not relevant to part–whole theory, this is reason to exclude them from mereology as such.

There is, however, one intransitive sense of 'part' which *is* intrinsic to the theory, but which, being easily definable in extensional terms, in no way constitutes an objection to extensional part–whole theory. This is the notion of an *immediate* part, definable in the vocabulary of even the weakest system of §1.4. x is an immediate part of y iff[18]

$$x \ll y \wedge \sim \exists z \ulcorner x \ll z \wedge z \ll y \urcorner$$

It is clear that this is applicable only where the individuals concerned are not all atomless, and further that it is itself a restriction of the transitive relation $\ll$. We conclude therefore that the objection concerning non-transitivity does not effect the legitimacy of extensional part–whole theory in accepting basic transitive part-relations.

3.2.3 Too Many Sums?

The general existence of sums or fusions is the feature of extensional mereology which has found least acceptance. Among those who have rejected it are Rescher and Chisholm,[19] as well as the authors mentioned in §2.9. It does not appear to be *intrinsic* or analytically part of the part-concept that to any arbitrary collection of individuals there is a sum individual including all these and nothing outside them.

[18] Cf. Husserl's definition of 'absolutely immediate parts' as 'such as may enter into *no* part of the same whole' (HUSSERL 1984: A262/B$_1$268 f., 1970: 469. 'Part' here obviously means 'proper part'. Findlay's translation wrongly has 'any' instead of 'no'.

[19] RESCHER 1955, CHISHOLM 1976: 219.

Someone disputing the sum axioms is not disagreeing with Leśniewski, Goodman and co. about the meaning of 'part', nor does he fail to understand what is meant by 'sum' or any of its synonyms.[20] He is disagreeing, as Goodman admits,[21] over the sense and/or range of application of the term 'individual'.

Goodman's use of the term 'individual' is frankly technical. Goodmanian individuals do not have to be connected, causally cohesive, or possess other properties which medium-sized dry individuals falling under sortal concepts normally possess. The intended analogy between extensional part–whole theory and extensional set theory plays a considerable part here. Just as any old collection of individuals can (*modulo* occasional paradoxes) be comprehended into an abstract set, so, argues Goodman by analogy, any old collection of individuals may (*without* threat of paradoxes) be considered to make up a sum individual. This individual can serve for some purposes as a substitute for the set for philosophers who deny the existence of abstract sets, such as Goodman or Leśniewski. Just as abstract sets may be comprehended of individuals which are ill assorted, and do not constitute a class, group, collection, or whatever in any everyday sense of the word, so a Goodmanian individual may have an odd assortment of parts, and may not be an individual, substance, or thing in any everyday sense of the word.

This analogy with sets illuminates Goodman's purpose, and illustrates well enough the measure of technical innovation involved; but it only constitutes a good analogy to the extent that the existence of arbitrarily membered sets is acceptable. We shall see in the next chapter that the existence of *concrete* (as distinct from *abstract*) pluralities of various kinds may be reasonably asserted, though the existence of arbitrarily membered pluralities must be more carefully argued for, since such entitites (for which we use the term 'class') appear to some extent to be the mere reflection of the existence of plural terms, and it has to be shown that this is more than just a *façon de parler*. Goodman's individuals have a similar appearance: they seem to exist just because there is a form of expression which requires a referent. The objection here is not that there are *no* sums, for clearly there are: rather it is the assumption that there are *arbitrary* sums

[20] As has been suggested to me in a letter by Audoēnus Le Blanc. One can understand the axioms and definitions of a theory without being convinced they are true.

[21] GOODMAN 1956 (1972: 155 f.)

which is in question. The requirement that, to be counted as an individual, something must possess exact identity conditions, is perhaps a little too strong: a great many of the things which we count as individuals possess identity conditions which are somewhat blurred at the edges, not least ourselves. However, for the things which we habitually count as individual substances, the identity conditions we possess are tied, as Wiggins has plausibly argued,[22] to a substance noun or sortal term, and the admission of Goodmanian sums does away with this general feature. Of course the defender of Goodmanian sums may point to the feature of extensional part–whole theory whereby identity consists in community of parts. However, as we shall see, this general condition is itself questionable.

For all the arguments on both sides, the opponent of sums would have a far better case if he could prove, as Russell did for sets, that their general and uncritical acceptance leads to contradictions. But no critic has managed to show this, for the very good reason that even the strongest extensional mereology is demonstrably consistent.[23] It is not the formal theory which is suspect, but its uncritical application to the world. We have only the uneasy feeling that sums are gratuitous entities, although they cannot be accused of leading to inconsistency. However, the ease with which Goodman, Leśniewski, and like-minded philosophers are able to accept them is not independent of the general rejection by such philosophers of abstract objects, and a decided predilection for materialistic ontologies. For someone who believes in universals, or mathematical objects, or possibilia, or Meinongian non-existents, that there could be sums bridging the gaps between the various ontological realms does not seem plausible, though there might be sense in talking about part–whole relations *within* the different realms.[24] On the other hand, it is again not clear what harm such sums could do which is not already inherent in the ontological pluralism presupposed. But sums undeniably look considerably less sinister when restricted within some sort of monistic ontology. Sums could be considered as the occupants of arbitrary portions of space or space–time, it being accepted in advance of course

[22] WIGGINS 1980: ch. 2.

[23] The simplest model of Mereology is the one-atom model. There are also proofs of the consistency of Mereology relative to the theory of real numbers (CLAY 1968—the idea goes back to Leśniewski) and to Protothetic (LEJEWSKI 1969).

[24] For part-relations between universals, see ARMSTRONG 1978: vol. II, 36 ff., and the brief discussion in §4.10 below. Part-relations among abstract objects have, however, been left outside the scope of this book.

that the existence of such arbitrary portions is itself acceptable. Here we have the pun on 'extension': extensional mereology works for extents, portions, or regions of extended space or space–time. If material objects are conceived simply as extended in space or space–time, as the occupiers of these portions, then the credibility of material objects which are arbitrary sums is enhanced by the credibility of the extended portions which they occupy. Similarly, anyone who considers that there is an amorphous prime matter, which exists purely by virtue of its occupation of space or space–time, will be likely to find arbitrary masses or heaps of the stuff as acceptable as arbitrary portions of the space or space–time it occupies. The point about such cases, which lends credibility to the sums in question, is that the associated predicates are *cumulative*: if a sum exists, the predicate which applies to the parts applies to the whole: any sum of portions of space–time is itself a portion of space–time, any sum of portions of a certain kind of stuff (whether primary or not) is itself a portion of that kind of stuff. Quine has pointed out the cumulative aspect of mass predicates,[25] and it would seem that masses of stuff seem best suited to form arbitrary sums, a line we pursue in the next chapter. Where a predicate is not cumulative, as 'is a man', we are inclined to look askance at putative sums. At best, many would want to say, what exists when we 'heap' together a number of men is not an individual, but rather a number of men: a class, plurality, or manifold of men. And this is itself not an individual, but a plurality of individuals. For someone who accept Goodman's nominalistic principle that there is 'no distinction of objects without distinction of content',[26] the acceptance of concrete pluralities would be reason to deny the existence of sums as well, or, perhaps, to claim that such pluralities are merely sums under another name. We shall see presently however, that it is possible for a sum or a plurality to occupy precisely the same portion of space–time as an individual and yet not be identical with it, so that Goodman's principle is to be rejected. And in general, once we move away from the simplistic view of matter or material objects as mere occupiers of space or space–time, we shall find the assumption that there are arbitrary sums less attractive, even if it cannot be shown to lead to contradictions.

Even the assumption that portions of space, or time, or space–time may always be summed to give another portion satisfying the

[25] QUINE 1960: 91.
[26] GOODMAN 1956 (1972: 158 ff.)

mereological sum axiom would not be universally accepted. Many would add the requirement that the portions in question all be *connected*. It is here worth recalling the interval model of §1.4, which showed that a finite number of intervals may have an interval as least upper bound without having a sum. On models of higher dimension, a least upper bound may not exist for a number of disconnected portions, though they may have upper bounds.

3.2.4 Mereological Extensionality

The last section showed that considerations in part–whole theory quickly lead to questions of general ontology, which is not surprising, given the highly general or formal nature of mereological concepts. In extensional part–whole theory there is a close tie between mereological concepts and the concept of identity via the Proper Parts Principle SA4 and the definition of 'part' SD1, giving rise in particular to the theorems

SCT7 $x = y \equiv \forall z \ulcorner z < x \equiv z < y \urcorner$

SCT71 $\exists z \ulcorner z \ll x \urcorner \vee \exists z \ulcorner z \ll y \urcorner \supset . \forall z \ulcorner z \ll x \equiv z \ll y \urcorner \supset x = y$

These correspond to two different meanings of 'individuals with the same parts are identical'. The first is in a sense quite unobjectionable, since 'part' is *defined* in SD1 as 'proper part or identical'. There is the supplementary question whether this is the best definition of the intuitive concept of proper-or-improper part. Consider the following alternative definition:[27]

SD15 $x \leq y \equiv (\exists z \ulcorner z \ll x \urcorner \supset \forall z \ulcorner z \ll x \supset z \ll y \urcorner) \wedge$
$(\sim \exists z \ulcorner z \ll x \urcorner \supset x \ll y \vee x = y)$

It follows by elementary logic and the transitivity of $\ll$ alone that

SCT72 $x < y \supset x \leq y$

but the proof of the converse implication requires SA4, the Proper Parts Principle. Without it, we cannot prove either of

SF12 $x \leq y \wedge y \leq x \supset x = y$

SF13 $\forall z \ulcorner z \leq x \equiv z \leq y \urcorner \supset x = y$

With this in mind, let us consider Rescher's next objection: it runs[28]

[27] I am indebted to the reader for Press for the helping me to clarify what is at stake here.

[28] RESCHER 1955: 10.

The extensionality property, which entails that wholes are the same if they possess the same parts, rules out those senses of 'part–whole' in which the organization of the parts, in addition to the mere parts themselves, is involved. Different sentences can consist of the same words.

We have noted the ambiguity inherent in the expression 'wholes are the same if they possess the same parts'. Rescher has in mind a thesis of the same form as SF13. On the other hand, he makes a thesis like SF12 one of his axioms, while arguing also that 'part' need not be reflexive, whereas both our relations $<$ and $\leq$ are reflexive. The prose seems to fit SCT71 better. However, it is the objection and the example which we wish to discuss. The objection is in our view perfectly correct. We might put it in this way. Certain individuals exist purely in virtue of a number of other individuals' existing, those which are its parts. These are sums. Then there are individuals which exist only when certain other individuals exist *and are related to one another in a certain way.* Take Rescher's example of linguistic objects. This is in one way rather unfortunate, since it speaks of what can be the case rather than what is the case, and also it is most readily understood as referring to abstract types rather than concrete tokens. We are not discussing the mereology (if there is one) of abstract objects (if there are any), so let us consider word tokens. Suppose we have a set of pieces of plastic formed like words, such as might be used for graphic displays or children's games. These are as concrete as one can get. Now in the box of words, suppose there are, among others, a token of the word 'cardinals' and a token of the word 'multiply'. While these two are lying around in the box with the other tokens their sum may exist, but there is no sentence in the most literal sense made up of them. If we now remove them and lay one before the other, we create, for example, a token sentence 'multiply cardinals', which may be construed as an imperative directed to arithmeticians, or a different imperative addressed to the Pope. Now Rescher's point is that this sentence is other than the sum of the parts, since the sum existed beforehand, but the sentence did not. And if we now reverse the order of the tokens we destroy the original sentence and create a new one, 'cardinals multiply', which is declarative, and could mean various things, for example, that the Pope is busy, or that their Eminences are busy at arithmetic or other activities. Whatever the blocks may be taken to mean, we have two different English sentences. It could be objected that these sentences exist at different times, and that this allows a mereologist of extensional leanings a way out via temporal parts (see the next section). But we can imagine the pieces to be semicircular, so that they form a ring, so that the two sentences exist simultaneously and which sentence we are reading depends on where we choose to start. This is the best way I can devise to rescue Rescher's example. While it is natural to say that the two sentences are composed of the same (token-) words, they do not obviously provide a counter-example to the Proper Parts Principle and SCT71, because it is not obviously wrong to say, for example, that one sentence has the fragment 'nals mult' as a part while the other does not.

To find a counter-example in Rescher's style we need two individuals which are clearly not identical yet which equally clearly have all the same parts, so that their difference is not a matter of the parts alone but also of the relations in which these are required to stand to one another; the same individuals must stand in two different sets of structural relationships at once. Examples spring to mind among pluralities. The Family Robinson may at the same time be the Basketball Team Robinson. But it is here tempting to identify the two. Or the Buildings Committee may have the same members as the Personnel Committee. A convincing natural example ought to come from individuals rather than pluralities, and here we must anticipate later discussion and mention Doepke's example of a person and that person's body.[29] What this and other examples suggest is that there is another way to generate counter-examples to extensionality than considering the relationships among the parts in the static sense of how they stand to one another. The difference between a person and a body is that the former only exists as long as certain kinds of process are going on in the body. These processes are not parts, nor are they relationships among the parts, though they are closely connected with such relationships. So when Rescher speaks of organization, we can understand that to mean anatomy, or physiology as well as anatomy.

It is usual among extensional mereologists to attempt to explain away such putative counter-examples either by claiming the identity of the objects concerned, or by claiming the non-existence of at least one, or by resorting to an ontology of four-dimensional objects. This last move is discussed below. An attempt to cope with the fact that different individuals may be constructed from the same parts by concatenating them in a different way, without invoking a four-dimensional ontology, and employing only the part–whole relation, is

[29] DOEPKE 1982.

made by Eberle. His favoured theory requires, however, the construction of 'relational individuals'. In order to do this, he needs a nominalist analogue of ordered pairs, and these are constructed by considering atoms to have 'sides', *which are nevertheless not parts*, and new individuals ('links') to consist of the left side of one atom and the right side of another.[30] The addition of the clause in italics seems to me to reduce Eberle's approach to absurdity. If an object has distinct sides, it has distinct parts, for where there are no parts, neither extension, nor shape, nor divisibility (*ergo* nor distinct sides) is possible.[31]

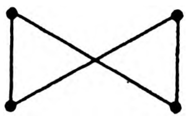
Certainly, if we wish to maintain the thesis PPP for substances in the Aristotelian sense, three-dimensional continuants which come into being, change, and pass away, then we land in difficulties, as an example of David Wiggins shows.[32] A cat, Tibbles, consists of a body, Tib, and a tail, Tail. But we cannot identify Tibbles with Tib + Tail because Tibbles could, through an accident, lose her Tail, but still exist, while Tib + Tail cannot lose Tail and continue to exist. Even if Tibbles fortunately never parts company with Tail, the essential possibility that she could do so is enough to distinguish her from the sum. But then, if Tibbles remains intact throughout her life, this is a case where every part of Tibbles is a part of Tib + Tail and vice versa, yet Tibbles $\neq$ Tib + Tail, which contradicts PPP. Two distinct material objects can then coincide spatially for their whole lives, yet not be identical. The defender of PPP may attempt the familiar move to four-dimensional objects to clear the objection. If, it will be said, the cat does not lose her tail, then the four-dimensional cat entity is identical with the sum of cat entity parts, whereas if the cat does lose her tail, she will be a proper part of this sum whose earlier temporal parts (before tail loss) are identical with the earlier temporal parts of the sum, and whose later temporal parts leave out the later temporal parts of the disconnected tail. It may be admitted that whether or not the four-dimensional cat-process is identical with the sum of cat-part processes is a contingent matter, but the extensionality principle is saved.

The answer does not avoid Wiggins's objection, since we can transpose this into the ontology of the defender. The difference between a four-dimensional cat-process and a sum of cat-part-

[30] EBERLE 1970: 93. [31] Leibniz, *Monadology*, §3.
[32] WIGGINS 1979. The example comes from William of Sherwood via Geach.

processes is that the sum's later phases can be grossly disconnected, while those of a cat-process cannot. The sum can branch out widely in a way in which the cat-process cannot. If the sum does not thus branch, it remains such that it *could* have branched, thereas the cat-process lacks this possibility. Hence the two may coincide spatio-temporally as four-dimensional objects, but differ in respect of this *de re* modal property. Of course, many defenders of four-dimensional ontologies do not like modality, though Wiggins has suggested the *de re* properties here involved are consonant with referential transparency.[33] The only way out for defenders of PPP is to deny modality *de re* altogether. This indeed Quine and others have done, but the issue is a wider one, not local to mereology.

We showed in § 1.4 that PPP follows on acceptably weak mereological assumptions from the Strong Supplementation Principle SSP (SA5). So if PPP is rejected, SSP must be rejected along with it. And indeed, if we look once again at the four-element model (see below)



which is a rough approximation to the state of affairs with the two-word ring sentence, we can see at once that SSP is here falsified. This model is indeed the simplest one to falsify both SSP and PPP, showing two different individuals compounded of the same proper parts.

Rejecting SSP gives no reason for rejecting the weaker WSP (SA3), which this model respects. The Weak Supplementation Principle, according to which an individual with a proper part has another proper part disjoint from the first, is much more plausible, so much so that, in cases of putative counter-examples where a disjoint supplement is lacking, we are more inclined to deny that the one object is a proper part of the other at all. That would suggest that WSP is indeed analytic—constitutive of the meaning of 'proper part'.

To abandon PPP is to leave behind the well-charted waters of extensional mereology. The relation $\leq$ is not anti-symmetric, no

[33] WIGGINS 1980: 110 ff. This presupposes Wiggins's own analysis of *de re* modality, but the analysis offered in Ch. 7 allows the same point to be made: $\Diamond Fx \wedge \sim \Diamond Fy \supset x \neq y$.

longer a partial order, but only a semi-order: reflexive and transitive. In terms of it one may define a symmetric predicate, the *coincidence* of parts:

SD16 $x \lesseqgtr y \equiv x \leq y \wedge y \leq x$

Individuals which thus mereologically coincide will be, at any time at which they coincide, perceptually indistinguishable from one another. They will occupy the same place at the same time, coincide spatially, or, as we shall say, be *superposed*. It is a question which will occupy us later whether all superposed objects must coincide mereologically.

The apparent oddity of this position, and the loss of algebraic neatness which rejection of PPP brings with it, are prima-facie reasons for looking askance at the rejection of extensionality of parts. We shall see, however, that the oddity disappears on closer acquaintance, and that the riches brought by rejecting PPP more than outweigh the loss of theoretical simplicity by virtue of their descriptive and explanatory power. From the standpoint of later chapters, extensionality of parts will look unacceptably ascetic. This applies in particular to Goodman's Principle of no difference of objects without difference of content: where 'content' means only 'parts', every exception to PPP is an exception to Goodman's Principle.

The issues surrounding the last problem, the move to an ontology of objects extended both in space and in time, are of sufficient importance to require more extended treatment.

3.3 The Flux Argument

According to the conceptual scheme inherent in our everyday thinking, one of the fundamental classes of object to be found in the world is that of things or substances, material objects extended in three spatial dimensions and enduring in time without being extended in time. Such material objects, of which material bodies like organisms provide a paradigmatic example, are contrasted with the events, states, and processes in which they are involved, which are in general both spatially and temporally extended. An object is temporally extended iff it has proper parts which are distinguished by the time of their existence or occurrence, as, for example, a race has earlier and later parts. The earlier parts or phases of a race are over, exist no longer, when the later parts are taking place. By contrast, the things or people running in the race (normally) exist throughout the race. Unlike the race, they have no temporal parts, only spatial ones. Unlike the race, they can change. One of the ways in which they can change is by gaining and/or losing parts. This distinction of physical objects into those which have temporal parts, which are often called 'events', but which we call, following Broad, *occurrents*, and those which do not have temporal parts, which we again follow Broad in calling *continuants*,[34] is a corner-stone of everyday common sense. It should not be given up unless there are compelling reasons to do so. It may be noted that we ourselves are continuants, so any revision involves revising our conception of ourselves. However, a number of philosophers have proposed abandoning the continuant/occurrent ontology in favour of one containing occurrents only, because of difficulties and puzzles to which the everyday ontology gives rise. These difficulties turn out to be mereological in nature, and concern the putative *flux*, the gain and loss of parts, of continuants.

The dialectic illustrating the urge, in explaining change in general and flux in particular, to move to an ontology in which the only physical objects are temporal in the sense of having temporal parts, is displayed in an argument put forward by a number of writers.[35] In fact the versions of the argument vary slightly from one writer to another, but we shall pick out a version which may be considered typical, and which is also useful for our purposes. The Flux Argument, as we shall call it, brings together a number of crucial ontological issues, and has elicited the most varied responses. It is almost a touchstone for finding out important facts about a philosopher's ontology. Let us first go through a form of the argument, and then see what it can lead to and has led to.

We take again poor Tibbles. Suppose we call her tail 'Tail' and the remainder of her 'Tib', as before. At a certain time, *t*, say, Tibbles is a perfectly normal cat with a tail, *felis caudata*. Then comes the accident in which she loses Tail, and at a later time *t'*, Tibbles, having survived, is tailless, *felis incaudata*.[36] Before the accident, Tib and Tibbles were obviously distinct, having different weights, shapes, and parts. After

[34] BROAD 1933: 138 ff.

[35] CARTWRIGHT 1975, CHISHOLM 1976: 157 f., HELLER 1984, HENRY 1972: 118 ff., THOMSON 1983, VAN INWAGEN 1981 are the modern sources I have used. As Chisholm notes, the problem of increase was posed by Aristotle in *De Generatione et Corruptione*, 321a, and there are discussions of this and similar problems in Abelard, Aquinas, Hobbes, Locke, Leibniz, and Hume, among others.

[36] I am assuming for the sake of argument that there is no vagueness about where the tail begins and ends anatomically. This simplifying assumption has no bearing on the flux argument.

the accident they share these and many more properties. So it seems plausible to assert both

(1) Tibbles $\neq$ Tib at t

and

(2) Tibbles $=$ Tib at t'

Since both Tibbles and Tib exist at both t and t', we have

(3) Tibbles at $t =$ Tibbles at t'

(4) Tib at $t =$ Tib at t'

whence by transitivity of identity it follows from (2, 3, 4) that

(5) Tibbles at $t =$ Tib at t,

contradicting (1).

What went wrong? Both in the (studiedly ambiguous) descriptive setting up of the argument and within it a number of assumptions have been made, each of which has a certain prima-facie plausibility, and yet each of which has been questioned at some time or other. There are suprisingly many:

(*a*) Material objects such as Tibbles exist—however they are to be construed.
(*b*) Material objects such as Tibbles and her parts are continuants.
(*c*) Material objects such as Tibbles may gain and/or lose parts without thereby ceasing to exist.
(*d*) Proper parts of material objects, such as Tib and Tail, exist even while still attached to (i.e. undetached from) their wholes.
(*e*) Identity is transitive.
(*f*) Identity is not relative to a sortal concept.
(*g*) Identity is not temporary, or relative to a time.
(*h*) Distinct material objects may not exactly occupy the same place at the same time.
(*i*) Distinct material objects may not share all their proper parts at the same time, provided they have proper parts (temporally relativized version of PPP!)

Where have philosophers parted company over the Flux Argument?

Assumption (*a*) is needed to get the argument going at all. None of the philosophers studied here denies it outright, although Chisholm regards objects with a flux of parts as being not objects, not existing, in the strict sense, but only in a derived sense.

Assumption (*b*) is used in the description setting up the argument, and is our chief concern. The outcome of the argument has been used by a number of philosophers as a reason for denying (*b*). We shall see in the next section how its rejection affects the argument. (*b*) has been denied by, among others, Quine, Cartwright, Henry, and Heller, and accepted by the others mentioned in footnote 35.

Assumption (*c*) is denied by Chisholm for objects which exist in the strict sense, *entia per se*. This is the thesis of mereological *constancy*, and we shall take issue with it below. Objects in a secondary or derivative sense, *entia per alio*, do undergo flux, but only in the sense that they are successively constituted by different *entia per se* which differ in their parts. Note that Chisholm does not deny that there are continuants: indeed, he strongly rejects the four-dimensional ontology of temporal objects. Only for someone who does accept continuants can the question of mereological essentialism in *this* form (the constancy of parts) arise, since flux of parts is a change, and strictly speaking only continuants can change. This will be gone into in greater depth below. Among the remaining writers we have in view, Geach, van Inwagen, and Thomson all accept (*c*).

Assumption (*d*) is denied by van Inwagen. There is a precedent for this way of thinking in Aristotle.

Assumption (*e*) has been questioned by Prior and denied by Garrett.[37] The effect of denying transitivity is also obtained if either (*f*) or (*g*) is denied, i.e. if relative identity is accepted.

Assumption (*f*) is denied by Geach.

Assumption (*g*) is denied by Grice and Myro.[38]

Assumption (*h*) is denied by Wiggins, Doepke, and Thomson,[39] *inter alia*, in our opinion correctly.

Assumption (*i*) is denied by Doepke and by us. Since the disposition of a material object's parts determines its place, we must also deny (*h*).

Working back through the denials, we may see how they block the argument to the contradiction.

Denying (*i*) or likewise denying (*h*) allows one to reject step (2) in the argument. We consider Tib and Tibbles to coincide mereologically at *t′*, but it would suffice to take superposition of the two as a fact. The positive reason for not identifying Tib and Tibbles at *any* time lies in Leibniz's Law: Tib and Tibbles are absolutely distinct since they differ in the properties they have at *t*.

[37] PRIOR 1965–6, GARRETT 1985.

[38] Cf. the reference to this unpublished work in DOEPKE 1982.

[39] WIGGINS 1968, 1980: ch. 1, DOEPKE 1982, THOMSON 1983.

Denying (*g*) or (*e*) blocks the inference to (5). That Tibbles = Tib at *t*′ then no longer entails that Tibbles = Tib at *t*.

Denying (*f*) means rephrasing the identities using sortal predicates, for example 'is the same cat as' and 'is the same (undetached) cat-part as'. What we then get is somewhat as follows: Tib is the same cat as Tibbles at *t*′ and the same cat-part as Tib at *t*, but we cannot thereby conclude that Tib is the same cat as Tibbles at *t*, or the same cat-part as Tibbles at *t*. Effectively, transitivity is blocked, as in the two previous cases.

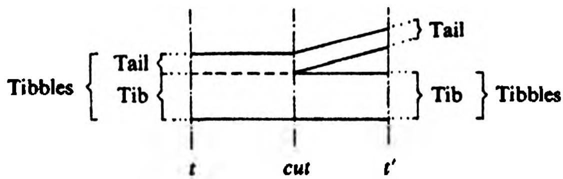
Denying (*d*) means denying that Tib (and Tail) exist at *t*, so the question as to that with which they are identical which exists at that time does not arise. But if Tib comes into existence at the time (call it *cut*) of the tail's being lost, how can it be identical with Tibbles, which existed beforehand? Van Inwagen accepts (*a*) and (*b*), the classical logic of identity, and (*h*). So he must either deny that something (Tib) comes into existence at *cut*, or move to a Chisholmian view, denying (*c*). He opts for the former. Tibbles gets smaller when Tail is cut off, but the only thing that comes into existence is Tail (if Tail is whole, otherwise various other bits come into existence.) Van Inwagen's position seems not only to fly in the face of common sense (which he recognizes), but to be unnecessarily radical in the light of the easier alternatives of denying (*h*) or (*i*). The consequences of denying the existence of undetached parts must surely be grave.[40]

The denial of (*c*) will occupy us at length below, and we reserve discussion of it until later. It may be noted that Chisholm's revision of our normal conceptual scheme is less radical than revising the logic of identity or the ontology of continuants, but is more radical than denying (*h*) or (*i*). Denying (*c*) blocks the argument again at step (2). Tibbles and Tib are not identical at *t*′, though they are very intimately related by both being constituted, at that time, by the same mereologically constant *ens per se*.

3.4 Fourth Dimension

Denying (*b*) is a favoured way out of the Flux Problem. Suppose we 'reconstrue' material objects as four-dimensional processes. Then the separation of Tib from Tail looks rather like the diagram below,

[40] VAN INWAGEN 1981 suggests that predications apparently about undetached parts can always be diverted to be about something else, but does not go about carrying out the enterprise.



where we suppress two spatial dimensions and let the temporal dimension be represented by the horizontal spatial direction.[41] Here we see a temporal section between *t* and *t′* of Tibbles, Tib, and Tail, or rather, of the corresponding processes, but we shall continue for the moment to call the processes by the same names as the continuants. The later part of Tibbles, between *cut* and *t′*, is identical with the later part of Tib, but its earlier part, between *t* and *cut*, is identical with the earlier part of Tib + Tail. So Tib and Tibbles do not share all their parts, differing in respect of their earlier ones. Thus accepting their non-identity does not entail rejecting mereological extensionality. It is also natural to understand a phrase like 'Tibbles at *t*' as denoting a momentary three-dimensional phase or time-slice of the four-dimensional object Tibbles, and this blocks the Flux Argument in the form given because (3) and (4) as thus construed are false. Temporal modification is removed from the predicates and instead built into the terms, so there is no need to tense mereological predicates or index them with times: they may be 'eternal', as they are usually understood in extensional mereology.

Note that the later part of Tib + Tail is spatially disconnected, whereas its earlier part is not. Tibbles remains connected throughout. What if Tibbles had remained intact? Then it would seem that Tibbles = Tib + Tail. Recall however our argument from the previous section. Tibbles is (thus reconstrued) a *cat*-process, and so cannot spread out in the way Tib + Tail can. Hence, although Tibbles and Tib + Tail *ex hypothesi* share all their parts, they are distinct—a counter-example to PPP. To counteract this, the defender of extensional mereology must claim that the notion of such *de re* modal properties is absurd or incoherent or in some other way unacceptable.

Assuming this claim is made, what advantages does the four-dimensional ontology offer? It is not just a matter of rescuing

[41] WOODGER 1937 uses just such diagrams in discussing the biology of natural fission and fusion.

extensional mereology, which is a relatively moderate gain. For one thing, there is no tampering with the logic of identity. For another, it does not fly directly in the face of common sense by flatly denying something that seems obvious, such as that continuants have undetached parts, or may undergo flux of parts. The revision is a transposition into a new ontological key which does not deny such facts, but reformulates them in a new language. For this ontology speaks its relative simplicity (look—no continuants), its use of a category (occurrent, event, process) which is already familiar, and the straightforwardness of its treatment of the relation between space-time and what fills it (compare the denial of (*h*)). Finally, there is the suggestive consonance of this ontology with the view of things found in most presentations of the physics of relativity theory. All of these advantages have recommended this ontology to a series of scientifically inclined philosophers from Whitehead onwards.[42]

It is only fair to outline the positive side of the four-dimensional ontology, because we are going to reject it as being scarcely intelligible to us. This might be merely a defect of our present modes of speech and cognition, one which could in time be overcome by enough work on the part of philosophers, scientists, and linguists. Our complaint is not that the four-dimensional ontology is inconsistent (if it is inconsistent, this fact is far from obvious), but that no one has begun to do the work required to make it understandable to we users of a continuant/occurrent ontology. Until it is shown that it can be made comprehensible, there is no reason to reject our present—largely Aristotelian—framework. In particular, the Flux Argument can be blocked by the much simpler expedient of denying assumptions (*h*) and (*i*).

A representative summary of what happens to material continuants in the process ontology is the following by Quine, which nicely illustrates several of the points made above:[43]

Physical objects conceived thus four-dimensionally in space–time, are not to be distinguished from events or, in the concrete sense of the term, processes. Each comprises simply the content, however heterogeneous, of some portion

[42] Notable representatives have included McTaggart, Russell, Carnap, Quine, and Smart. Strongly against it are Broad, Prior, Geach and Chisholm, *inter alia*. See also HACKER 1982, which brims over with good sense. Hacker calls the four-dimensional picture a 'metaphysical quagmire' (4) and doubts whether we could speak four-dimensionalese (5).

[43] QUINE 1960: 171.

of space–time, however disconnected and gerrymandered. What then distinguishes material substances from other physical objects is a detail: if an object is a substance, there are relatively few atoms that lie partly in it (temporally) and partly outside.

Quine's last point here is simply wrong. It suggests that material substances are simply those which gain and lose relatively few atomic parts. But for objects in constant flux, such as organisms, there are probably very few atoms which stay with them throughout their careers.

Chisholm has attempted to show that three arguments which have been offered in favour of the ontology of occurrents are unconvincing. Here, in summary, are the arguments and Chisholm's rebuttals:[44]

(1) The argument from spatial analogy. Chisholm points out that, far from there being an analogy between space and time, there is an important disanalogy at just the crucial point: 'One and the same thing cannot be in two different places at one and the same time. But one and the same thing can be at two different times in one and the same place.'[45] The point is not conclusive, since the defender of temporal parts may use his thesis to argue against the disanalogy. But he cannot then use the argument to argue for his thesis without circularity.

(2) The argument from change. How can Phillip be drunk last night and sober this morning? For him to be both is contradictory. So it is different temporal parts of him of which we predicate drunkenness and sobriety. This point is easily coped with: rather than say that one stage of Phillip is (timelessly) drunk and another stage is sober, we simply say that Phillip was drunk last night and is sober this morning! Either we uses tenses, as in natural language, or we relativize our predicates to times.

(3) The argument from flux (not the same as the Flux Argument above!): Quine uses temporal parts to solve Heraclitus' problem about bathing in the same river twice: 'Once we put the temporal extent of the river on a par with the spatial extent, we see no more difficulty in stepping into the same river at two times than at two places.'[46] Chisholm replies that, since not any sum of river-stages is a *river-*

[44] CHISHOLM 1976: appendix A. [45] Ibid.: 140.
[46] QUINE 1960: 171.

process, we must say what conditions such a sum must satisfy in order to be a river-process. He finds that this either again presupposes continuants (river banks, human observers), or a theory of absolute space, or finally requires the introduction of a technical term 'is co-fluvial with', which can only be understood to mean 'is part of the same river as'. So we are no further forward.

The last point touches the nerve of the reform suggestion. To be successful, the suggestion must show how to eliminate *all* singular and general terms denoting continuants, and *all* predicates and other functor expressions for which singular or other terms denoting continuants are argument expressions. All proponents of a process ontology, with the possible exception of Whitehead, indulge in a form of double-talk when it comes to giving concrete examples. So we have talk of river-stages, or stages of Phillip that are drunk or sober, and so on. Quine talks happily of 'conceiving' or 'construing' continuants four-dimensionally. But this is not simply redescription, for when something is redescribed, *it* gets a new description. Reconstrual, taken seriously, is rejection. Continuants literally disappear from our ontology, leaving something else in their stead. To describe what is introduced, we need a completely new language. So it is cheating to talk of *river*-stages, of stages of *Phillip*, of *cat*-processes. Even if we allow such talk—as a temporary measure—it is not just cheating but *false* to talk of bathing in a river-stage, of a Phillip-stage being drunk. It cannot be right to change the subject and leave the predicate unmodified and still think one has a true sentence. Likewise, Cartwright talks of the distinct *careers* of four-dimensional objects.[47] But only continuants, like generals and opera singers, have careers A four-dimensional object does not *have* a career—at best it *is* a career. But if continuants disappear from our ontology, it is not a career *of* anything. The double-talk is an attempt to smooth over the difficulties involved in replacing continuants by keeping the respectable familiarity of the old while reaping the benefits of the new. The strangeness of the four-dimensional objects is almost always underestimated in the literature. Where it is not, in the philosophy of Whitehead, we are confronted with an obscurity which has become proverbial.

The problems of understanding the process ontology arise in part because we naturally use our existing conceptual scheme involving

[47] CARTWRIGHT 1975: e.g. 167.

both continuants and events to try and step beyond it to one without continuants at all, and it is questionable whether we can succeed in leaving continuants behind. Even scientists who use four-dimensionalese in their work relapse into being Aristotelians when they talk about their books, apparatus, families and—themselves. To be really thorough, one would have to replace all this talk as well. If Strawson is right,[48] this is impossible, since reference to anything is ultimately parasitic upon reference to perceptually prominent continuants—bodies and persons. Perhaps it is possible to transpose Strawson's requirements regarding objectivity and reidentification into a process ontology. In that case, even if we are in fact unable to work ourselves completely into such an ontology, it would be in principle possible for other sentient beings or perhaps alien human cultures to live with such an ontology, engage in commerce and science, and flourish. Whether this is possible seems to me to be not a priori decidable: we should have to wait until we confronted such a culture—and then we should have the translation problem to solve.

None of this shows the *impossibility* of a process ontology, simply its alien nature. For not only do continuants go, so also do the most familiar of occurrents, the events which involve continuants, in particular changes in continuants. For in the four-dimensional ontology, as Geach has stressed,[49] there may be timeless variation, but there is no change. Change consists in an object having first one property (or accident, as I should prefer to say) and then another, contrary one. But processes have all their properties timelessly, in the sense that what have different properties are different temporal *parts* of a process, and not the whole process. This is no more change than the fact that a poker is cool at one end and hot at the other is a change.[50]

The credentials of the four-dimensional ontology are often considered to hang together with those of physical science, in particular relativity theory.[51] But the evidence that relativity theory forces us to abandon the ontology of continuants and events is slight and circumstantial. It is true that Minkowski diagrams represent time as simply another dimension along with the spatial ones, but we cannot

[48] STRAWSON 1959: ch. 1. [49] GEACH 1965 (1972: § 10.2).

[50] MCTAGGART 1921–7: §§315–16. Cf. GEACH 1965 (1972: 304).

[51] Cf. for example SMART 1972, who claims that the Minkowskian picture of special relativity and the calculus of individuals together make up 'a metaphysically neat picture' (13).

argue from a diagram, which is only a convenient form of representation. A closer examination of the concepts and principles of relativity, however, shows that they rest squarely on the ontology of things and events. A *world-line* is a sum of *events*, all of which involve a single *material body*; any two events on the same world-line are *genidentical*. That which cannot be accelerated up to or beyond the speed of light is something with a non-zero mass. But only a continuant can have a mass. In like fashion, the measuring rods and clocks of special relativity, which travel round from place to place, are as assuredly continuants as the emission and absorption of light signals are events. Nor does relativity entail that large continuants have temporal as well as spatial parts. It simply means that the questions as to which parts large continuants have at a given time have no absolute answer, but depend on fixing which events (such as gains and losses of parts) occur simultaneously. Whether body of gas A detaches itself from a large star before, after, or simultaneously with the falling of body of gas B into the star, may depend on the inertial frame chosen.[52]

In view of these problems, we suggest that rejection of the old ontology be postponed until such time as the promised better alternative is in a more liveable state. The Flux Argument enjoins then that we reject extensional mereology, but the minor inconvenience this brings is nothing compared with the conceptual upheaval involved in rejecting continuants.

3.5 Applications of Extensional Mereology

If extensional mereology is rejected as a universal theory of part and whole, it is nevertheless worth asking whether there are more restricted domains within which all its principles may be truly applied. Of course, any abstract mathematical structure which is a model of extensional mereology, i.e. is a complete Boolean algebra without zero, is trivially such a region, but we are concerned with concrete applications, that is, applications where the individuals of the domain are not abstracta. Without claiming exhaustiveness, it seems plausible to suggest the following candidates: spatial, temporal, and spatio-temporal extents; the prime stuff or occupier of such extents; occurrents; and finally, masses and pluralities in general. Of all these

[52] For further arguments that relativity theory does not compel rejection of the ontology of continuants and occurrents, cf. MELLOR 1981: 128–32.

candidates, perhaps only the first group is really uncontroversial. In particular there appears to be nothing against taking spatio-temporal extents to fulfil the laws of classical extensional mereology. The only—big—question is whether there are such extents. That is a question which lies outside the scope of this book; but were it be answered affirmatively, then we should have a straightforward application. Regarding the prime occupier of extents, which Quine for example called the 'content of a portion of space–time', we may have doubts as to whether there is any such, in view of the possibility of superposition which will be mentioned below. As to matter in the traditional sense, as the occupier of space (as against space–time), it is again far from clear that anything answers the description. *If* there should be such stuff, however, since it is continuant (one cannot exactly say *a* continuant), we need the considerations to be found in Part II. Occurrents seem to have some prospects of fulfilling at least the extensionality principle, whether or not arbitrary sums are allowed. For them it is important to define the notions of spatial and temporal part. As to masses (in general, not just masses of prime matter) and classes or pluralities, it turns out that neither calculi of individuals nor the more ample resources of Mereology serve all the purposes we have even for an extensional part–whole theory, since we shall discover that there are different senses of 'part' according to whether we are talking of a relation between individuals, between classes, or between masses. This is a rather different criticism from the foregoing: the point is not that extensional part–whole theories are here misleading, but rather that they have several different, but analogous, applications. The connections between the different analogous senses of 'part' thus finding employment are sufficient to prevent there from being a single, overarching sense of 'part' which covers all of them, despite their appealing formal parallels. The way to meet this criticism is not to abandon extensional part–whole theory as such (although we do not attempt to defend it for individuals), but rather to reduplicate its structure in a peculiar way. This rather radical suggestion cannot be made clear until the requisite notions of mass and class have been introduced. The applications of extensional mereology to occurrents, masses, and classes will be discussed in the next chapter. The status of matter and its mereology must wait until temporal notions are introduced in Part II.

4 Occurrents, Classes, and Masses

In this chapter we discuss the mereology of three kinds of object for which it is reasonable to suppose the extensionality principle PPP holds: occurrents, classes, and masses. We have already encountered the former in the previous chapter. Classes and masses provide the most plausible concrete interpretations of classical extensional mereology. The ontological categories of class and mass have been relatively neglected by comparison with that of individuals, and we shall need to introduce them carefully. The concept of class is not that found in contemporary set theory, but what we contend to be the old-fashioned notion of a plurality of individuals; this view too will need some preliminary explanation.

4.1 Occurrents

Occurrents comprise what are variously called events, processes, happenings, occurrences, and states. They are, like continuants, in time, but unlike continuants they have temporal parts. The concept of temporal part will be more precisely defined below. We adopt Broad's term 'occurrent' rather than resort to the expedient, often found in philosophical discussions, of stretching the term 'event' to cover processes and states.

These are numerous general ontological problems concerning occurrents which we do not touch on because they do not directly involve mereology.[1] Throughout this section and the next, we shall simply assume that there are occurrents, and ignore reductionistic attempts to eliminate or reduce them.[2] Since occurrents are distinguished from continuants precisely in having temporal parts, the arguments of the last chapter show that continuants cannot be reduced to occurrents, and it appears that the converse is the case as well. *In general* neither continuants nor occurrents have any onto-

[1] The issue of identity conditions for events is central to much recent analytic action theory—probably to its detriment. Like ANSCOMBE 1979 I do not think there is such a creature as a general identity condition for events, any more than there is one for material objects. Cf. SIMONS 1982*e*.

[2] Cf. the arguments against the reductionism of HORGAN 1978 by ALTMAN *et al.* 1979. This is not to deny that there are good points in both HORGAN 1978 and HORGAN 1981.

logical, epistemological, or referential priority over the other. We need both.[3]

Because occurrents have temporal parts, they enter into predications involving temporal reference in a quite different way from continuants. Characteristic of a continuant is that at any time at which it exists, it is present as a whole, and not just in part. So when a continuant has first one property and then another, contrary property, it is the whole continuant which has the properties and not different parts of it, whereas with occurrents we can always refer such temporary properties down to temporal parts. To express that a continuant has a property at a certain time we need to modify the predicate temporally. We write this form of atomic predication using subscripts:

$F_t c$

corresponding roughly to the English '*c* is *F* at *t*'. This is subject to exceptions, such as logical predications involving timeless predicates like identity or class membership, but it holds for a large class of predicates. In particular, it holds for the predicate 'is part of', and we shall make use of such temporal modification in coming chapters.

By contrast, the time of an occurrent is so to speak built into it.[4] The form of atomic predication suited for describing events is simply

Fe

where temporal characteristics are absorbed in the predicate. If we compare the sentences[5]

Caruso was in San Francisco on 18 April 1906
The Great Earthquake was in San Francisco on 18 April 1906

we see that the first ascribes Caruso a location *at* a time, whereas the second ascribes the earthquake *both* a location *and* a time. In the second, but not the first, we may recast the proposition as a

[3] So, for instance, the view of MARTIN 1978: ch. 1 that reference to continuants such as Brutus may be replaced by reference to Brutus events—i.e. events involving *Brutus*—is crassly inadequate until it can be shown that individuation and characterization of Brutus events can proceed in total independence of individuation and characterization of Brutus. The mere introduction of hyphenated expressions into an already existing language is not enough. STRAWSON 1959: ch. 1 argues for the priority of continuants as objects of reference, but cf. the criticisms in MORAVCSIK 1965, DAVIDSON 1980: 173 ff.

[4] By this we do not mean that the time of an occurrent is essential to it. That is a more difficult issue. Cf. the account in FORBES 1985: ch. 8, §§ 5 f., which is mentioned in § 7.5 below.

[5] The example and the discussion of it follow ROBERTS 1979.

conjunction. In the second, but not the first, we may replace 'was' by 'occurred' or 'took place'. The second involves two binary predicates, the first a single ternary predicate, 'ξ was in η on ζ'. Whereas Caruso could and did move around and change, the earthquake could do neither. It did not have its location at a time: it simply had it.

When we come to speak of mereological relations among occurrents we shall accordingly not need to modify the mereological predicates temporally as we shall need to do for continuants. It is this which recommends occurrents to philosophers wishing to apply the full extensional mereological axioms to the physical world.

The relation of occurrents to time is direct and intimate. Their relation to space is usually less direct, in that their location is given by that of the continuants which participate in them. In this way they seem to be the 'duals' of continuants, which have a direct relation to space and an indirect relation to time.[6] By virtue of involving extended continuants, occurrents may be scattered and have spatial as well as temporal parts. Perhaps not all temporal occurrents involve continuants; for instance, the change of intensity of a magnetic field is an event, but may be the field is not a continuant.[7] In any case, the location of an occurrent can only be determined because there are continuants from which we take our bearings at different times. Dually, the date of happenings, whether or not they involve continuants, is reckoned by taking some event as fixed and relying on local cyclical processes to enable us to measure the temporal gap.

Where occurrents are positive in both duration and spatial extent, they may have parts which are neither purely spatial nor purely temporal, in addition to their spatial and temporal parts. In a football match, the first half of play is a temporal part, the events taking place in one half of the field make up a spatial part, and the part played by one of the players is neither purely spatial nor purely temporal. These ideas can be more precisely defined.

[6] Cf. WIGGINS 1980: 25–6, n. 12 on the duality, which is however not perfect. Continuants occupy space but persist through time. Dual to continuants would be objects which had temporal but no spatial parts. ZEMACH 1970 claims there are such objects, which he calls 'processes', and further objects, which he calls 'types', which have neither spatial not temporal parts. For my part, I have been unable to see that there are either processes or types, nor would I agree with Zemach that either continuants alone or events alone would provide a scientifically acceptable ontology.

[7] The example is due to Myles Brand. A field is of course not a material object, but we might plausibly claim it is a continuant. The example supposes—tendentiously—that a change in field intensity is a real and not a mere Cambridge change.

We let $e, f, e', \ldots$ be variables for occurrents, subject to the vocabulary of tenseless mereology and the axioms of at least the minimal extensional theory. How far we can go beyond this will be discussed below. Denote the spatio-temporal location of a given occurrent e by 'spn[e]' and call this region its *span*. We may say an occurrent is *at* its span, *in* any larger region, and *covers* any smaller region. Now suppose we have fixed a frame of reference so that we can speak not merely of spatio-temporal but also of spatial regions (places) and temporal regions (times). The *spread* of an occurrent, (relative to a frame of reference) is the space it exactly occupies, and its *spell* is likewise the time it exactly occupies. We write 'spr[e]' and 'spl[e]' respectively for the spread and spell of e, omitting mention of the frame. It is possible to define analogous senses of 'at', 'in', and 'covers' for these kinds of occupation.

An occurrent whose spell is a single moment is *momentary*; one whose spread is a single point is *punctual*; and one which is both is *atomic*. Atomic occurrents are the point-events of relativity theory. It is doubtful whether there really are any, but very small and short-lived events approximate them. An occurrent is *connected* (spatially connected, temporally connected) according to whether its span (spread, spell) is topologically connected. A football match is not temporally connected because of the half-time break. The part of a football match consisting of the actions of the two goalkeepers in a single half of the match is usually spatially disconnected.

Using the notions of spread and spell, we can define those of temporal and spatial part of an occurrent. A *temporal part* of an occurrent is a part including all simultaneously occurring parts of it:

$$e' <_{\mathrm{T}} e \equiv e' < e \wedge \forall f \ulcorner f < e \wedge \mathrm{spl}\,[f] < \mathrm{spl}\,[e'] \supset f < e' \urcorner$$

A *phase* of an occurrent is a temporally connected temporal part of it, and a *slice* is a phase of zero duration. The definition of *spatial part* is analogous:

$$e' <_{\mathrm{s}} e \equiv e' < e \wedge \forall f \ulcorner f < e \wedge \mathrm{spr}\,[f] < \mathrm{spr}\,[e'] \supset f < e' \urcorner$$

A *segment* of an occurrent is a spatially connected spatial part of it, and a *section* is a segment of zero width in one dimension.

There appears to be no objection to taking spatial, temporal, and spatio-temporal regions to be subject to the principles of the full classical theory. This could then be used to lay the mereological basis for a natural geometry and chronometry. For this there is no need to deny, as Whitehead did, that there is a unique maximal region, although we do not know whether this is finite or infinite in extent. There is clearly by definition no such thing as superposition of distinct regions, and so no reason to deny the Proper Parts Principle for regions. Finally, the totally homogeneous nature of regions places no conceptual obstacles in the way of accepting the unrestricted existence of arbitrary sums of regions.

A somewhat different picture emerges when we consider occurrents. For these are not in general mere occupiers of space-time. No less than continuants, they come in natural and artificial kinds, which rules out their classification in pure mereological or even pure spatio-temporal terms. It is, by comparison with the case for continuants, fairly obvious that the superposition of occurrents entails neither their identity nor their mereological coincidence. To take an example from Davidson,[8] the warming up and the rotating of a metal sphere may have the same span, but they are clearly distinct, and it is not even clear whether they overlap mereologically. The ease with which occurrents can be superposed has suggested to Brand that superposition be used as a criterion for distinguishing continuants from occurrents: no two continuants may be superposed, whereas occurrents may be superposed, and are identical only when they are *necessarily* superposed.[9] We cannot use this criterion, since we take continuants to be superposible as well. Even the sense of 'being in the same place at the same time' is different according to whether we are speaking of continuants or occurrents. We should rather speak of occurrents having the same span, or at least having phases with the same span. This does not rule out the application of PPP to occurrents, however, so until a convincing counter-example turns up, it is worth taking the mereology of occurrents to be extensional. One kind of putative counter-example is given in action theory by certain cases of what is called *level generation.*[10] Consider the following: Princip's crooking his finger (basic action), his pulling the trigger, his discharging the firearm, his firing the fatal shot, his killing the archduke, his setting off the war. All these are sometimes regarded as distinct actions, though some or all of them have the same brute physical parts. But in this case we do not need to give way: the alternative theory, according to which we here have only one event which parades under numerous distinct

[8] DAVIDSON 1980: 178 f. Cf. also 125.

[9] BRAND 1976: 144 ff., BRAND 1981, BRAND 1984, ch. 3. For criticisms of Brand see HORGAN 1980, SIMONS 1981*b*.

[10] On level generation, see GOLDMAN 1970, HORGAN 1981: 461.

and logically non-equivalent descriptions, is in any case much more plausible.[11]

If we thus incline to extensionality of parts for occurrents, the sum principles are a quite different matter. Occurrents so separated in space-time that there could be no causal link between them surely cannot be of a single nature, cannot form a sum. On the other hand, it is plausible to suppose they may nevertheless be common parts of a more embracing whole, i.e. that they may have an upper bound. Indeed, the existence of a unique maximal upper bound appears perfectly acceptable. If we accepted the process ontology we should call it 'the Universe', but a better title is 'the Career of the Universe'. If we accept that the Universe is expanding, then we accept it as a continuant, since expansion is a form of real change. Note that 'the Universe' in this everyday sense is not the same as U in the classical theory.

In summary, it appears reasonable to suppose that occurrents fulfil, not the full classical axioms, but rather the following weaker set from §1.4: SA0–3, 6, 16, and 23.

Since occurrents, unlike continuants, cannot first have and then lack a property, they cannot change, not even in relation to other things. In particular, therefore, they cannot move. That is not to say that they are stationary and fixed, but rather that the opposition of motion and rest has no application to them. There are, however, usages which seem to attribute change to occurrents, such as

> The argument began calmly, but soon became heated
> The wedding moved from the church to the bride's parents' house

These are connected with the fact that we may say of an event while it is going on 'It is at present calm', 'It is now in the church', and later 'Now it is heated', 'It is now no longer in the church, but in this house', and so on. This runs exactly parallel to the sorts of thing we can say about continuants while they are in existence. One of the disputants is first calm, then he becomes angry; the wedding guests are first in the church, then they move elsewhere. Why can continuants change and

[11] There are several attitudes to such events. One attitude says they are all the same. Another says they are all different. A third says some are parts of others, so some may be identical and others not. The last and most flexible position appears to be the only safe general course: without a universal recipe for deciding identities, one must approach each case on its merits.

temporal objects not, although many of the latter involve or are changes?

It is sometimes said that changes change, for example, that an acceleration is a change in velocity, which is itself a change in position. But this is muddled thinking: velocity is not a change; it is a vector magnitude associated with a change, the motion of a continuant, and acceleration is another vector magnitude associated with the change and the first magnitude. A change in a magnitude, vector or scalar, is a measure of a real change, but is not itself a real change. In any case, if changes were to change, would they not require a second time, alongside the first, in which to change?

Some predicates are true of their subjects by virtue of other predicates' being true of *parts* of these subjects. Socrates was snub-nosed because his nose was snub; Table Mountain is flat-topped because its top is flat. In general, if $b < a \wedge Fb$, then $\lambda x \ulcorner \exists y \ulcorner y < x \wedge Fy \urcorner\urcorner$ is true of *a*. We call a predication of a whole inherited from a predication about one or more of its proper parts a *local* predication. A predication about an occurrent may be temporally local, that is, inherited from a predication about temporal parts. There are also spatially local predications, inherited from predications about spatial parts. These are possible for both occurrents and continuants, and are indeed the only kind of local predications for the latter. So, to take again McTaggart's example, a poker may be hot-tipped and cool-handled, in that its tip is hot and its handle cool. This complex state constitutes a variation, in that the poker has different temperatures in different parts; but this is not a change, since the contrary properties do not claim the same part of the poker and so may be simultaneously instantiated. If, on the other hand, we suppose the poker uniformly hot, and then uniformly cool, the contrareity of the properties and the fact that they have the same subject entails that the times at which the poker has them must be different. But we must take this to mean that the poker changes, because it has no temporal parts to which we could more intimately assign the contrary properties. By contrast, an argument which is first calm and then heated does have temporal parts which can receive the contrary predicates 'calm' and 'heated'; its early phases are calm, and its later phases are heated, and predications about the argument mentioning this are, if they have the whole argument as subject, temporally local. (There are of course, in addition, non-local predications about it—for example, that it lasted twenty minutes.) So it is not the *whole* argument that is first calm and then heated; since these properties do not claim the same object, they do not constitute a change.[12] For the same reason, occurrents do not move; rather they have some phases in one place, others elsewhere. This particular case has been forcefully argued by Dretske.[13]

It is now commonplace to distinguish between real changes and mere Cambridge changes.[14] An object 'undergoes' a Cambridge change when some proposition concerning it changes in truth value. But this need not involve the object itself changing. That five ceases to be the number of John's children is not a change in five; that a politician or a film-star gains or loses in popularity is not a change in them, nor is someone's attaining majority. In the Cambridge sense, objects which are not spatio-temporal at all, like numbers, can change, and so can spatio-temporal objects which do not exist at the time in question, as when a soldier is posthumously decorated. Pinning down the more important concept of a real change is more difficult. Certainly qualitative alterations are real changes, but so are substantial changes. Since the bogus Cambridge changes usually or always involve switches in the truth value of relational propositions (those involving ages being only an apparent exception), Cambridge changes are sometimes called *relational* changes. But this appears mistaken, since there are real relational changes, such as changes in the relative positions and distances of several bodies. These are not changes *in* the objects so much as changes *among* them. Perhaps the only characteristic that all real changes have in common is that they all involve their subjects in some *causal* manner. A chameleon's change in colour not only involves movement of pigments in its skin, it alters the skin's capacity differentially to absorb and reflect incident light, which is the point of the whole affair. Even the relational changes of distance and position of objects affect their relative causal powers, such as their mutual gravitational attraction and the time it would take for light to go from one to another.

As we loosely formulated the idea of Cambridge change, temporal objects are subject to Cambridge changes, in that the same temporal object may be spoken of in a sentence whose truth value varies with

[12] MELLOR 1981 makes the same point: 110ff.

[13] DRETSKE 1967: he draws the cautious moral that ordinary usage is not always to be trusted. So it isn't.

[14] GEACH 1969: 71 f.; 1972: 321 f. The epithet 'Cambridge' derives from the fact that this criterion of change was employed by a number of Cambridge philosophers, including Russell and McTaggart.

time, for example: 'It was twenty years ago today that Sergeant Pepper taught the band to play'. It is only in this sense that the action of an assassin in shooting a public figure later *becomes* a murder, when the victim dies. But since mere Cambridge change is not really change, this does not affect our position that occurrents do not change.

4.2 Activities and Performances

Occurrents enter into action theory at the very basis, since many actions and all acts are events. Not all actions are events if we count deliberate omissions and forbearances as actions, but provided we confine ourselves to actions in which the agent actively *does* something (acts), then these are all events and therewith occurrents. Mereological considerations occasionally play a part in action theory,[15] and we here show how they can be used to give a simple explanation of an old distinction.

Action theorists frequently distinguish between two different sorts of action, namely *activities* and *performances*.[16] The difference is a well-founded one, showing itself in the different tense and aspect behaviour of action verbs. Languages mark the distinction in various ways: in English, to which our discussion is confined, it concerns the continuous and perfective aspects. The main difference is that for activity verbs such as 'weep', 'laugh', and 'talk', the continuous of a tense entails the perfect of the same tense (with a minor reservation to be noted):

John is weeping → John has wept (on this occasion)
John was laughing → John had laughed (on that occasion)

whereas for performance verbs such as 'wash' or 'build a house', the continuous of a tense entails the *negation* of the perfect of the same tense:

John is washing → John has not yet washed (on this occasion)
John was building his house → John had not (yet) built his house

Discussion of the distinction goes back to Aristotle, though it has been much intensified in recent years. It is worth noting that the same

[15] Cf. THALBERG 1977: ch. 4f., THOMSON 1977, who both champion the middle position on event identity (cf. note 11 above) and hold that, e.g., Princip's pulling the trigger is only part of Princip's firing the gun. In this case the part–whole analysis is resistible: cf. HORNSBY 1980: ch. 2.

[16] VENDLER 1967: ch. 4, 'Verbs and times', KENNY 1963: ch. 8, 'States, performances and activities', POTTS 1965. Cf. also various essays in ROHRER 1980.

distinction can be made in the case of at least some occurrents which are not actions, where we may speak of changes with a culmination (birth, for example, or fission of an atom) and changes without a culmination, such as motion.[17] But most attention has rightly been focused on the distinction as it applies to actions.

In the formulation of the entailments we had to add 'on this/that occasion' because a simple perfect aspect tends to mean 'at *any* previous time'. So without qualification, that John is washing is by no means incompatible with John's having washed, for example, the previous day. That we have to specify particular occasions shows that the main point of the distinction is ontological rather than purely linguistic.

Apart from this distinction, others which have been mentioned are that performances *take* (a certain) time, for example, it takes ten minutes to climb the staircase, whereas activities *go on for* a time, for example, someone may sing for ten minutes, and that performances but not activities may be interrupted before completion, since performances, but not activities, may be brought to completion.

What does the difference between performances and activities amount to, and how can it be used to explain the differences between the two kinds of verb? One suggestion by Joy Roberts is that we should take activity sentences to have an underlying logical form in which the verb is nominalized and the resulting nominal made the subject of a sentence of the form '$F_t n$', that is, one ascribing a property to a continuant at a time, whereas performances have a logical form in which the nominal fits the subject position of sentences ascribing properties to events, i.e. of the form 'Fn'.[18] Ingenious though this proposal is, there are a number of objections which can be raised against it. In the first place, the notion of an underlying logical form is a dubious one which should be avoided if possible, especially if the underlying form appears to use linguistic constructions like nominalization which are arguably less basic than the one appearing in the surface form. Secondly, there is the fact that nominals like 'John's dressing' in fact have two uses: they may denote either a performance itself or the *characteristic activity* leading up to completion of the performance. More importantly, Roberts's proposal amounts to

[17] Among them GABBAY and MORAVCSIK 1980; what is said here could be extended to what they term 'processes' and 'mere events'. We have restricted attention to actions merely because they provide the clearest examples.

[18] ROBERTS 1979.

treating activities as continuants. But activities are not continuants, since they have temporal parts. If John's weeping were a continuant, then it would all be present at any time at which it is going on, which is false, since at any such time some phases are past and others yet to come. And if activities are not continuants, the theory that we somehow treat them as though they are lacks all motivation.[19]

A more satisfying explanation for the difference between activities and performances and their connection with the aspect of verbs can be provided by means of mereological considerations, and to this end we introduce a number of concepts.[20]

A predicate is *dissective* iff it applies to all parts of anything to which it applies:

$$\text{diss}(F) \equiv \forall xy \ulcorner Fx \wedge y < x \supset Fy \urcorner$$

and *antidissective* if it applies to no proper parts of anything to which it applies:

$$\text{antidiss}(F) \equiv \forall xy \ulcorner Fx \wedge y \ll x \supset \sim Fy \urcorner$$

The predicate 'weighs less than 2 gm' is dissective, while 'is a circle' is antidissective. The absolute property of dissectiveness is less useful than a relativized version: F is G-dissective iff it applies to all parts satisfying 'G' of those things to which it applies:

$$\text{diss}_G(F) \equiv \forall xy \ulcorner Gy \supset . Fx \wedge y < x \supset Fy \urcorner$$

For instance, 'water' is not absolutely dissective, since there are parts of any quantity of water which are not water. Where a predicate is dissective down to some but not all parts, we shall informally call it *partly* dissective. Relative dissectiveness is often part-dissectiveness and vice versa. A predicate is finally *phase-dissective* iff it applies to all phases of whatever it applies to. Phase-dissectiveness is dissectiveness relative to the relation of being a phase of.

The nominals characterizing activities are partly phase-dissective. Any sufficiently long phase of a weeping is itself a weeping, etc. The

[19] Another flaw is that Roberts claims that 'For object sentences, the progressive tenses have the same conditions as the respective simple tenses' (ROBERTS 1979: 180). This means that 'John's weeping is loud' and '*John's weeping is being loud' have the same truth conditions, whereby so do 'John weeps loudly' and 'John is weeping loudly'. But the simple tense in both activity and performance verbs connotes habit or repetition (KENNY 1963: 175). In fact, for certain predicates, of which 'is loud' is one (KENNY 1963, hence our asterisk), there is no genuine progressive at all; cf. Roberts's ill-formed '*John is being hungry', '*The earthquake is being 6.5' (179).

[20] These definitions are in the style of LEONARD and GOODMAN 1940, but do not exactly follow theirs.

nominals characterizing performances are, on the other hand, anti-dissective. No proper part of a dressing is itself a dressing, no proper part of a circle drawing is a circle drawing, etc. A dressing is a complete dressing or it is no dressing at all (we are here overlooking the above-mentioned ambiguity in nominals like 'dressing').

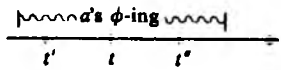
To link these simple mereological characteristics with the aspect of verbs, we need an account of the truth conditions of sentences involving verbs in certain aspects. We need only make this account sufficiently detailed for our purposes, and do not have to go into every last detail. We give these conditions only for the present tense: other tenses can be handled similarly. We use '$[t, t']$' for the closed temporal interval between t and t'.

The truth conditions for aspects are then as follows:

Continuous Aspect

> a is ϕ-ing at t iff for some t', t'' such that t' is earlier than t and t'' is later than t, there is a (sm) ϕ-ing by a which covers $[t', t'']$.

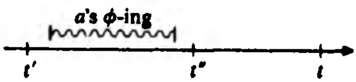
We may illustrate this as follows:



Perfective Aspect

> a has ϕ-d at t iff for some t', t'' such that t' is earlier than t'' and t'' is not later than t, there is a (sm) ϕ-ing by a in $[t', t'']$.

We can illustrate this by the diagram below. That is, a has ϕ-d at t iff it



ϕ-s in some interval ending not later than t. In this case we are interested in the most recent episode of ϕ-ing if there are several. (Cf. the remarks about 'on this occasion'.)

We now turn to proving the aspect laws for activity and performance verbs (present tense only, with time reference explicit).

Suppose that a is ϕ-ing at t. Then there exist t_1 and t_2, with t later than t_1 and earlier than t_2, such that a's ϕ-ing covers $[t_1, t_2]$. There are two cases to consider. Case 1: ϕ-ing is an activity. Then ϕ-ing is partly phase-dissective. $[t_1, t]$ is a phase of $[t_1, t_2]$, so *provided* it is long enough for the phase of a's ϕ-ing over $[t_1, t]$, to be itself a ϕ-ing by a, there is a ϕ-ing by a at and therefore in $[t_1, t]$, whence by the truth conditions for the present perfect, a has ϕ-d at t. Case 2: ϕ-ing is a performance. a's ϕ-ing covers $[t_1, t_2]$ so there is a temporal region τ (usually an interval) containing $[t_1, t_2]$ such that a's ϕ-ing is at τ. Any interval $[t_3, t_4]$ contained in τ with t_4 not later than t is a proper part of τ and therefore, since ϕ-ing is antidissective, there is no ϕ-ing by a in any such interval (previous episodes of ϕ-ing outside τ are, we agreed, not considered). Hence a has not ϕ-d at t, and will only have ϕ-d (on this occasion) on or after the end of τ, assuming the ϕ-ing is completed.[21]

4.3 Plural Reference

Since modern predicate logic replaced the traditional logic of terms, the semantic relation of reference has occupied the attention of philosophers and logicians almost exclusively in one special form only, namely singular definite reference. The paradigm expressions serving to refer to single individuals are proper names like 'Bertrand Russell'; but in many logical theories, excepting Russell's own, definite descriptions are treated as belonging to the same syntactic category as proper names. We follow tradition in calling such expressions 'singular terms'. In their number we also reckon certain indexical expressions like 'him' and 'that man', although the semantic peculiarities of indexicals are not our interest here. The expression 'singular term' contrasts traditionally with 'general term'. General terms include in particular common nouns like 'horse' and common noun phrases like 'small white cloud'. A stock explanation of the difference between singular and general terms was that singular terms can denote only one thing, whereas general terms can denote any number of things, or none at all. This account leaves much to be desired, but in most modern treatments the issue of how singular and general terms differ simply does not arise, since, following Frege and Russell, general terms are regarded as mere parts of predicates like 'is a

[21] Matters are in fact more complicated than this, because John can both be drawing a circle (hence not yet have drawn it) and have just drawn a circle (hence no longer be drawing it), in that he may simultaneously start drawing circles with each hand and finish one before the other. But these complications affect the verb rather than the performances concerned, each of which is a separate performance in its own right. The analysis fits if we take more specific verb phrases like 'draw a circle with his left hand'.

horse', which do not designate or denote, but are true of or apply to any number of individuals, or none at all. In this respect Leśniewski remains closer to the traditional analysis of atomic sentences, as did Schröder before him. In the informal explanations given as to how to interpret the names and functors of Ontology,[22] 'names' are said to include not only proper names and definite descriptions, but also common nouns and common noun phrases, and even some adjectives. The true position of common nouns in a natural language like English or German is captured neither by orthodox predicate logic nor by Ontology. Common nouns are neither mere dependent parts of predicates nor of a kind with proper names; they form a third basic syntactic category, along with sentences and definite terms, and a formal language which adequately characterizes their role will accordingly be syntactically more complicated than either predicate logic or Ontology.[23] This is not our principal interest here. We are concerned to contrast singular terms not with common nouns but with another kind of term, namely *plural* terms.

Natural languages abound with plural terms; they include plural proper names, like 'Benelux', plural definite descriptions, like 'the authors of *Principia Mathematica*', and multifarious plural indexicals, like 'these books' or 'they'. In addition there are terms formed by conjoining other terms, like 'Lennon and McCartney', 'the Montagues and the Capulets', 'Henry VIII and his wives'.

Plural terms in English differ from singular in two ways: syntactically and semantically. The more important difference is semantic. To explain it, we must first distinguish syntactic singularity and plurality from semantic singularity and plurality. A term is syntactically singular (plural) if the verbs it fits in a grammatical sentence must be singular (plural) where this difference in number is linguistically observed. Thus 'Socrates' is syntactically singular and 'Lennon and McCartney' is syntactically plural, because 'Socrates is singing' and 'Lennon and McCartney are philosophizing' are grammatical, while '*Socrates are singing' and '*Lennon and McCartney is philosophizing' are ungrammatical. According to this test, some English terms are

[22] Cf. LEJEWSKI 1960: 17, HENRY 1972: 17.

[23] On common nouns cf. LEWIS 1970, FITCH 1973, SIMONS 1978, LEJEWSKI 1979, GUPTA 1980, SIMONS 1982*d*. Gupta's approach is in many ways the most suggestive, since it offers an explanation as to why common nouns appear to be dispensable in extensional logic (cf. the demonstration in a Leśniewskian context by Lejewski), namely that their different logical behaviour only shows up in modal contexts.

syntactically indeterminate as to number; for instance 'the orchestra', since both 'the orchestra is playing' and 'the orchestra are playing' are grammatically acceptable. The syntactic distinction between singular and plural is not a universal feature of natural languages. Some languages, such as Chinese, manage very nicely without it; others, such as Sanskrit, make a tripartite distinction between singular, dual, and plural (more than two); Lyons reports that in Fijian there is even a trial number.[24] Logically, the syntactic distinction is unimportant and even inconvenient,[25] and we prefer to consider formal languages where it is not made.

A term is semantically singular if it designates one object, and semantically plural if it designates more than one object. 'Socrates' is semantically singular, and 'Lennon and McCartney' is semantically plural. A term which is either semantically singular or semantically plural we call *referential*. A term which is not referential, i.e. one which does not designate anything at all, we call *empty*. In normal use, syntactically singular and plural terms are intended to be semantically singular and plural respectively, but both kinds may be empty, as 'Pegasus' and 'Holmes and Watson' testify. From now on, we stipulate that 'singular' and 'plural' shall mean 'semantically singular' and 'semantically plural' respectively. In Leśniewski's Ontology the variable names may be empty, singular, or plural. The constant name 'Λ' may only be empty; by virtue of the fact that the theses of Ontology are true even in the empty domain, no name in Ontology is guaranteed to be referential. The singular terms (constant and variable) of classical predicate logic must be singular on all interpretations; in the free logic of Lambert, van Fraassen *et al.* or in Lejewski's theory of non-reflexive identity,[26] terms may be singular or empty; in my logic of plural reference there are neutral terms, which are like those of Ontology, 'singular' terms, which may be singular or empty, and 'plural' terms, which may be plural or empty.[27] For most logical purposes it is convenient to use only neutral terms, as Leśniewski clearly recognized instinctively. What is important is the recognition that designation may be not merely a function, as in classical logic, or a partial function, as in free logic, but in general a relation. The term 'the

[24] LYONS 1968: 283.

[25] SIMONS 1982*c*. Cf. §8 of SIMONS 1985*d*.

[26] LEJEWSKI 1967*b*.

[27] Cf. SIMONS 1982*c*, §3, and SIMONS 1985*d*, which extends further the embedding procedure of §2.8 above to account for such an option.

authors of *Principia Mathematica*' designates *both* Russell *and* Whitehead, that is, it designates Russell *and* it designates Whitehead, and no one else. It is not *true* of each of them; that holds rather of the predicate 'is an author of *Principia Mathematica*', nor does it designate the set {Russell, Whitehead} as this is normally understood. How can a set write, or co-operate in writing a book? A set is an abstract individual, and cannot put pen to paper or exercise any other causal influence. At best, its members can do that. If anyone objects that my use of 'designate' is deviant, my reply is that it may not reflect the predominant tradition on reference, which has concentrated almost exclusively on singular reference to the detriment of everything else, but that does not justify giving it the emotive label 'deviant'; if someone wants to use another term, say 'denote', exclusively for singular reference, that is fine, as long as I can use 'designate' in my way.[28]

4.4 Pluralities: Groups and Classes

If the linguistic phenomenon of plural reference is relatively unproblematic, a more difficult question is whether there are plural *objects*, objects that are essentially not one thing but many things. Russell, for instance, thought in 1903 that there are: the operation corresponding to the use of 'and' in forming lists, which he called 'addition of individuals' or 'numerical conjunction', and the objects thus formed, of which numbers other than one are assertible, which he called 'classes as many'. Russell called individuals 'terms'; non-individuals, including classes as many, he called 'objects'.[29] The notion of a class as many can also be found in the work of Cantor and Bolzano.[30] Russell counted among classes both classes as many and (as a limiting case) individuals, but denied the existence of any null class, rightly in my view.

Classes are the ontological counterparts of referential terms, singular and plural, and it is just for this reason that the question of the existence of pluralities arises; surely the recognition of plural reference as a relation between one term and several individuals whereby we refer to all of them at once, obviates the need to recognize

[28] Cf. SIMONS 1982*b*: 166–8. [29] RUSSELL 1903: 43, 55 n.

[30] BOLZANO 1981: §§82 ff., 1975: §3 f. (from which Russell took the concept of numerical conjunction), CANTOR 1932*b*: 443 (1967: 113). For further historical remarks cf. SIMONS 1982*c*. §1.

classes as a distinct kind of object over and above the several individuals severally referred to? The assumption that the one term corresponds to one object is an unwarranted carry-over of the idea that terms have a many–one relation to their referents, which applies only to singular terms.

This argument overlooks the innocuousness of classes in the sense here meant, for which sense I have also used the term 'manifold'.[31] A class is not something over and above its several members, and the members *are* the class. Someone who admits that there is more than one individual *thereby* admits that there is a class of more than one individual. In particular, a class of several individuals is not a new, higher-order, abstract individual. A class of several concrete individuals is itself a concrete particular,[32] though not a concrete individual. This conception of classes, as 'low-brow' collections rather than 'high-brow' abstract individuals, fits the linguistic phenomenon of plural reference rather than the requirements of foundations of mathematics.

One reason why classes seem to be higher-order individuals rather than lower-order pluralities is that we speak, in the singular, of *a* class, *one* class, *this* class, even where the class has more than one member. But the presence of the grammatical singular is not a sufficient indication that we are referring to an individual. For one thing, the grammatical singular is used in English to refer to particular masses, such as 'this water', 'the gold in this ring', and these, we shall argue, are not individuals. In a case like 'the class of men in this room', the complete term is a complex one made up of a *collective noun* in the singular plus a general noun (phrase) in the plural. English abounds in collective nouns, both general-purpose ones like 'class', 'group', and 'collection' and lexically specific ones like the famous 'murmuration' (of starlings) or 'exaltation' (of larks).[33] They all serve the function of referring collectively to a number of objects while the noun phrase is singular. This means that we have the grammatical means available to pluralize the collective noun phrase and apparently refer to several collections, as in 'the groups of people began to converge'. Other languages with plurals possess the same device.

Let us call a term like 'the collection of jewels in this box' a *group*

[31] SIMONS 1980, 1982*b*, 1982*c*.

[32] As are the aggregates of BURGE 1977.

[33] Cf. SPARKES 1975.

term.[34] The noun 'collection' and its like we call *collective nouns.* The object, if there is one, referred to by a group term is a *group*. The ontological question facing us is whether there are groups. Prima facie this question is trivial; if someone asks us whether there are flocks of birds, herds of cattle, packs of wolves, bunches of grapes, orchestras, committees, and battalions, the answer is surely positive. If the question is whether there are such groups *over and above* or *in addition to* individual birds, cattle, etc., we must answer more carefully. Of course there would be no orchestras if there were no musicians, and when we have got suitable musicians organized and playing together we do not need to add some further ingredient to obtain an orchestra. All the same, an orchestra is not simply a number of musicians. If we consider at random several musicians, the chances are overwhelming that they do not constitute an orchestra; they may be in different continents, or live at different times. An orchestra is made up of musicians who regularly come together to play together; for that to happen they must have overlapping lives, and must be sometimes in one room together while they play. Such conditions on its members are *constitutive* of an orchestra; if they are not fulfilled, we have no orchestra. There are also other conditions, requiring a certain distribution of instruments, depending on the kind of orchestra and the pieces they wish to play. For the most part, the constitution conditions of groups are lax enough to allow fluctuation of membership. If one violinist leaves and another takes her place, the orchestra is not thereby destroyed and replaced by another. There are limits on this procedure of replacement: we cannot at once replace all players, or all but one, and still have the same orchestra. It would at most be a new orchestra with the old name. But given the possibility of replacement, groups may continue to exist even though over time they undergo a complete change of membership. They can thus 'outlive' their members: the Hallé orchestra is well over a century old, and all its original players are long since dead. The same goes for many other kinds of group. Such groups are *relatively* independent of their personnel.

The limiting case of a group is a class. A group is several objects fulfilling certain constitution conditions. The existence of the group members alone is insufficient to guarantee the existence of the group. In the case of a class, it is sufficient, since a class is simply the several

[34] BLACK 1971 calls them 'ostensibly singular plural referring expressions'.

objects; there are no further constitutive conditions.[35] The relation of member to class is tenseless, like identity, so classes cannot fluctuate in membership even if their members exist at widely different times. So classes may be adequately handled by a tenseless, extensional theory like Leśniewski's Ontology.

If the existence of groups is not in doubt, that of classes, precisely because they are a limiting case, is less secure. Leśniewski, who called such classes 'distributive' by contrast with 'collective classes' or mereological sums, thought that all reference to classes was eliminable, a sentence of the form '*A* is a member of the class of *b*s' being simply an unnecessarily elaborate way of saying '*A* is a *b*'. However, to prove that reference to classes is eliminable we have to show that it is eliminable in all contexts. Some predicates concerning classes prove easier to eliminate than others. For example 'the *a*s outnumber the *b*s' may be familiarly defined in terms of the existence of a bijection between the *b*s and a subclass of the *a*s but not vice versa,[36] and this definition can be so written that '*a*' and '*b*' appear only in contexts of the form '$c \varepsilon a$', '$c \varepsilon b$'. But there are other predicates true of groups, like 'surround' or 'disperse', for which an elimination does not readily suggest itself. In some cases where ineliminability seems assured, for instance 'the *a*s cover an area of $100\,m^2$' (it being possible that *a*s overlap, for instance), it would be open for a Leśniewskian to claim that the subject is the *collective* class or sum of *a*s; but this way out is not available for a true proposition like 'if there are 10 *a*s, there are 1,023 classes of *a*s', and if the *a*s are not at least weakly discrete (cf. Chapter 2, MD16, MT20) distinct classes of *a*s may have the same sum. Such examples strongly suggest the ineliminability of plural reference. If we write them out in Leśniewskian language—here is the

[35] WITTGENSTEIN 1913: 'Mankind is a class whose elements are men; but a library is not a class whose elements are books, because books are part of a library only by standing in certain spatial relations to one another—while classes are independent of the relations between their members.' The details are wrong, since books may be taken out of libraries, and not just any lot of books in the requisite spatial relation constitutes a library; but the main point is right.

[36] For the expression of this in Lesniewskian language, cf. SOBOCINSKI 1954: 42, n. 12. Similar eliminations are possible for other statements of numerical proportion, e.g. 'There are 50% more *as* than *bs*' as 'for some relation *R*, the domain of *R* is the *as* and the converse domain of *R* is the *bs*, and every *a* bears *R* to exactly two *bs* and every *b* bears the converse of *R* to exactly three *as*'.

shorter 'if there are two *a*s, there are three classes of *a*s':

$$\Sigma bc^{\ulcorner} b \varepsilon a \wedge c \varepsilon a \wedge b \not\simeq c \wedge \Pi d^{\ulcorner} d \varepsilon a \supset d \simeq b \vee d \simeq c^{\urcorner\urcorner} \supset$$
$$\Sigma bcd^{\ulcorner} b \sqsubset a \wedge c \sqsubset a \wedge d \sqsubset a \wedge b \not\simeq c \wedge c \not\simeq d \wedge d \not\simeq b \wedge$$
$$\Pi e^{\ulcorner} e \sqsubset a \supset e \simeq b \vee e \simeq c \vee e \simeq d^{\urcorner\urcorner}$$

the quantification binding non-singular variables cannot be replaced by quantification binding singular variables only.

4.5 Analogies Between Ontology and Mereology

There is a notable duplication of algebraic structure in Ontology and Mereology, although this is to some extent masked by the forms of expression normally employed in the latter. As we pointed out in §3.1, both are closely related to Boolean algebra: Tarski showed that Mereology is like extended Boolean algebra save for the null element, while Lejewski has shown how to interpret Boolean algebra within Ontology.[37] The principal algebraic difference between Ontology and Mereology is that whereas general Mereology is neither atomistic nor atomless, Ontology, taking the functor '$\sqsubset$' to correspond to the part-relation $<$, is atomistic.

It may sound odd to talk of 'atoms' in Ontology, but we might remember that the Greek word for 'individual' is 'ἄτομος'; the concepts 'atom' and 'individual' are, etymologically at least, closely related.[38] We can show that the similarity is algebraic as well as merely etymological. Consider, for instance, the following uninterpreted formula

$$a \sqsubset b \equiv \Sigma c^{\ulcorner} c \sqsubset a^{\urcorner} \wedge \Pi c^{\ulcorner} c \sqsubset a \supset c \sqsubset b^{\urcorner}$$
$$\wedge\, \Pi cd^{\ulcorner} c \sqsubset a \wedge d \sqsubset a \supset c \sqsubset d^{\urcorner}$$

One interpretation of this which makes it true is to take '$\sqsubset$' *to mean the same as* 'ε', in which case the formula is simply Leśniewski's original axiom for Ontology. However, another interpretation can be obtained by making use of a mereological definition which says that

$$a\, \alpha\tau\, b \equiv a\, \varepsilon\, \mathrm{pt}(b) \wedge \Pi cd^{\ulcorner} c\, \varepsilon\, \mathrm{pt}(a) \wedge d\, \varepsilon\, \mathrm{pt}(a) \supset c\, \varepsilon\, \mathrm{pt}(d)^{\urcorner}$$

and taking '$\sqsubset$' *of the uninterpreted formula to mean the same as* '$\alpha\tau$', which as a binary predicate means 'is-an-atom-of'. The formula thus

[37] TARSKI 1935, LEJEWSKI 1960–1.

[38] In SCHRÖDER 1890–1910, the term 'individual' ('Einzelne') is used for what are now called Boolean atoms.

interpreted is not a theorem of general Mereology. It always holds from left to right, and from right to left also in atomistic or atomless Mereology, but the hybrid model given in §1.6 gives a counter-example to the right–left implication.

Conversely, consider the effect of taking the primitive of Ontology to be '$\sqsubset$'. Lejewski notes[39] that, in doing so, one can use the following axiom

$$a \sqsubset b \equiv \Sigma c \ulcorner c \sqsubset a \urcorner \wedge \Pi c \ulcorner c \sqsubset a \supset \Sigma d \ulcorner d \sqsubset c \wedge d \sqsubset b \wedge \Pi e f \ulcorner e \sqsubset d \wedge f \sqsubset d \supset e \sqsubset f \urcorner\urcorner\urcorner$$

defining 'ε' by

$$a \varepsilon b \equiv a \sqsubset b \wedge \Pi c d \ulcorner c \sqsubset a \wedge d \sqsubset a \supset c \sqsubset d \urcorner$$

which is of course perfectly analogous to the definition of '$\alpha\tau$' in terms of '$<$',

$$a < b \equiv a \, \varepsilon \, \mathrm{pt}(b)$$

$$a \, \alpha\tau \, b \equiv a < b \wedge \Pi c d \ulcorner c < a \wedge d < a \supset c < d \urcorner$$

The important thing about Ontology is that its axioms guarantee that if anything exists at all, then an individual object exists; in particular

$$\mathrm{E}a \supset \Sigma b \ulcorner b \, \varepsilon \, a \urcorner$$

whereas of course the analogous formula in Mereology (if anything exists then an atom exists)

$$\mathrm{E}a \supset \Sigma b \ulcorner b \, \alpha\tau \, a \urcorner$$

is only true on atomistic assumptions.

What makes it possible to show these similarities and differences is the fact that Ontology allows plural as well as singular (and empty) terms, and so enables the functor '$\sqsubset$' to be given a non-trivial interpretation. What the similarities appear to show is that there is a certain sense in which *both* '$\sqsubset$' and '$<$' are part–whole relations: both are partial orderings with suprema, 'almost closed' under certain Boolean and extended Boolean operators. But of course, because *individuals* have parts (in the sense of '$<$') while (distributive) *classes* have parts (in the sense of '$\sqsubset$'), and individuals are the members of the classes, the two senses must be kept apart. As Ontology on the one hand shows, it is possible to develop a theory of '$\sqsubset$' without a theory of '$<$', and as the calculus of individuals shows, it is possible to develop a theory of '$<$' without a theory of '$\sqsubset$', although one has to

[39] LEJEWSKI 1958: 164.

resort to axiom schemata for predicates or use abstract sets to express the general principle of the existence of fusions. But since in the sense of '<', only individuals, and not pluralities (classes with two or more members) can be or have parts, the role of plural reference in Mereology is a background one.

It is important to notice that there is a sense of 'are part of' used in connection with classes which coincides with the sense 'are some of', that is, class inclusion. We say for instance that the husbands in this city are part of, are some of the married people in this city. The pronunciation of 'some' is important here: it should be given the stressed pronunciation [s ʌ m]. There is an unstressed use of 'some', pronounced [səm] or [sm̩], and occasionally also written 'sm' in philosophical and linguistic texts. This unstressed 'sm' is simply a mass/plural counterpart of the singular indefinite article 'a(n)', and is found in sentences like 'There are sm apples in the bowl', 'There is sm milk in the jug' (compare 'There is an [ən] apple on the table'.) The stressed 'some', like the 'one' in 'one of', is not an indefinite article.

The analogy between '$\sqsubset$' and the part–whole predicate '<' can be made more perspicuous by noting the following parallels: the formulae given in each column are theses of Ontology and Mereology respectively:

O1	$a \sqsubset b \supset \mathrm{E}a \wedge \mathrm{E}b$	M1	$a < b \supset \mathrm{E!}a \wedge \mathrm{E!}b$
O2	$\mathrm{E}a \supset a \sqsubset a$	M2	$\mathrm{E!}a \supset a < a$
O3	$a \sqsubset b \wedge b \sqsubset c \supset a \sqsubset c$	M3	$a < b \wedge b < c \supset a < c$
O4	$a \sqsubset b \wedge b \sqsubset a \supset a \cong b$	M4	$a < b \wedge b < a \supset a = b$
O5D	$a \triangle b \equiv \Sigma c \ulcorner c \sqsubset a \wedge c \sqsubset b \urcorner$	M5D	$a \circ b \equiv \Sigma c \ulcorner c < a \wedge c < b \urcorner$
O6	$a \sqsubset b \equiv \Sigma c \ulcorner c \triangle a \urcorner \wedge \Pi c \ulcorner c \triangle a \supset c \triangle b \urcorner$	M6	$a < b \equiv \Sigma c \ulcorner c \circ a \urcorner \wedge \Pi c \ulcorner c \circ a \supset c \circ b \urcorner$
O7	$\Sigma a \ulcorner \mathrm{E}a \wedge \phi a \urcorner \supset \Sigma b \ulcorner \mathrm{E}b \wedge \Pi c \ulcorner c \triangle b \equiv \Sigma d \ulcorner \mathrm{E}d \wedge \phi d \wedge c \triangle d \urcorner\urcorner\urcorner$	M7	$\Sigma a \ulcorner \mathrm{E!}a \wedge \phi a \urcorner \supset \Sigma b \ulcorner \mathrm{E!}b \wedge \Pi c \ulcorner c \circ b \equiv \Sigma d \ulcorner \mathrm{E!}d \wedge \phi d \wedge c \circ d \urcorner\urcorner\urcorner$

These theses show how both relations are, in their different ways, part–whole relations, the class inclusion relation $\sqsubset$ just as much as the usual relation between individuals. It is worth recalling that the Galilean proof that there are as many squares as natural numbers was once treated as a paradox, on the grounds that is showed that a whole need not be greater than its parts. In the nineteenth century, it was common to call class inclusion a relation of part to whole, and even now the standard German for 'subset' is 'Teilmenge', 'part-set'.

Another important point is that whereas the theses O4 and O7 of Ontology are thoroughly uncontroversial, the corresponding theses M4 and M7 of Mereology are by no means so (see §3.2.3 for M7 and §3.2.4 for M4.) By contrast, O4 tells us merely that classes are extensional, and O7 assures us of the existence of unions. O4 is thus partly constitutive of the notion of class, and O7 can hardly be doubted.

If we make serious use of the availability of plural subjects in Ontology, we notice quickly that predicates and other functors may be defined which generalize those we can use of individuals, and are analogous to them in logical structure. The most obvious predicate is that of strong identity '$\cong$', which can be defined as

$a \cong b \equiv a \sqsubset b \wedge b \sqsubset a$ (cf. Chapter 2, LD7 for an equivalent definition)

in analogy with the definition of singular identity

$$a = b \equiv a \varepsilon b \wedge b \varepsilon a$$

Similarly, functors of singular and strong difference may be defined respectively as

$$a \neq b \equiv a \varepsilon a \wedge b \varepsilon b \wedge \sim (a = b)$$

$$a \ncong b \equiv a \sqsubset a \wedge b \sqsubset b \wedge \sim (a \cong b)$$

Where there is identity there is difference, and where there is difference there is number. If $a \neq b$, then a and b are two—two individuals. Where $a \ncong b$ then a and b are also two, but not necessarily two individuals. They might be two pluralities, or one plurality and one individual. A predicate meaning 'is a plurality' may easily be defined as

$$\text{plur } a \equiv \Sigma bc \ulcorner b \varepsilon a \wedge c \varepsilon a \wedge b \neq c \urcorner$$

Since we have agreed to use the term 'class' to mean 'individual or plurality', we do not need to define a new predicate 'is-a-class', since the predicate 'E' already does this work, the following being a thesis:

$$\text{E}a \equiv \text{E!}a \vee \text{plur } a$$

So, if $a \ncong b$, we can say, quite generally, that a and b are two *classes*. A special case is where a and b are individuals. Despite the prejudices of most philosophers, 'one' does not mean the same as 'one individual', nor does 'the same' mean 'the same individual'. So a plurality is in one sense one, and in another sense many. It is one class (as distinct from other classes), but at the same time is (i.e. is composed of, has as members) many individuals. Lejewski reports that Leśniewski could attach no sense to the term 'set' as this occurs in the set theory of Zermelo.[40] This is understandable, as Zermelo's sets may be members of higher-order sets, and so are (abstract) individuals, yet apart from the empty set they manage to have members as well.

The reason for this confusion lies in overlooking the *analogies* of the formal concepts of identity, difference, and number, analogies we claim extend to the formal concept *proper part*. On the mistaken assumption that whatever is one is one individual, classes (which are also one) are also individuals. Note that we are *not* here using 'individual' to mean '*Urelement*' whereas this seems to be the meaning that Leonard and Goodman attach to the word. Leśniewski did not use the word 'individual', but rather the Polish and German equivalents of 'object' (*przedmiot*, *Gegenstand*). We have not used this word because we prefer to have it in reserve to cover all of individuals, masses and classes. An individual is in our view anything which can be the subject of a true singular count predication which is neither disguisedly plural nor disguisedly mass, for example, 'Hans is a man'. Every individual falls under what have been called sortal nouns. This is certainly a different meaning of 'individual' from that used by Leonard and Goodman, who, according to Sharvy (see below) use the term in fact for *masses*. We do not go so far as to restrict the term only to connected objects: 'country' is a good sortal and 'Denmark is a country' is true, but Denmark is not connected.

The sense in which a plurality is many means that it cannot under any conception be a member of a class. The only way to make sense of the abstract sets of Frege, Zermelo, and the modern tradition is to conceive the membership relation as just one more relation between individuals.[41] In this way sets may be thought of as individuals which *represent*[42] or *comprehend*[43] classes.

[40] LEJEWSKI 1967*c*. On how Leśniewski might have handled sets, cf. LEJEWSKI 1974, 1985. A different approach is suggested in SIMONS 1982*d*.

[41] This is the conclusion of STENIUS 1974.

[42] Cf. this idea in STENIUS 1974, but before him in LÖWENHEIM 1940 and again in SIMONS 1982*c*, also the remarks in § 4.10.

[43] Cantor speaks of 'Zusammenfassung zu einem Ganzen'. Cf. the comparisons in SIMONS 1982*c* and 1982*d*. In his later work Cantor clearly distinguished sets (*Mengen*) from pluralities (*Vielheiten*); cf. above all the correspondence with Dedekind: CANTOR 1932*b*.

We said before that number terms have different meanings for individuals and for classes, though there are analogies between them. First of all we recall the definition

$$E!a \equiv \Sigma c \ulcorner c \varepsilon a \urcorner \wedge \Pi cd \ulcorner c \varepsilon a \wedge d \varepsilon a \supset c \varepsilon d \urcorner$$

We then define numerical functors taking one-place predicates as arguments:

$$I_1(\phi) \equiv \Sigma a \ulcorner E!a \wedge \phi a \wedge \Pi b \ulcorner E!b \wedge \phi b \supset b = a \urcorner\urcorner$$

$$I_2(\phi) \equiv \Sigma ab \ulcorner E!a \wedge \phi a \wedge E!b \wedge \phi b \wedge a \neq b \wedge \Pi c \ulcorner E!c \wedge \phi c \supset c = a \vee c = b \urcorner\urcorner$$

and so on, the series being indefinitely continuable. We may read '$I_n(\phi)$' as 'there are (exactly) n individuals which ϕ'.

The analogous functors may be obtained by a simple alteration:

$$C_1(\phi) \equiv \Sigma a \ulcorner Ea \wedge \phi a \wedge \Pi b \ulcorner Eb \wedge \phi b \supset b \cong a \urcorner\urcorner$$

$$C_2(\phi) = \Sigma ab \ulcorner Ea \wedge \phi a \wedge Eb \wedge \phi b \wedge a \ncong b \wedge \Pi c \ulcorner Ec \wedge \phi c \supset c \cong a \vee c \cong b \urcorner\urcorner$$

and so on, replacing 'E!' by 'E' and '=' by '≅'. So we may read '$C_n(\phi)$' as 'there are (exactly) n classes which ϕ'. The former definitions may be considered as special cases of the latter, according to the scheme 'there are n individuals which ϕ iff there are n classes which have one member and ϕ'. If we define a functor K of predicate conjunction as

$$K \langle \phi, \psi \rangle a \equiv \phi a \wedge \psi a$$

then this schema may be expressed as

$$I_n(\phi) \equiv C_n(K \langle E!, \phi \rangle)$$

We can similarly treat other numerical functors such as 'there are n pluralities which ϕ', 'there are n pairs which ϕ' as restrictions in this sense of the most general functor C_n.

4.6 Mass Reference and Masses

It seems to me that we can give reasons both for the parallels underlying the two systems and the differences between them, namely the difference in controversy value and the previously mentioned atomicity of Ontology. To do this, we need to consider a kind of predication which is dealt with neither by Ontology nor by orthodox predicate logic, namely mass predication. It is by now well enough known that natural languages contain common nouns which, in contrast to count nouns like 'horse', 'table', and 'star', are mass nouns, for example 'beef', 'water', 'furniture', and 'wood'. Some nouns, such as 'cake' and 'coffee', admit of both mass and count uses. Syntactic criteria for mass nouns include that they admit 'much' and 'a little', and resist 'many' and 'a few'. Where mass nouns are used in the plural (except those that are already plural, like 'oats') the sense is either that of a kind or brand of the commodity in question, or that of a certain standard portion.[44]

Sentences containing mass nouns enter into logical relations as readily as those containing count nouns, for example:

All water from that spring is radioactive
The water in this glass is from that spring

∴ The water in this glass is radioactive

The expression 'the water in this glass' is what we call a mass *term*, by contrast with the mass *noun* 'water in this glass'. Attempts to cope with mass predication within standard predicate logic have usually taken the form of replacing straightforward mass predications like 'The water in this glass is from that spring' by predications referring to lumps, chunks, portions, or bits, which are taken to be individuals, and hence fit to be quantified over. We may call this standard approach.[45] Not many philosophers have dared to challenge the millennia-long prejudice of Western philosophers in favour of the singular, or to suggest that predicate logic is limited in its applicability. Leśniewski's Ontology, no less than standard predicate logic, has no avowed place for mass terms, and it seems most likely that proponents of Ontology would adopt the standard approach. A further defensive bulwark against a special status for mass terms is provided by the general sum or fusion principle of Mereology, which guarantees, given a number of portions of some stuff, a unique maximum portion, containing all of that stuff, for instance all the world's water. Quine's hybrid approach to mass nouns treats them in the standard way in predicate position, but in subject position as referring to this maximal portion.[46]

Of those who have been prepared to assert the autonomy of mass predication, I should mention Henry Laycock and Richard Sharvy.

[44] Cf. the survey in PELLETIER 1979 for these and other points.

[45] Notable proponents of the standard approach are PARSONS 1979, MONTAGUE 1979.

[46] Cf. QUINE 1960, §20.

The former has carried the argument to the other side and argued the primacy of stuff,[47] while the latter has suggested, not in jest, that even such a language as English might have no true count nouns.[48] Sharvy's faith in the existence of masses, as distinct from individual portions, enables him brilliantly to extend Russell's theory of descriptions so that it applies to mass predication.[49] We shall have occasion to make use of his insights below. One indication that all is not well with the standard approach is provided by the fact that the sentence

Most of the world's gold is yet to be mined

is by no means equivalent with

Most of the world's portions of gold are yet be mined[50]

since there may be many small portions already above ground and fewer large portions below ground. To get round this, the defender of portions must claim that he does not confine 'portion' to maximal connected portions, but allows arbitrary sub-portions and perhaps arbitrary sums as well. But, although we know that as a matter of fact there are smallest possible portions of gold, namely gold atoms, this is not something which could be known a priori and should not therefore have any part in the *logical* analysis of such predications. Had gold been indefinitely divisible, then any two portions of gold would contain the same number of sub-portions, no matter how much gold they contained, and equivalence would not be guaranteed. Given the mere possibility of any indefinitely divisible stuff, the equivalence is at best a fortunate accident for the defenders of portions. One of the advantages claimed for Ontology is its ontological neutrality, and this advantage can be claimed to carry over to Mereology in that this is neutral on the question of atomism. The success of a logical analysis of mass predication ought not to be dependent on non-logical truths.

If mass predication cannot be treated as a form of singular predication, the subjects of such predications must be something other than individuals. They are sometimes called 'quantities',[51] but as 'quantity' also has connotations of measure I prefer to call them 'masses'. It may seem as though simply by using the grammatical singular and plural—'mass', 'masses'—we have already admitted that

[47] LAYCOCK 1972, 1979. [48] SHARVY 1978.
[49] SHARVY 1980. [50] Cf. MCCAWLEY 1981: 436.
[51] The usage stems from H. CARTWRIGHT, whose 1963 is one of the rather few modern works to take non-singular reference seriously.

masses are individuals, if there can be more than one of them. But this objection confuses grammer with logic: it is right too to say that there is more than one class, but that doesn't make classes individuals. The fact that a natural language like English has only one kind of singular and plural means that terms designating classes or masses first have to be artificially modified to singular before they can be pluralized. But because terms for individuals do not need such modification, it is easy to get the idea that only individuals can be denoted by grammatically singular terms. It is one of the fundamental conclusions of this chapter that the grammatical distinction between singular and plural is *almost completely* skew to the ontological distinction mass/count, and the latter's further division into individuals and pluralities. There can be more than one mass, as well as more than one class. The water in this cup is one mass, the water in that cup another. The water in both cups is a third mass, which is the sum of the other two.[52]

Where then does the difference lie between masses and individuals? The difference between individuals and classes was that a class could be many individuals. A mass is, as we should expect, neither one individual nor many. This does not entail that a mass be indefinitely divisible into sub-masses of the same kind. The units of furniture and footwear are the individual chairs, tables, etc. and the individual boots, shoes, etc. respectively. But mass nouns are not sortals: they provide in general no principle for counting the items falling under them.[53]

If masses are neither one nor many individuals, it follows that there is no logical relation between single masses corresponding to 'ε' or '$\sqsubset$', where these are understood to mean 'is one of' and 'are some of' respectively. The logical relation which masses enter into apart from identity, is that which we may express by '*is* some of'. The water in this glass is some of the water from that spring; the rice in this bowl is some of the food in this bowl; and so on. The expression 'is some of' can here be replaced by 'is part of', just as, for plurals, 'are some of' could be replaced by 'are part of'. Sharvy has pointed out that 'is some of' cannot be treated as equivalent to 'is some (sm)'. To take his best example: the water in my whisky-and-water is some of (part of) my whisky-and-water, but it is not sm whisky-and-water.[54] We shall see presently how such sentences may be expressed in a formal language. Once again, plurals show an exact parallel. The whites in a racially

[52] This use of 'the' is that of SHARVY 1980: cf. below.
[53] Cf. GRIFFIN 1977, ch. 3 on sortals for a more detailed picture.
[54] SHARVY 1983*b*: 235. There are similar examples in SHARVY 1980.

mixed school class are part of a multiracial group, but they are not a multiracial group, which must contain members from at least two different races.[55]

It is not accidental that 'some of' and 'part of' are interchangeable both for masses ('is some of') and classes ('are some of'). We noted already that 'are some of', as represented in Ontology by '$\sqsubset$', has the characteristic formal properties of a part–whole relation. It is not surprising that 'is some of' shares these properties. To see this, we need to consider how we could interpret M1–7 so that they characterize the relation 'is some of'. In Mereology the functors 'E!', '$=$', and '$<$' involve the existence of individuals, in the sense that if any sentence 'E!a', '$a = b$' or '$a < b$' is true, then individuals exist. But mass predication is independent of assumptions about the existence of individuals. M1–7 can be divorced from this interpretation. To see this, let the variables now range over *masses*, take '$<$', the 'is some of' relation, as primitive, and *define* existence and identity predicates as follows:

$$a = b \equiv a < b \wedge b < a$$

$$\text{E!}\,a \equiv \Sigma b \ulcorner b = a \urcorner$$

It is then plausible to say that M1–7, so interpreted, characterizes the 'is some of' relation on masses. Since they are structurally exactly like the equivalent formulae of Mereology they are, as we should expect, neutral on the atomicity question, whereas O1–7 belong to Ontology, which asserts that if anything exists, then individuals exist. So we see now that there are in fact not two, but *three* possible interpretations of a part–whole predicate characterized by any analogue of O1–7: a class interpretation, a mass interpretation, and an individual interpretation. Furthermore, the principles M4 and M7 are *not* controversial for masses. If the rice in this bowl is some of the food in this bowl and vice versa, then the rice in this bowl *is* (the same stuff as) the food in this bowl. And if there is *any* water, then there is that maximal mass containing all the water there is.

I said that the existence of masses must be presumed to be logically independent of the existence of individuals. We may envisage a universe consisting of various basic kinds of stuff, mixed together in different places in varying proportions, but nowhere showing the abrupt discontinuities and unified structures characteristic of the

[55] Here 'class' obviously has its academic meaning.

paradigm individuals—organisms, heavenly bodies, physical objects—of our world. It is not just that *we* should be unable to distinguish any individuals in this world—that would be an epistemological, not a metaphysical difference. Rather there would actually *be* no individuals. Someone who, thinking of such a world, attempted to 'generate' individuals by considering particular parcels or portions of the stuff there would be inventing logical fictions or producing logical constructions in order to justify his own prejudices. The paradigm individuals in our world are *not* logical constructions, although there *may* be other individuals (the state, legal persons, characters in novels) which are logical fictions or constructions. But these are not exactly paradigm individuals.

Given that in our world there exist *both* individuals *and* masses, what is the relationship between them? The fundamental relationship is one which we may call *constitution*. Aristotle recognized the existence of such a relationship when he correctly said that we should properly say of a bed not that it is *wood*, but that it is *wooden* (of wood), that a statue is not *bronze*, but *bronzen* (of bronze), and so on.[56] The wood and the bed are distinct entities: the wood existed before the bed, and may outlast it. The wood *constitutes* or *makes up* the bed. Thus the two may be in the same place at the same time without being identical. Constitution is an asymmetric relation.[57] Interestingly, it may be said to hold not only between masses and individuals, but also between classes and individuals. A toy fort may be made up of certain toy bricks (and nothing else). Once again we see the parallels between mass and class. Each brick may be *made of* wood, and therefore *made up of* a certain mass of wood. So the toy fort is itself made up of that mass of wood which is the sum of the masses making up the bricks which make up the fort. Likewise, each brick is made up of certain cellulose and other molecules, these are made up of certain carbon and other atoms, so the fort is made up of the carbon and other atoms in all bricks. We can thus see the way in which constitution is transitive.

It seems to me that the interpretation which does justice to *all* the principles of Mereology, one which renders them all uncontroversial, is one which Leśniewski never considered, namely a mass interpretation. The first person to my knowledge to have stated this clearly is

[56] Aristotle, *Physics* VII. 3, 245ᵇ.
[57] Cf. GRIFFIN 1977: 164; DOEPKE 1982: 55.

Richard Sharvy. As he puts it:[58]

'a part of' . . . cannot be replaced by 'some of', which I claim is the proper mereological and generally logical understanding of the mass and plural sense of 'part of'. It is in fact the only sense in which the calculus of individuals has a true interpretation that comes close to coinciding with an intended interpretation. The temptation to read the calculus of individuals as having something to do with nominalism comes from Goodman's tacitly treating the variables of his system as ranging over quantities of matter in the first place, rather than over individual substances (although he misleads us by calling those objects 'individuals').

Elsewhere, Sharvy accuses the proponents of the calculus of individuals of an unwarranted materialism.[59] That is, the principles of Mereology, including M4, M7, and even perhaps M3, are plausible for masses, but not for individuals: to assume they hold for individuals is to treat these as though they were mere masses of matter. The differences are well marked linguistically. Notice the difference in acceptability of the following:

He took some of the gold
He took part of the gold
He took some of the apples
?He took part of the apples
*He took some of the table
He took part of the table

By contrast:

He took all of the gold
*He took the whole gold
He took all of the apples
*He took the whole apples
?He took all of the table
He took the whole table

but notice

He took the whole batch of gold
He took the whole batch of apples

If we examine the use of the predicate 'is *a* part of' in everyday language, that is, that predicate which holds between individuals, we

[58] SHARVY 1983*b*: 236. My thanks to Richard Sharvy for sending me an earlier draft of the material in SHARVY 1983*a*, 1983*b*, and answering queries arising.
[59] SHARVY 1980: 618 f.

find that some of the principles of Mereology fail for it. In particular, the existence of general sums is not assured. The four wheels of a car are parts of it (each is part of it), but there is not a fifth part consisting of the four wheels.

Now the existence of senses of 'a part of' which diverge from those determined by the axioms of Mereology does not of itself prove that Mereology is 'wrong'. It shows simply that certain interpretations of the constants of Mereology are not appropriate. However, I see no hope of there being any relation between individuals satisfying all the principles of Mereology. Accepting the appropriateness of the mass interpretation, one could attempt one as follows:

> Individual *a* is a part of individual *b* iff the matter making up *a* is part of the matter making up *b*

Certainly one can define a very general part–whole relationship between individuals in this way, but there is no guarantee that there will be an exact match between this and the 'is part of' relation, because there are cases where mass *m* is part of mass *n* where we do not want to say that there are individuals *a*, *b* such that $m =$ the matter of *a* and $n =$ the matter of *b*. The assumption that this is so is precisely what critics of Mereology disagree with. The matter of this stone may be part of the matter of all the world's stones, but that does not mean that this stone is a part of an individual made up of all the world's stones, for there is no such individual.

At one time, I thought the difference between individuals and classes on the one hand and masses on the other could be characterized by saying that for masses there is no distinction between distributive and collective classes: roughly speaking, Ontology and Mereology fall together into a single theory. This view no longer seems to me to be correct. After all, we have argued that it makes as much sense to speak of one mass, two masses, many masses, as it does to speak of one individual, etc., or one class, etc. In other words, plural reference and the relation 'is one of' can just as well apply to masses and classes as to individuals. We may see this with the help of concrete examples. Suppose we take sm gin *g* and sm tonic *t* and mix them together in a glass to form sm gin-and-tonic *a*. Then *a* is one of the items of which 'is gin and tonic' is true, whereas *g* and *t* are not. Let us define a nominal term *b* as follows:

> *c* is one of $b \equiv c$ exists and *c* is gin and tonic

We can approximate *b* in English by 'the masses of gin and tonic in the world'. Then clearly *a* is one of *b*, but neither *g* nor *t* is one of *b*, whereas *g* is *some of* (part of) one of *b*, and *t* is some of (part of) one of *b*, namely *a*. The term *b* is *a plural* term designating all masses of gin and tonic. We must distinguish this from the *singular* term denoting the world's gin and tonic. Let this term be '*A*'. It follows that *a* is some of *A*, and therefore that each of *g*, *t* is some of (is part of) *A*. Also *A* is one of *b*, but there is many another item which is one of *b* in addition to *A*, although *each* such item is part of *A*. We must note that it is possible also to read 'some of' (but not 'part of') so that whatever is some of *A* is gin and tonic. But this is because this sense of 'some of' includes 'sm'. In *this* sense

> *c* is some of the world's gin and tonic (*A*) iff *c* is part of the world's gin and tonic and *c* is sm gin and tonic

We may acknowledge this use of 'some of' (which applies also to classes) without giving up the wider 'part of' relation.

Exactly similar considerations apply to classes. Suppose we take sm white children *w* and sm black children *b* and put them together in a multiracial school class *m*. Then *w* are sm of (are part of) *m*, and *b* are sm of *m*, and therewith both are part of the maximal class *M* consisting of all school children in mixed classes. But *m* is one of the classes of which 'is a multiracial class' is true, whereas neither *w* nor *b* is a multiracial class. The plural expression 'the multiracial classes in the world' approximates a plural term *r* satisfying

> *c* is one of *r* iff *c* exists and *c* is multiracial (is a multiracial class)

It is possible also to use the vocabulary of Ontology and Mereology for these class terms.

In each case we must note an essential difference between the functors of Ontology on the one hand and those of Mereology on the other. Ontology has to do with singular and plural. Singular and plural (one and many) applies equally well to all categories: individuals, classes, and masses. The connection between the two, which prevents the singular/plural distinction from being *completely* skew to that between individual and class is that, putting it picturesquely, 'class' is the plural of 'individual'. Many individuals *are* a class. A class is many individuals.

It is possible now to see why the standard approach to mass terms is so compelling. If several objects or items can fall under a mass noun, then it seems to follow that these several objects must be many of which each such object is one. Because the singular/plural (one/many) distinction coincides *for individuals* with the *individual/plurality* distinction, it is thoroughly natural to think that this means that masses are individuals. A similar remark applies to the 'abstract individual' approach to classes.

Nevertheless, accepting classes and masses as *sui generis* objects does not involve a radical revision of logic. Predicate logic and Ontology have been previously tied to the interpretation of singular terms as designating individuals. Once it is realized that being a (designating) singular term and designating an individual are not the same thing, we can see that predicate logic and Ontology as such are in need of no revision: all that needs to be done is to widen their application.

4.7 Outline of a Comprehensive Theory

We outline here one way in which all the previous considerations may be brought together.[60] We take the rules and axioms of Ontology to be defined analogously for *three* primitives 'ε_I', 'ε_C', and 'ε_M', with all the usual defined functors. The three categorical predicates 'is an individual', 'is a class', and 'is a mass' are simply '$E!_I$', '$E!_C$', and '$E!_M$'. Further, the connection between classes and individuals is given by taking as axiom

CMA1 $\Pi ab\ulcorner a =_C b \equiv a \cong_I b\urcorner$

which implies

CMT1 $\Pi a\ulcorner E!_C a \equiv E_I a\urcorner$

showing that a class is simply one or more individuals.

We define a part–whole relation '$<_C$' on classes simply by

CMD1 $\Pi ab\ulcorner a <_C b \equiv a \sqsubset_I b\urcorner$

while a further part–whole relation '$<_M$' for masses may be obtained by adopting suitable mereological theses such as M1–7 (adapted to masses). By virtue of the usual theses of Ontology, these principles apply to the part–whole relation '$<_C$' as well. But the algebra of '$<_C$' must be atomistic, while that of '$<_M$' need not be.

It should further be stated axiomatically that no class (and therefore

[60] For an approach which similarly gives both mass and plural predication their due, makes numerous good informal points, and is more developed semantically than this, see LINK 1983, which also provides references to further literature. The present approach was developed independently of Link in SIMONS 1983*b*. Comparisons must await another occasion.

no individual, individuals being unit classes) is a mass:

CMA2 $\sim \Sigma a \ulcorner \mathrm{E!}_{\mathrm{C}} a \wedge \mathrm{E!}_{\mathrm{M}} a \urcorner$

The existence of unions, required by O7, is guaranteed even where we have plural class terms, because we can employ the definitions

CMD2 $a\,\varepsilon_{\mathrm{I}} \cup \langle \phi \rangle \equiv a\,\varepsilon_{\mathrm{I}}\, a \wedge \Sigma b \ulcorner \mathrm{E}_{\mathrm{I}}\, b \wedge \phi b \wedge a\,\varepsilon_{\mathrm{I}}\, b \urcorner$ (union of a predicate)

CMD3 $\varepsilon_{\mathrm{C}}\{b\}\, a \equiv a\,\varepsilon_{\mathrm{C}}\, b$ (monadic predicate derived from 'ε_{C}')

CMD4 $a\,\varepsilon_{\mathrm{I}}\,\mathrm{Un}[b] \equiv a\,\varepsilon_{\mathrm{I}} \cup \langle \varepsilon_{\mathrm{C}}\{b\} \rangle$ (union of *b*s)

Here, for '$a\,\varepsilon_{\mathrm{I}}\,\mathrm{Un}[b]$' to be true, '*a*' must be a singular individual term, but '*b*' may be either a class or an individual term.

We now introduce a predicate corresponding to the notion of *constitution* or *making up*: '$a \Rightarrow b$' may be read as '*a* makes up *b*'. It should be remarked that a more adequate theory of constitution must wait until a later chapter, when we deal with tensed sentences about continuants. We shall then be able to explicate further the properties of constitution which we are here simply stating. It is governed by the following principles:

CMA3 $a \Rightarrow b \supset \mathrm{E!}_{\mathrm{M}}\, a \wedge \mathrm{E!}_{\mathrm{I}}\, b$

CMA4 $\mathrm{E!}_{\mathrm{I}} a \equiv \Sigma b \ulcorner \mathrm{E!}_{\mathrm{M}}\, b \wedge b \Rightarrow a \urcorner$

CMA5 $b \Rightarrow a \wedge c \Rightarrow a \supset b =_{\mathrm{M}} c$

Since no individual is a mass, it follows that making up is irreflexive and asymmetric:

CMT2 $\sim (a \Rightarrow a)$

CMT3 $a \Rightarrow b \supset \sim (b \Rightarrow a)$

Now we define the most general part–whole relation among individuals which we found could be accepted without controversy:

CMD5 $a <_{\mathrm{I}} b \equiv \Sigma cd \ulcorner c <_{\mathrm{M}} d \wedge c \Rightarrow a \wedge d \Rightarrow b \urcorner$

Since, it is not axiomatically guaranteed that every mass makes up an individual, the following are not theorems:

CMF1 $a \Rightarrow b \wedge a <_{\mathrm{M}} c \supset \Sigma d \ulcorner c \Rightarrow d \urcorner$

CMF2 $a \Rightarrow b \wedge c <_{\mathrm{M}} a \supset \Sigma d \ulcorner c \Rightarrow d \urcorner$

The following theses regarding '$<_{\mathrm{I}}$' may easily be derived using the properties of '$\Rightarrow$' and '$<_{\mathrm{M}}$':

CMT4 $a <_{\mathrm{I}} b \supset \mathrm{E!}_{\mathrm{I}} a \wedge \mathrm{E!}_{\mathrm{I}}\, b$

CMT5 $\mathrm{E!}_{\mathrm{I}}\, a \supset a <_{\mathrm{I}} a$

CMT6 $a <_I b \wedge b <_I c \supset a <_I c$

Defining CMD6 $a <>_I b \equiv a <_I b \wedge b <_I a$

We get CMT7 $a <>_I b \equiv \Sigma c \ulcorner c \Rightarrow a \urcorner \wedge \Pi c \ulcorner c \Rightarrow a \equiv c \Rightarrow b \urcorner$

further CMT8 $a \Rightarrow b \wedge b <_I c \supset \Sigma d \ulcorner a <_M d \wedge d \Rightarrow c \urcorner$

and CMT9 $a \Rightarrow b \wedge c <_I b \supset \Sigma d \ulcorner d <_M a \wedge d \Rightarrow c \urcorner$

Defining CMD7 $a \ll_I b \equiv a <_I b \wedge \sim (b <_I a)$

we get CMT10 $a \Rightarrow b \wedge b \ll_I c \supset \sim (a \Rightarrow c)$

CMT11 $a \Rightarrow b \wedge c \ll_I b \supset \sim (a \Rightarrow c)$

These properties of '$<_I$' are all plausible and desirable. Because of our caution in the admission of individuals corresponding to masses, the properties of '$<_I$' lack the algebraic simplicity of '$<_M$' or '$<_C$'. For those who are happy that Mereology applies to individuals as it stands, we can recommend as axiom the formula:

CMF3 $E!_M a \supset \Sigma b \ulcorner a \Rightarrow b \wedge \Pi c \ulcorner a \Rightarrow c \supset c =_I b \urcorner \urcorner$

from which, using the definition of '$<_I$', we can guarantee the application of all of M1–7 to '$<_I$'. Without this axiom, there are two distinct notions of overlapping for individuals: having a common individual part:

CMD8 $a \circ_I b \equiv \Sigma c \ulcorner c <_I a \wedge c <_I b \urcorner$

and having common matter:

CMD8* $a \circ_I^* b \equiv \Sigma cd \ulcorner c \Rightarrow a \wedge d \Rightarrow b \wedge c \circ_M d \urcorner$

The former entails the latter, but not vice versa, though with the simplifying axiom they coincide.

We mentioned earlier that there are senses in which a class may make up an individual, and a mass may make up the matter of a class. To define these notions, we first of all define the sum or fusion of a number of masses:

CMD9 $a \varepsilon_M \text{sum}[b] \equiv a \varepsilon_M a \wedge E_M b \wedge$
$\Pi c \ulcorner c \circ_M a \equiv \Sigma d \ulcorner d \varepsilon_M b \wedge c \circ_M b \urcorner \urcorner$

then the part-masses of a class:

CMD10 $a \varepsilon_M \text{pm}[b] \equiv a \varepsilon_M a \wedge b \varepsilon_C b \wedge \Sigma c \ulcorner c \varepsilon_I b \wedge a \Rightarrow c \urcorner$

so the mass making up the matter of a class is simply

sum[pm[b]]

the sum of the part-masses of the class. Under the simplifying assumption above this means that to every class there corresponds a unique *individual* which is its sum. This is the sum normally found in Mereology.

Finally, a class makes up an individual if its matter makes up that individual:

CMD11 $a \Rightarrow_C b \equiv \text{sum}[\text{pm}[a]] \Rightarrow b$

Within the framework of the system sketched, we can show how Sharvy's generalized theory of definite descriptions can be accommodated.[61] For individuals, the theory is simply Russell's and needs little comment. For classes and masses, Sharvy employs the fact that characteristic class and mass nouns are *cumulative*, that is, any sum of parts (in the sense of '$<_C$' or '$<_M$') falling under the noun likewise falls under the noun. Now not all predicates true of classes and masses are cumulative, for instance 'has ten members', 'weighs ten grams', but where more than one thing falls under such a predicate the corresponding definite description is empty. On the other hand, where predicates are cumulative, more than one object can fall under them and the corresponding definite descriptions still denote: we can say of more than one mass that it is coffee in this room, but '*the* coffee in this room' denotes the sum of all such; more than one class is a class of men in this room, but 'the men in this room' denotes the union of all such. Sharvy's chosen notation for 'the x such that $\phi(x)$' is '$\theta x \cdot \phi x$':[62] it is an advantage of Ontology that such descriptions may be given direct nominative definitions rather than be defined in context:

CMD12 $a\, \varepsilon_C\, \theta x^{\ulcorner}\phi x^{\urcorner} \equiv a\, \varepsilon_C\, a \wedge \phi a \wedge \Pi b^{\ulcorner}\phi b \supset b <_C a^{\urcorner}$

CMD13 $a\, \varepsilon_M\, \theta x^{\ulcorner}\phi x^{\urcorner} \equiv a\, \varepsilon_M\, a \wedge \phi a \wedge \Pi b^{\ulcorner}\phi b \supset b <_M a^{\urcorner}$

If we look at the exact analogue for individuals:

CMD14 $a\, \varepsilon_1\, \theta x^{\ulcorner}\phi x^{\urcorner} \equiv a\, \varepsilon_1\, a \wedge \phi a \wedge \Pi b^{\ulcorner}\phi b \supset b <_1 a^{\urcorner}$

we see that in certain circumstances this will agree with the Russellian description:

CMD15 $a\, \varepsilon_1\, \iota x^{\ulcorner}\phi x^{\urcorner} \equiv a\, \varepsilon_1\, a \wedge \phi a \wedge \Pi b^{\ulcorner}\phi b \supset b =_1 a^{\urcorner}$

namely, when there is only one individual a such that ϕa. But there are circumstances when the two will differ. If a table is formed by pushing two smaller tables together, 'the table' in the sense of 'θ' is the large

[61] SHARVY 1980.

[62] In SHARVY 1980, 'μ' is used instead of 'θ', but since 'μ' is used in formal number and recursion theory for a minimum function, Sharvy now prefers 'θ'.

table of which the other two are parts, while 'the table' in the sense of 'ι' does not denote. Both analyses seem to me to be genuine alternatives, and the situation therefore more complicated than Sharvy portrays it. This can be brought out by noting the following definitions

CMD16 $a \, \varepsilon_C \, \iota x \ulcorner \phi x \urcorner \equiv a \, \varepsilon_C \, a \wedge \phi a \wedge \Pi b \ulcorner \phi b \supset b =_C a \urcorner$

CMD17 $a \, \varepsilon_M \, \iota x \ulcorner \phi x \urcorner \equiv a \, \varepsilon_M \, a \wedge \phi a \wedge \Pi b \ulcorner \phi b \supset b =_M a \urcorner$

which are the class and mass analogues of Russell's descriptions. Sharvy is right that expressions like 'the men in this room', 'the coffee in this room' are to be analysed using 'θ', but if we add the singularizing adaptors 'class of' and 'mass of', to get 'the class of men in this room', 'the mass of coffee in this room', then a similar ambiguity arises as did for the table example. One possible reading is the 'θ' or maximal reading: the class of men in this room $=_C$ the class of *all* of the men in this room, the mass of coffee in this room $=_M$ the mass of *all* of the coffee in this room. But the singularization allows a Russellian, 'ι' or uniqueness reading as well.[63]

4.8 Extending the Analogy?

According to the previous section, mereological concepts apply not in one but in three analogous senses, corresponding to the three categories of concrete particular: individuals, classes, and masses. Is this the limit of the analogy? A range of further candidates may be generated by picking up a possibility unexploited in the previous section. We said that a plurality is in one sense many and in another sense one: it is many individuals and one class. As one class, it may be one of several classes fulfilling a certain condition. So we may have $a \, \varepsilon_I \, c$ and $b \, \varepsilon_C \, c$. Now these classes c are also many classes, but in being *just these* classes they also may be considered to have a certain identity and unity which is analogous to, but distinct from, the kind of identity and unity their member classes have. It seems reasonable to suppose that the unity, etc. of c stands to that of b as that of b stands to that of a. And once we have taken two steps, it appears arbitrary to stop there. Rewriting 'ε_I' and 'ε_C' as 'ε_1' and 'ε_2' respectively, we can then iterate the construction, having a chain like $a \, \varepsilon_1 \, b \, \varepsilon_2 \, c \, \varepsilon_3 \, d \, \varepsilon_4 \ldots$. Here we have

[63] There are other uses of 'the' covered neither by Sharvy's theory nor by mine, but we leave these out of consideration here.

an open ascending hierarchy of meanings of 'is one of', which allows us to define analogous meanings of identity and inclusion: for each $i \geqslant 1$ we have

$$a =_i b \equiv a\,\varepsilon_i\, b \wedge b\,\varepsilon_i\, a$$
$$a \simeq_i b \equiv \Pi c \ulcorner c\,\varepsilon_i\, a \equiv c\,\varepsilon_i\, b \urcorner$$
$$a \sqsubseteq_i b \equiv \Sigma c \ulcorner c\,\varepsilon_i\, a \urcorner \wedge \Pi c \ulcorner c\,\varepsilon_i\, a \supset c\,\varepsilon_i\, b \urcorner$$

and so on. This allows us to interpret classical extensional mereology atomistically at each level: $\sqsubseteq_i$ is just $<_{i+1}$. But since we can also have plural mass terms, and sentences like 'The water in this glass is one of the masses of water drawn from that spring today', which are regimented in the form $a\ \varepsilon_M\ b$, the same kind of open hierarchy can be built beginning with masses rather than individuals.

The prospect of such open hierarchies raises in acute form the question of how seriously we should take the existence of referents corresponding to terms at each of these levels. The higher we go, the more tenuous appear the existence claims of such referents.

The suggestion in §4.3 was that plural reference is not everywhere eliminable, which encourages one to accept the existence of pluralities. At the same time, it was allowed that pluralities are innocuous; they do not crowd the world, because if the individual A exists and the individual B exists, then so does the class of A and B, but to say the latter is to say no more than the former. The functor 'class of' is a *singularizer*, but unlike those obtained from collective nouns like 'flock' or 'congregation' it functions in no other way, *as used by logicians*. It is a mere syntactic adaptor and nothing more. In real life, even the most neutral terms like 'group', 'collection', 'class', and 'set' tell us something more than that we have to do with more than one object. The pure singularizer is a logicians' invention. By contrast, unadorned plurals are a grass-roots feature of ordinary language. To extend the analogy from pluralities to pluralities of pluralities and so on, we need the artifice of pure singularization, which is one argument against accepting classes of classes and so on. It should be stressed that we are not here taking 'class' in the mathematician's sense of abstract sets, so we are not here querying whether a mathematician could get by without sets of sets: that is a quite different issue. The question here is whether there are pluralities of pluralities, and, as usual in ontology, we must take our initial bearings from language. The raw evidence from English suggests there is scant need for talk about classes of classes. While nominal conjunction of singulars, as in 'Romeo and

Juliet', yields a term for a plurality of individuals, conjoined plurals, as in 'the Montagues and the Capulets', do not clearly yield a plurality of pluralities: in some cases it appears to designate merely the *union* of the two pluralities, which is itself just another plurality of individuals; in other cases the terms appear to designate not the mere pluralities but rather the *families*, which are groups. We have nothing against classes of groups, groups of groups, classes of groups of groups, and so on. It is only the formal case of classes of classes which appears unnecessary. So on balance I think the analogy should be kept within the bounds of the last section.[64]

4.9 Parts of Groups

Because collective nouns forming group terms are not thus mere dummies, we can have physical hierarchies of groups, like a line of circles of heaps of stones, or institutional ones, like an association of leagues of clubs. In such cases, the additional constitutive conditions imposed at each level in the hierarchy places restrictions on what can be counted as a part of the group in question. The concept *part* is therefore no longer purely formal, but has a material admixture, as when used with respect to structured individuals to apply to components or other salient parts. It will therefore not usually satisfy the full classical principles. In particular, the same plurality of individuals may simultaneously satisfy two or more sets of group-constituting conditions. The groups may therefore coincide in membership without being identical—extensionality goes. The extensional classes making up groups at any time may be looked on as the matter of the groups.

The 'is one of' as holding between individuals and classes is the purely formal concept of singular inclusion, but when applied to groups this expression can have a tensed or temporally relative meaning. For instance, the same sentence, 'John is one of the Directors' may be uttered falsely before John's election to the Board, and truly after. The expression 'the Directors' thus designates those who are *now* Directors in this case (it need not, but can and often does work like this.) So the group designated by 'the Directors' is a collective *continuant*: it persists over time and its membership may

[64] This is also the conclusion of SIMONS 1982*b*.

fluctuate. With this and with tensed inclusion, we move outside the ambit of Part I and into that of Part II.

4.10 Further Possible Applications of Extensional Mereology

Suppose we accept properties, expressed by predicates, of which classes are the extensions. Then we can reflect back the mereological structure of classes into one among properties. We can define

$$F < G \equiv \exists xFx \wedge \forall x\ulcorner Fx \supset Gx\urcorner$$

If ext (F) is the extension of F, then

$$F < G \equiv \text{ext}(F) \sqsubset \text{ext}(G)$$

Since the syntactic category of the predicates is S/N, that of the '<' defined is S/(S/N) (S/N), so we do not have a part-*relation* so much as a syntactic analogue of one. To get a part-relation among properties, we should have to take these to be abstract individuals denoted by abstract singular terms, so that

$$Fx \equiv x \text{ has } F\text{ness}$$

Then we can define the obvious relation (equivocating on '<') by

$$F\text{ness} < G\text{ness} \equiv F < G$$

or in general

$$\phi < \psi \equiv \exists x\ulcorner x \text{ has } \phi\urcorner \wedge \forall x\ulcorner x \text{ has } \phi \supset x \text{ has } \psi\urcorner$$

This is of course extensional inclusion. It can be defined analogously for relations of any number of places. All these functors will exhibit the formal properties of the part-relation < in a suitably analogous form.

Traditional logic recognized not only extensional inclusion but intensional inclusion as well. Predicates traditionally have an intension, which is a concept, and these stand in inclusion relations and all the other mereological relations definable in terms of inclusion. The intension of a concept is traditionally the conjunction of simple (atomic) concepts which are its characteristic marks ('Merkmale') and one concept is included in another iff all the marks of the first are marks of the second.[65] The relation between intensional and extensional inclusion is captured in the following version of the Canon of

[65] The part-concept among properties described by ARMSTRONG 1978: vol. ii, 36 ff. is rather wider than this. Nevertheless, he recognizes this as a special case: 'P stands to P & Q as the blade of a knife stands to the whole knife' (36).

Inverse Variation of Intension and Extension:

$$\text{int}(F) < \text{int}(G) \supset \text{ext}(G) < \text{ext}(F)$$

However one might wish to interpret intension nowadays—and the old atomistic sums will not wash[66]—as long as one can find a way of talking about intensional inclusion, we may expect some version of the Canon to hold, giving an interesting crossing of two mereological structures among concepts.[67]

Some kind of intensional part–whole relation might be applied to propositions, considered as the abstract bearers of truth and falsity. A promising one is the following:

$$p < q \equiv \forall a \ulcorner a \text{ makes it true that } p \supset a \text{ makes it true that } q \urcorner$$

where the quantifier ranges over all possible truth-makers. This would then define a form of implication, though what properties this had would depend on the principles governing the relation of making true.[68]

A more immediate possibility for applying mereology among abstract objects looks to (high-brow) set theory. Zermelo–Fraenkel set theory, with '$\subset$' (inclusion) as the part-predicate, fails to model classical extensional mereology, because there is a null set and no universal set. Thus not all predicates yield a sum ('$\xi = \xi$' does not). In a grounded cumulative hierarchy of sets, however, the sets generated at any given level will provide place for a mereology provided we exclude the empty set. This is not surprising, since they model a complete Boolean algebra, and we just delete the zero to model mereology.

A more direct way of reproducing mereological structure within sets is to make use of Cantor's idea that a set is an abstract individual which *comprehends* a class.[69] Let **I** be a fixed class of individuals, and suppose that for each subclass $a \sqsubseteq \mathbf{I}$ we assign an individual $[a]$ to represent or comprehend a, adding one for the empty class, in such a way that there is exactly one individual per class and one class per individual. Call the individuals representing classes in this way *sets*, and define the relation $<$ on sets in the obvious way:

$$X < Y = \forall ab \ulcorner X = [a] \wedge Y = [b] \supset a \sqsubseteq b \urcorner$$

Since the representing relation is arbitrary, provided only that the

[66] Cf. FREGE 1884: §88.

[67] Cf. the lattice-theoretic account in §7 of SIMONS 1986c.

[68] Cf. MULLIGAN *et al.* 1984: §6.

[69] Cf. note 43 above.

restrictions are observed, and the cardinality is correct (there must be 2^c sets if I has cardinality c), the induced mereological structure on sets may have nothing intrinsically to do with relations among the sets (which may be arbitrary individuals, abstract or concrete) apart from their correlation with classes.

This is an instance of a general phenomenon. Suppose we have some objects A on which a classical extensional mereology is defined under the relation $<$ (so A must have cardinality $2^c - 1$ for some positive c). Let B be another class with the same cardinality as A, and let $f: A \rightarrow B$ be an arbitrary bijection between the two. Then f induces a mereology on B under the relation $<'$ defined in the natural way

$$\forall xy \varepsilon B \ulcorner x <' y \equiv f^{-1}(x) < f^{-1}(y) \urcorner$$

Once again, any actual relation coextensive with $<'$ need have nothing to do with part–whole in any natural sense.

Since it is possible thus to generate mereologies cheaply, one should be careful that the extension of existing concepts of part and whole to new areas does not degenerate into a game. Enough has been said here to give an idea of the possibilities. Since most of these concern abstract objects of some sort, they strictly lie outside the scope of this book, which is concerned with concrete objects. It is to these that we now return.

Part II

Mereology of Continuants

> In the state of living Creatures, their Identity depends not on a Mass of the same Particles; but on something else. For in them the variation of great parcels of Matter alters not the Identity
>
> Locke, *An Essay concerning Human Understanding*, book II, ch. 27, § 3

The problems taken up in this part are among the most pressing of those left aside by extensional part–whole theory as described in Part I. We shall consider what happens to part–whole theory when temporal considerations are taken into account, and discuss some of the issues arising. Matters of modality will be left until the final part.

The interaction of part–whole issues with those of time varies according as we consider continuants on the one hand or occurrents on the other. Questions such as 'Was A ever part of B?', 'When was A part of B?', 'When did A have an α as part?' are significant only for continuants. We can ask of a certain brass lever if and when it was part of the Flying Scotsman, but we cannot ask the same thing of the Battle of El Alamein with respect to the North Africa Campaign: at best we can ask when the battle took place, and whether it was indeed part of the campaign. It is precisely because occurrents—events, states, and processes—are extended in time and have temporal parts, whereas this is not the case for continuants, that such questions are significant for continuants alone, and for them we need to introduce temporally modified mereological predicates.

Their introduction does not compel us to give up extensionality of parts, but the only way to hang onto this is to suppose that all true continuants are like Locke's masses of matter, mereologically *constant*. We shall confront this position and claim it is untenable. That leaves us with the problem of giving an account of the mereology of continuants in flux, and of their relationship to ones which are not.

5 Temporary Parts and Intermittent Existence

There are a number of mereological issues which are relevant to the ontology of continuants, some quite crucially. Firstly there is simply the question as to what happens when we attempt to extend the notions developed in the tenseless extensional mereology of Part I to a temporally modified part–whole relation. Many of the complications arise because continuants may have *temporary* parts, be mereologically variable. We examine some of the implications of this view, and defend it against criticisms by Chisholm. We also argue that mereological considerations lead to the perhaps surprising view that continuants need not exist continuously, and show how this in turn leads to a novel solution to the Ship of Theseus problem.

5.1 Continuants

A continuant is an object which is in time, but of which it makes no sense to say that it has temporal parts or phases. At any time at which it exists, a continuant is wholly present. Typical continuants come into existence at a certain moment, continue to exist for a period (hence their name) and then cease to exist. Physical bodies, including human beings, are prime examples of continuants. Most of the continuants with which we are familiar have a life which, no matter how short, covers an interval. But we do not rule out a priori the possibility of continuants which exist only for a moment, though it is very likely there are none except as idealized possibilities. Imagine two parallel lines in a plane which begin one metre apart and move apart from one another at a speed of 1 m/sec., while two other parallel lines square to the first begin 3 m apart and move towards one another at the same speed. Both sets of lines begin to move at the same instant. Then one second later the four lines form a square with sides 2 m where they overlap. This square configuration exists only instantaneously, but it exists nevertheless. We further do not require that a continuant have a continuous life: it turns out that there are philosophically interesting continuants which exist intermittently.

We use the semi-technical 'continuant' in preference to 'thing', 'substance', or 'object'. We reserve the latter as a formal term covering anything at all. Not everything which we regard as a continuant is what would normally be called a (material) thing, object, or substance. In particular, there are numerous continuants, such as smiles, knots, and waves, which are disturbances in substances rather than substances. Nevertheless, typical continuants (those which do not exist only momentarily) display an important formal property which Aristotle took to be characteristic of (first) substances: they may have contrary properties at different times in their lives, and yet it is the whole continuant, and not just part of it, which has the different properties successively.

It follows that a sentence ascribing an attribute to a continuant must in general indicate the time at which the continuant has the attribute. Two ways of meeting this requirement are open. Natural languages tend to operate with devices like tenses and temporal adverbs, which have an indexical element, taking the time of utterance as datum. The other way is to suppose provided a system of dates for giving names to times, so the general form of predication about a continuant c is '$\phi(c)$ at t', where ϕ is a tenseless sentence frame. For various reasons, not all of them argued here, we prefer the latter alternative. First, it is simpler. Secondly, we wish to consider predications where temporal reference is significant alongside those (such as identity predications) where it is not. Finally, we shall thereby be better able to get a better perspective on the differences between continuants and occurrents. We leave it to the interested reader to investigate how similar ideas can be formulated in a tensed language.

Although we shall concentrate mainly on continuants which are individuals, there are plural and mass continuants as well. A committee, orchestra, or species is a group which comes into existence, continues to exist, then ceases to exist. These are all groups which are, in the sense of 'part' developed in Chapter 4 for pluralities, mereologically variable: they change in membership. Pluralities designated by plain plural terms, i.e. classes or manifolds, do not have to be continuants. For instance, the events leading up to the First World War are no continuant. Even where the members of a class are themselves continuants, the class itself is not. The kings and queens of England should in all likelihood not be considered as a plural continuant. This is because the purely tenseless predicate 'is one of' of membership is, like identity, a purely formal predicate, and hence immune to temporal modification. A mass continuant would be something like the wine in a certain bottle, which came into existence as part of the wine in one or more vats, and will probably cease to exist when broken down in various digestive systems, or perhaps by turning before then into vinegar. It is to be noted that, although we identify this mass by reference to its period of being all together in the bottle, its existence does not coincide with its period of being gathered together: the same wine exists even after it has been poured into several glasses. It is significantly less easy to find a clear example of a mereologically variable mass continuant. The water in the River Salzach *might* afford an example. However, we seem more inclined to say the water in the Salzach today is *different* from that yesterday, rather than that it is the same water with some difference in parts. Mass terms tend to connote mereological constancy, since it is in terms of conservation of matter (mass term standing in for mass substance terms) that we account for mereological variation. This is one reason why mass terms so plausibly fit the tenseless extensional mereology of the previous chapter.

5.2 Temporary Parts

It is a belief deeply woven into common sense that many material objects may, within certain limits, gain and/or lose parts without prejudice to their identity and continued existence. We call such objects *mereologically variable*, and contrast them with objects for which the gain or loss of the least part spells doom; these we call *mereologically constant*. Exact definitions of these concepts follow. The belief in the mereological variability of most of the substances with which we are acquainted is a key one in the metaphysics of Aristotle and other realists such as Locke. Yet numerous philosophers, including Abelard, Leibniz, Hume, Reid, and most recently Chisholm,[1] have found the belief to be either false or at any rate sloppy. In examining this issue, we operate on Moore's principle that the onus is on those who wish to deny a common-sense belief to prove their case. We shall conclude that the reasons for hanging on to the common-sense view are better than those for giving it up or revising its significance.

For the purposes of presenting the problems of flux in continuants, there is no need for a sophisticated theory of time. We shall assume that the variables t, t', t'', . . . range over temporal instants, which we

[1] See the references in CHISHOLM 1976: 145, 221.

think of as ordered by a dense linear ordering. It would complicate matters unnecessarily to consider intervals rather than instants,[2] or to consider the relationist position according to which times are abstracted from events. One thing at a time. We introduce a predicate modifier taking these instant variables as arguments, corresponding to the temporal preposition 'at'. Modification of a simple predicate by the modifier 'at *t*' is signified by subscripting the variable to the predicate sign. The logical constants, including identity, and all expressions, including predicates, defined purely in terms of the logical constants, are such that the modifier 'at *t*' cannot significantly be applied to them. Nor can it be attached to singular terms, although it will occur within descriptive singular terms. Similarly, predicates defined in terms of a formula where all occurrences of temporal variables occur bound (as in CTD10 and CTD12 below) will not be further modifiable. There are philosophical reasons for taking these decisions to be the right ones, but we shall not here go into them, as that would lead us too far afield. We adopt the convention that *unmodified* occurrences of *modifiable* predicates are to be understood as being existentially quantified, i.e. as meaning 'at some time', according to the schema

CTDS $Fa_1 \ldots a_n \equiv \exists t \ulcorner F_t a_1 \ldots a_n \urcorner$

where, to avoid scope problems, we restrict F to being a *simple* predicate symbol (which may however be defined). We shall for the present confine temporal variables to the roles of modification and being quantificationally bound; mereological, ordering, and other relations among times will not be considered, though they would be necessary to take account of further issues such as coming to be, ceasing to be, growing, and diminishing.

The formal system here sketched is called CT (for 'continuants and times'). We let a, b, c, a_1, ... be continuant parameters and x, y, z, x_1, ... (bound) continuant variables. The underlying logic we suppose to be a two-sorted version of that underlying the free system F of Chapter 2, allowing for necessary adjustments to vocabulary. Since we want quantificational and identity theory for times as well as continuants, we assume that the axioms and rules of the underlying logic apply to both continuant and time terms. So the first axioms,

[2] For an explicit treatment of time intervals using mereological concepts, see NEEDHAM 1981, whose mereological part we discussed in §2.9.4 above.

rules and definitions of CT (after CTDS) are:

CTA 0–5 as FA0–5, modulo vocabulary

CTR 1–2 as FR1–2, similarly

CTD 1–2 as FD1–2, similarly

We now consider mereological vocabulary. Definitions falling under CTDS will not be given specially. We take '$<_t$' as primitive, and leave it deliberately open at this stage whether improper parts are identical with their wholes: in the light of discussion in Chapter 6 it will transpire that '$<_t$' can be given more than one interpretation. For the moment, the partial characterization given in the axioms to follow suffices to introduce the important points we wish to consider. We may define analogues of two familiar notions in the obvious way:

CTD3 $a \ll_t b \equiv a <_t b \wedge \sim b <_t a$

CTD4 $a \circ_t b \equiv \exists x \ulcorner x <_t a \wedge x <_t b \urcorner$

To deal adequately with continuants we need, alongside the timeless logical notion of existence, a temporal notion as well, corresponding to the sense in which it is true of Troy that it existed in the twelfth century BC but not that it existed in the twelfth century AD. For this purpose we introduce the predicate 'Ex', which, in contrast to 'E!', is temporally modifiable. The two are connected by the principle that, for continuants, to exist (E!) is to exist (Ex) at some time, which we embody in an axiom

CTA6 $\mathrm{E!}a \equiv \exists t \ulcorner \mathrm{Ex}_t a \urcorner$

which yields, by CTDS

CTT1 $\mathrm{E!}a \equiv \mathrm{Ex}\, a$

We do *not* have a tensed identity predicate. There is no sense which can be given to the question 'When was *a* identical with *b*?' which makes it a question about identity. We further stipulate here that 'Ex' and its modifications are not significantly predicable of times: expressions like '$\mathrm{Ex}_t t$' are not well formed. We are now in a position to give axioms for mereological notions comparable to minimal extensional mereology without the extensional principle SA6:

CTA7 $\mathrm{Ex}_t a \supset a <_t a$

CTA8 $a <_t b \supset \mathrm{Ex}_t a \wedge \mathrm{Ex}_t b$

CTA9 $a <_t b \wedge b <_t c \supset a <_t c$

CTA10 $a \ll_t b \supset \exists x \ulcorner x \ll_t b \wedge \sim x \circ_t a \urcorner$

In each case we have suppressed an initial universal quantifier binding the temporal variable. So far, the system appears not too dissimilar from the system F of Chapter 2. We can mention some obvious theorems:

CTT2 $\mathrm{E!}a \equiv a < a$

CTT3 $\mathrm{E!}a \equiv a \circ a$

CTT4 $\mathrm{E!}a \equiv \exists x \ulcorner x \circ a \urcorner$

CTT5 $a < b \supset \mathrm{E!}a \wedge \mathrm{E!}b$

which seem to promise enough similarities with the untensed system F to make us feel that much the same theme is under discussion. However if we look again, the parallels begin to break down. For instance, the formula analogous to that expressing transitivity in F:

CTF1 $a < b \wedge b < c \supset a < c$

does not follow from the axioms given so far, for when fully expanded it reads:

CTF1′ $\exists t \ulcorner a <_t b \urcorner \wedge \exists t \ulcorner b <_t c \urcorner \supset \exists t \ulcorner a <_t c \urcorner$

which is false if the times when a is part of b do not overlap those when b is part of c. The theorem

CTT6 $\exists t \ulcorner a <_t b \wedge b <_t c \urcorner \supset \exists t \ulcorner a <_t c \urcorner$

is of course a simple consequence of CTA9 by predicate logic. Further, as a result of the critical discussion in Chapter 3, we do not want mutual containment to entail identity, and so define a *coincidence* predicate as follows:

CTD5 $a <>_t b \equiv a <_t b \wedge b <_t a$

giving us such theorems as

CTT7 $\mathrm{E!}a \equiv a <> a$

CTT8 $a \ll_t b \equiv a <_t b \wedge \sim (a <>_t b)$

Coincidence is an equivalence relation for continuants existing at any particular time, in that

CTT9 $\mathrm{Ex}_t a \supset a <>_t a$

CTT10 $a <>_t b \supset b <>_t a$

CTT11 $a <>_t b \wedge b <>_t c \supset a <>_t c$

and coincidence is connected with identity by

CTT12 $a = b \supset . \mathrm{Ex}_t a \vee \mathrm{Ex}_t b \supset a <>_t b$

We are in a position to give definitions of mereological constancy (MC) and mereological variability (MV):[3]

CTD6 $\mathrm{MC}a \equiv \forall tt' \ulcorner \mathrm{Ex}_t a \wedge \mathrm{Ex}_{t'} a \supset \forall x \ulcorner x <_t a \equiv x <_{t'} a \urcorner\urcorner$

CTD7 $\mathrm{MV}a \equiv \sim \mathrm{MC}a$

It follows from this, trivially, that

CTT13 $\sim \mathrm{E!}a \supset \mathrm{MC}a$

which is as we should expect: only continuants which *exist* (at all) have parts that can vary.

Before bringing these concepts to bear on the issue of whether mereologically constant objects have some kind of superiority or priority over mereologically variable ones, we shall, to get the feel of the new concepts, consider how far we can go in carrying over the ideas developed for untensed mereology to this version. There are appreciable differences.

A relatively minor point concerns how to define disjointness. We consider three possible disjointness predicates. The obvious one is

CTD8 $a \,{}_1\wr_t\, b \equiv \sim a \circ_t b$

Objects are disjoint at a time in this sense if (but not only if) either or both does not exist then. This has the odd consequence that Socrates is disjoint from Quine in 1646. Now of course there *are* dated truths concerning objects not existing at the date in question, for example, that Socrates was not alive in 1646, or that Quine was to be alive 300 years later. But it would seem odd to give a list of pairs of objects disjoint from one another on, say, 1 July 1646, and include the pair Socrates, Quine on this list. To take account of this we may define a second disjointness predicate:

CTD9 $a \,{}_2\wr_t\, b \equiv \mathrm{Ex}_t a \wedge \mathrm{Ex}_t b \wedge \sim a \circ_t b$

In fact, a similar point could have been made with respect to the free extensional system F, for there $\sim \mathrm{E!}a \vee \sim \mathrm{E!}b \supset a \wr b$, but the point was too minor to mention. For those intent on preserving the parallels to previous systems, neither of the two disjointness predicates so far defined is a contradictory to $\circ$ in the sense of $a \wr b \equiv \sim (a \circ b)$: both notions are sub-contrary to $\circ$. A true contradictory may be defined:

CTD10 $a \,{}_3\wr\, b \equiv \forall t \ulcorner a \,{}_1\wr_t\, b \urcorner$

(or equivalently, replacing '${}_1\wr$' by '${}_2\wr$'.)

[3] SHORTER 1977 uses the terms 'component identical' and 'component changing'.

exists along with and coincides with *b*. Yet *a* and *b* may have no parts in common, so the sum leaps back into existence with a brand new set of parts. One may be forgiven for thinking that we are wishing these strange objects on the world with our symbolism if we take them seriously. To avoid them we can again define a modifiable predicate by analogy with Pr:

CTD15 $c\mathrm{Su}_t ab \equiv \mathrm{Ex}_t c \wedge \forall x \ulcorner x \circ_t c \equiv x \circ_t a \vee x \circ_t b \urcorner$

where again the notions are connected by

CTT16 $c\mathrm{SU}ab \wedge \mathrm{Ex}_t c \supset c\mathrm{Su}_t ab$

and we leave diachronic questions open. But there is an intuitive pull in another direction, requiring the *simultaneous* existence of a sum's summands at any time at which it exists. For this we may define a new predicate

CTD16 $c\mathrm{SM}ab \equiv \forall t \forall x \ulcorner x \circ_t c \equiv \mathrm{Ex}_t a \wedge \mathrm{Ex}_t b \wedge (x \circ_t a \vee x \circ_t b) \urcorner$

with the consequence

CTT17 $c\mathrm{SM}ab \supset \forall t \ulcorner \mathrm{Ex}_t c \equiv \mathrm{Ex}_t a \wedge \mathrm{Ex}_t b \urcorner$

This notion of sum does not of itself avoid intermittently existing sums, for if *a* and *b* are already intermittently existing objects, which sometimes exist at the same time and sometimes do not, then the sum comes into and goes out of existence as well. But if there are no other intermittently existing objects, none is introduced as a sum by this definition (assuming we add the appropriate existence axiom).

There is accordingly room for grades of caution in admitting the existence of sums: we may admit SM sums and not SU sums, though if we admit the latter we admit the former, as long as at least two objects ever exist simultaneously. Indeed, it is a theorem that

CTT18 $c_1\mathrm{SU}ab \wedge c_2\mathrm{SM}ab \supset \forall t \ulcorner \mathrm{Ex}_t a \wedge \mathrm{Ex}_t b \supset c_1 <>_t c_2 \urcorner$

—whenever objects exist simultaneously, all their SM sums and all their SU sums coincide. Conversely, if one of *a*, *b* exists at a time at which the other does not, then at such times no SM sum exists, but an SU sum does, assuming an axiom ensuring the existence of SU sums. An SM sum is inherently *frail*: it exists only as long as both of its summands, whereas an SU sum may outlast any particular one of its summands, though of course not all of them. However we view the *existence* of sums in general, there would appear to be room for both of the *concepts* of sum here introduced.

Matters become more interesting when we consider the generalized notion of sum, because the time variables play a significant role. Pre-analytically, we should expect a sum of, say, schoolteachers to be an object comprised of all and only persons who are schoolteachers. This simple-sounding requirement shows itself, however, to be ambiguous. Do we mean all persons who are schoolteachers at any time, or all who are alive at the time of utterances, or all who are alive and are schoolteachers at the time of utterance (i.e. excluding retired schoolteachers and schoolteachers-to-be)? To cover the most important alternatives, we define two new expressions. The first is a cross between a quantifier and a predicate: it takes one nominal argument and binds one nominal variable. The expression 'SU' will again be used, for reasons which will become obvious. '$a\mathrm{SU}x\ulcorner A\urcorner$' may be read as '$a$ is a sum of objects such that A', and is defined as follows

CTD17 $a\mathrm{SU}x\ulcorner A\urcorner \equiv \forall x\forall t\ulcorner x \circ_t a \equiv \exists y\ulcorner A[y/x] \wedge x \circ_t y\urcorner\urcorner$

provided that A does not contain 't' free. This covers both the case where we consider the sum of objects which are Fs at some time or other, and those which are Fs at some given time, for

CTT19 $a\mathrm{SU}x\ulcorner Fx\urcorner \equiv \forall x\forall t\ulcorner x \circ_t a \equiv \exists y\ulcorner Fy \wedge x \circ_t y\urcorner\urcorner$

it being remembered that 'Fx' abbreviates '$\exists t'\ulcorner F_{t'}x\urcorner$'. So in the sense of SU, a is a sum of schoolteachers iff, at any time, a comprises all those objects which exist at that time and are, were or will be schoolteachers at some time or other. This sum may exist at a time when no one is a schoolteacher, provided at least one ex-teacher or teacher-to-be is alive.

Similarly we may sum those things which are schoolteachers at a given time, say t', by

CTT20 $a\mathrm{SU}x\ulcorner F_{t'}x\urcorner \equiv \forall x\forall t\ulcorner x \circ_t a \equiv \exists y\ulcorner F_{t'}y \wedge x \circ_t y\urcorner\urcorner$

Again, at times other than t' there does not have to be a current F for a to exist: only something existing which was or will be F at t'.

Neither of these notions captures the meaning intended for a sum of things which are F at the time in question (the then current Fs). For this we need a new predicate-cum-quantifier, this time binding two variables at once. The expression '$a\mathrm{SUM}^{x}_{t}\ulcorner F_t x\urcorner$' may be read as '$a$ is a sum of Fs', but it has a different definition, showing the ambiguity of the vernacular expression:

CTD18 $a\mathrm{SUM}^{x}_{t}\ulcorner F_t x\urcorner \equiv \forall x\forall t\ulcorner x \circ_t a \equiv \exists y\ulcorner F_t y \wedge x \circ_t y\urcorner\urcorner$

SUM differs from SU with respect to the binding of the temporal variable. It is accordingly SU which is the generalization of the binary sum (also written 'SU'), since

CTT21 $cSUab \equiv cSUx^{\ulcorner}x = a \vee x = b^{\urcorner}$

but we cannot apply SUM to the open sentence '$x = a \vee x = b$', since this cannot be significantly temporally modified. In the sense of SUM, a sum of schoolteachers is an object which, at any time, is comprised of just those persons who are schoolteachers at that time. So at a time when no one is a schoolteacher, no SUM sum exists, although if there are future or former schoolteachers alive then, an SU sum could exist.

For those who are disposed, as I am, to deny that there are mereological sums or heaps of schoolteachers in any case, we must caution that the problems of temporal sums are not avoided by this denial, which applies only to this example. In particular, exactly analogous problems arise for pluralities: 'the collection of all schoolteachers' exhibits precisely the same ambiguities, which in view of Chapter 4 is to be expected. For reasons we gave there, we are on stronger ground in asserting the general existence of sums when considering pluralities and masses than when considering individuals. In the case of mass terms however, the tendency is very strong to understand 'all the world's salt' to mean only present salt, although it might—at a pinch—refer to all the salt there ever was, is, or will be. This tendency to prefer the SUM concept for mass terms may be connected with their operating at a level closer to mereological constancy, as we shall see.

Mereological notions are of relevance to our individuative practices. It matters, in finding out what Fido is, that we can in principle at any time decide with reasonable certainty for most things whether they are part of him, overlap him, or are disjoint from him at that time. But we have not finished deciding what Fido is when we can pick him out from his surroundings (and from what he surrounds) at any one time. For an object like Fido does not stand mereologically still. Things that were once part of him are shed, new things and stuff gets incorporated. Fido is subject to change of constituent matter, or 'metabolism'.[5] Knowing what Fido is depends on knowing roughly within what limits such metabolism or flux of parts is tolerable. The kind of object does not determine which particular parts it has, but it usually does set limits to what *sort* of parts it (typically) has, what these are made of, and how they normally stand in relation to one another.

[5] The German word for metabolism, '*Stoffwechsel*', is particularly apt.

For objects which are especially important to us, such as ourselves, the study of such part–part relationships even has a name: anatomy. This applies in particular, however, to the objects of a kind taken at a particular time, whereas for individuation we are also interested in the diachronic story.

5.3 Chisholm's *Entia Successiva*

If some of the issues raised so far have had more to do with trying out how far we can adapt the ideas of extensional mereology to the tensed context than to real-life problems, we have still learnt that taking account of time introduces considerable complications into mereology. It is precisely this sort of complication which must be taken into account if we are to give an adequate account of the mereological aspects of continuants. Many of the complications stem from the assumption that some objects can have parts at one time which they do not have at another. If this assumption were wrong, or needed importantly modifying, then we should need to think again. There is, however, at least one philosopher who agrees with us in rejecting the four-dimensional account of change, who accepts that we pre-theoretically describe substances as having some of their parts temporarily, yet who insists that a genuine continuant is mereologically constant. The finely wrought theory of Roderick Chisholm[6] examines all the issues in this area, and comes to a very different conclusion from us, namely that mereologically variable continuants are not primary substances, but rather logical constructions out of mereologically constant ones, and that talk of temporary parts is loose. The issue is not an abstract one. If Chisholm is right, the vast majority of those things we regard as substances, including such apparently exemplary cases as organisms, are in fact logical constructions.

Chisholm's mereology is based on '$\ll$' rather than '$<$', and does not presuppose the existence of arbitrary sums. He does not clearly mark the distinction between tensed and untensed part–whole vocabulary. If we consider the axioms he gives for part and whole, they may be formulated as follows (using the variables of CT but the symbol '$\ll$' as

[6] CHISHOLM 1973, 1976: appendix B.

understood in F rather than in CT):

RCA1 $x \ll y \wedge y \ll z \supset x \ll z$

RCA2 $x \ll y \supset \sim y \ll x$

RCA3 $x \ll y \supset \Box (E!y \supset x \ll y)$

RCA4 $x \neq y \supset \Diamond (E!x \wedge E!y \wedge \sim \exists z \ulcorner x \ll z \wedge y \ll z \urcorner)$

On the basis of this vocabulary Chisholm introduces disjointness:

RCD1 $x \wr y \equiv x \neq y \wedge \sim \exists z \ulcorner z \ll x \wedge z \ll y \urcorner \wedge \sim x \ll y \wedge \sim y \ll x$

which is equivalent by predicate logic to the definition

$$x \wr y \equiv \sim \exists z \ulcorner z < x \wedge z < y \urcorner$$

on putting $x < y \equiv x \ll y \vee x = y$.

Two further notions, '*w* is strictly made up of *x* and *y*' (we write '*w* sm *xy*') and '*x* is strictly joined with *y*' (we write '*x* sj *y*') are defined thus:

RCD2 $w \,\mathrm{sm}\, xy \equiv x \ll w \wedge y \ll w \wedge x \wr y \wedge \sim \exists z \ulcorner z \ll w \wedge z \wr x \wedge z \wr y \urcorner$

RCD3 $x \,\mathrm{sj}\, y \equiv \exists w \ulcorner w \,\mathrm{sm}\, xy \urcorner$

The first of these is simply the notion of a disjoint sum. Because Chisholm does not accept the existence of arbitrary sums it does not follow, as it would in ordinary extensional mereology, that any two disjoint individuals are strictly joined. Chisholm regards individuals as strictly joined when there is no third individual between them.[7] This however introduces topological considerations, so it is wrong to say, as Chisholm does, that joining and disjoining are forms of mereological change;[8] rather they are part-mereological, part-topological changes. The criterion for strict joining is also unfortunate in that it implies that if *x* and *y* are strictly joined but not in contact, they could become disjoined in virtue of the passage between them of a third object, which is not *per se* a change in either or a change in their direct relations to one another. If the passage of a third object between the other two is brief, and they undergo no essential mereological changes in the meantime, then the question arises as to whether the disjoint sum which went out of existence with the insertion of the third object is the same as that which comes into existence when it was removed. If it is the same, we have an intermittently existing sum. This problem of intermittence is one which Chisholm himself poses for the case of a toy fort which is built from toy bricks, taken apart, and then

[7] CHISHOLM 1976: 153. [8] Ibid.

reassembled with the same bricks in the same positions.[9] This objection is *ad hominem*, since Chisholm gives reasons for being dissatisfied with the usual ontology which consist in mentioning just such problems. However, there is no doubt that it is important for numerous reasons to subjoin topological concepts such as those of touching or being in the interior of something to those of mereology.[10]

The axiom RCA3 is taken by Chisholm as expressing a principle which is vital for his treatment, the principle of mereological essentialism. We shall discuss such modal principles in greater detail in Part III below. Important here is that Chisholm expresses the principle, or something like it, in another way, namely[11]

> For every *x* and *y*, if *x* is ever part of *y*, then *y* is necessarily such that *x* is part of *y* at any time that *y* exists

This gives an essential or *de re* necessary property of *y*. Anticipating the discussion of such properties in Chapter 7, we symbolize it as

RCA3′ $\exists t \ulcorner x \ll_t y \urcorner \supset \Box (\mathrm{E!}y \supset \forall t \ulcorner \mathrm{Ex}_t y \supset x \ll_t y \urcorner)$

This is a stronger principle than RCA3 (where we understand the latter in such a way that the untensed predicates mean what they do in CTDS.) Rather than try to puzzle out the connections between tensed and untensed vocabulary in Chisholm, which he does not make clear, we shall simply go ahead and modify his axioms to allow temporal modification. The first two may be modified simply by subscripting '≪' with '*t*' and binding the whole initially by '∀*t*'. The results we may call RCA1′ and RCA2′ respectively. The fourth axiom does not concern us here, so we leave it aside. If we now define untensed occurrences of non-logical predicates to be eliminable in favour of tensed ones as in CTDS, then RCA1–3 follow from RCA1′–3′ provided we add some uncontroversial assumptions. We show this for just the first case.

RCT1 $x \ll y \wedge y \ll z \supset x \ll z$

Proof

(1) $x \ll y \wedge y \ll z$	Assumption
(2) $\exists t \ulcorner y \ll_t z \urcorner$	1, CTDS
(3) $y \ll_{t'} z$	2, Assumption

[9] Ibid: 90.
[10] Cf. TILES 1981: §§ 8 ff. for an application of such an extension.
[11] CHISHOLM 1976: 149.

(4)	$\mathrm{Ex}_{t'}\, y$	3, CTA8
(5)	$\mathrm{E!}y$	4, $\exists$ in, CTA6
(6)	$\exists t \ulcorner x \ll_t y \urcorner$	1, CTDS
(7)	$\forall t \ulcorner \mathrm{Ex}_t\, y \supset x \ll_t y \urcorner$	6, RCA3′, $\Box$ out, 5
(8)	$x \ll_{t'} y$	7, $\forall$ out, 4
(9)	$x \ll_{t'} z$	8, 3, RCA1′
(10)	$x \ll z$	9, $\exists$ in, CTDS

While it is perhaps understandable that Chisholm does not make quite clear the relationship between tensed and untensed mereological predicates, the relationship between RCA3′ and our interpretation of RCA3 makes it clear that the former is to be preferred. The force of RCA3′ is that if any objects ever stand in a purely mereological relationship to one another (where a relationship is *purely* mereological when it can be defined solely in terms of mereological primitives and logic), then this relationship is *essentially permanent* in the sense that it holds as long as all the objects involved exist, and only comes to an end when one or more ceases to exist. This is presumably one reason why Chisholm calls both coming-to-be ceasing-to-be *mereological* changes—in our view unhappily (cf. the remark about joining and disjoining above).

It must be stressed that mereological essentialism applies, according to Chisholm, not to common or garden continuants, but only to continuants properly so-called (*entia per se*) and their parts properly (i.e. strictly and philosophically) so called. The mereological constancy of such objects is a simple consequence of RCA3′. Mereologically variable objects are not continuants in a strict and philosophical sense, but rather what Chisholm calls *entia successiva* or *entia per alio*.

We first sketch Chisholm's theory of *entia successiva*, and discuss some of the difficulties in its formulation. Roughly speaking, an *ens successivum* is a series of *entia per se*, each of which is mereologically constant, and which 'stands in for' or 'does duty for' the *ens successivum* at a particular time. The idea is not dissimilar from that which Russell applied to get bodies and persons as logical constructions out of series of sense data, although Chisholm is of course not a phenomenalist. Chisholm first defines direct succession between stand-ins, then defines succession (which is relative to the kind of object in question, be it a table, cat or whatever) as the ancestral of the direct succession relation. Two *entia per se* constitute the same successive α if they stand in the relation of α-succession. This gives us diachronic identity conditions for *entia successiva*. Finally, certain particular properties possessed by *entia per se* (roughly, those characterizing its present physical state, but the details may be omitted here) are inherited by the *entia successiva* for which they stand in. This applies in particular to part–whole. Chisholm explains the coincidence of mereologically variable continuants such as cats and the mereologically constant entities which do duty for them by simply *defining* the parts of an *ens successivum* at any time to be those of its stand-in. The apparent fact that continuants change parts is to be explained by saying that different successive stand-ins may (permanently) *have* different parts.

We note that *only* mereologically constant objects can stand in for others, and that mereological sums must themselves strictly be constant. Without mereological constancy, there is nothing to stop mereological sums from being mereologically variable. To see this, take once again poor Tibbles, who was introduced by Wiggins to discuss precisely the point at issue.[12] Tail is her tail, and Tib the remainder. Tibbles coincides with the mereological sum Tib + Tail, and what is more, continues to do so, despite changes in the parts of both Tib and Tail. For we introduced the individuals Tib and Tail, so to speak, on the back of Tibbles, Tib as the body of the cat, Tail as its tail. With blood circulation and the like, Tib and Tail toss certain parts back and forth. But the body of a cat and its tail metabolize parts in such a way that their sum keeps in mereological step with Tibbles herself. Now Wiggin's point is to show that Tibbles is not identical with Tib + Tail, despite their constant coincidence, since Tibbles, but not the sum, possesses the modal property of *being such that it could continue to exist even if Tail were annihilated.* (Here it is clear that the sum is being understood as the frail SM sum of the previous section.) Chisholm's reply to Wiggins[13] makes it instructively clear that he is, at least for purposes of argument, taking body and tail as mereologically constant parts, since he remarks that, although Tibbles may coincide with Tib + Tail today, she may have coincided with some other mereological sum yesterday, such as Tib + Tain, or Tip + Tail. Pressed, he would see the mereologically variable body and tail as themselves *entia successiva* and their sum as only a loose and popular,

[12] WIGGINS 1979. NOONAN 1980: 23 makes the same point. He calls the part of any cat apart from its tail a 'puss'. Sometimes, for example in the case of a Manx cat, a puss can also be identical with a cat.

[13] CHISHOLM 1979: 385.

and not as a strict and philosophical sum, which may only involve constant objects. We get closer to this idea by considering, not the body and tail, but the masses of matter making up body and tail respectively at any given time. We should argue, on the contrary, that the difference between the sum of a body and tail on the one hand (i.e. usually an integral cat) and the sum of the matter composing a cat's body and the matter composing its tail on the other (i.e. the matter composing the cat) is not to be explained in terms of two different notions of sum, a loose and popular one versus a strict and philosophical one, but rather in terms of the difference between the kinds of entities summed. However, this begs the question against Chisholm, so we shall for the moment only work with sums of mereologically constant objects, which are acceptable to him. Chisholm gives, in two different places, different definitions of the notion of kind-relative succession between *entia per se*.[14] We use the variable 'α' as schematic for kind-words ('cat', 'table', etc.) in place of Chisholm's concrete examples. The earlier definition goes as follows:[15]

x is at t a direct α-successor of y at t' = Df (i) t does not begin before t'; (ii) x is an α at t and y is an α at t'; and (iii) there is a z, such that z is a part of x at t and a part of y at t' and at every moment between t' and t, inclusive, z is itself an α.

If, for example, through the interval t', y is a table which does not lose or gain any part, and x is the whole which arises by joining or disjoining a piece to y at the first moment of t' (or, indeed, by simultaneously joining one piece and disjoining another), then x is at t a direct table-successor of y at t'. The common part of x and y, say w, is, if x arises simply by joining, y itself, or, if x arises by disjoining or by simultaneous joining and disjoining, w is the part of y minus the part which is disjoined (there will in general be other such parts, but we have in each case picked the largest such). What is wrong with this account is that w is not throughout a table. Chisholm argues that it is, as follows:[16]

If we cut off a small part of a table, we may still have a table left. But if the thing that is left is a table, then, since it was there before, it was then a table that was a proper part of a table.

[14] CHISHOLM 1976: 99 and 1979: 386.
[15] CHISHOLM 1976: 99. [16] Ibid.

The argument is not logically valid. Were it correct, then Shakespeare's Prince Hal could have argued as follows (cf. *Henry IV, Pt. 2,* act IV, sc. v):

If the King dies, then we may still have a King (namely myself, the Heir Apparent). But if that person is a King, then since he was there before, he was then a King who was the eldest son of a King.

Hal was not King while his father was alive, but *became* King on the death of his father, and could have become King earlier, had his father died earlier. Similarly, the object which becomes a table was not a table beforehand, but rather a proper part of a table which was destined to become a table on detachment of the relevant part, and could have become the table earlier had the part been chipped off earlier. The point is not new, having been made before by Wiggins and Quine, though in neither case against Chisholm.[17]

Chisholm later gives a different definition of succession as follows (we adjust the symbolism to harmonize with our rendering of the earlier version):[18]

x directly takes over from y as an α at t = Df y and x have a part in common; either y is a part of x or (sic) y ceases to be an α at t; or x is part of y and x begins to be an α at t

x is a direct α-successor of y at t = Df x is an α during t; and at the first moment of t, x directly takes over for y as an α

y is a direct α-predecessor of x at t = Df x is an α during t; and at the last moment of t, x directly takes over for y as an α

Chisholm then defines predecessors and successors in terms of finite chains of objects which are both predecessors and successors of the right kind. As he points out, the definitions are compatible with both fission and fusion of αs. Unfortunately, it seems that they are not

[17] WIGGINS 1968: 95. QUINE 1981 has, as ever, a nice formulation of the point: 'A table contains a graded multitude of nested or overlapping physical objects each of which embodies enough of the substance to have qualified as a table in its own right, but only in abstraction from the rest of the molecules. Each of these physical objects would qualify as a table, that is, if cleared of the overlying and surrounding molecules, but should not be counted as a table when still embedded in a further physical object that so qualifies in turn; for tables are meant to be mutually exclusive. Only the outermost, the sum of this nest of physical objects, counts as a table' (92 f.). We might quarrel with the penultimate point only: if we make a large table by pushing two smaller tables together, it does not seem senseless to say we have a table which is a proper part of a table. But this is clearly a different kind of case from the one under consideration.

[18] CHISHOLM 1979: 386.

compatible with the simple case where an α simultaneously gains and loses a piece. If $a+b$ is an α, and b disjoins from a at the same time as c joins a, so that $a+c$ is an α, then $a+b$ should be the α-predecessor of $a+c$, and $a+c$ the α-successor of $a+b$; but this is not allowed by the definitions unless we know that a is an α throughout, so it connects $a+b$ and $a+c$ in a chain. However, we argued above that it need not be the case, and in general will not be the case, that a is an α all the time. In this case a will in general never be an α, being too bound up with other parts throughout. Chisholm's definitions only work, then, if they assume a false principle.

It could be that there are ways of patching up definitions to take the objection into account, but I am more interested in an essential feature of Chisholm's account which I find counter-intuitive, namely that each of the stand-ins for what we ordinarily call an α (dog, cat, table, ship, etc.) is itself, indeed is in the primary sense, an α. This is important: the 'is' here is the 'is' of predication and not the 'is' of constitution.[19] For if it were the 'is' of constitution, i.e. if we read 'a is an α' as meaning 'a constitutes (makes up) an α',[20] then we are entitled to ask, 'Which α?' But this question can only be answered by invoking the whole machinery of stand-ins to individuate the *ens successivum*, for this derives its identity from those of its stand-ins. Were this not so, then there would be nothing to be said for the claim that the stand-ins are ontologically prior; we should rather be investigating the relations between different, intimately related, but nevertheless ontologically coeval objects—which is indeed our position, but not Chisholm's. So the machinery must be invoked to explain itself if the 'is' in 'This stand-in is a table' is the 'is' of constitution, and we get no further forward. So the stand-in really *is* itself a table. The primary objects of which we predicate things like 'is a man', 'is a table' are mereologically constant, and the rest of our usage is a mere 'playing fast and loose with identity': when we say that the man before us has lost a lot of hair in the last year we are using 'man' loosely. What we strictly should say is that the man who stands in today for the same successive man has less hair than the man who last year stood in for him (him?). I am at one with David Wiggins in thinking that Chisholm is here skirting dangerously close to the four-dimensional ontology:[21] despite the latter's disclaimer, and his insistence that the relation of a successive

[19] On the 'is' of constitution cf. WIGGINS 1980: 30ff.
[20] Cf. the converse in GRIFFIN 1977: 165.
[21] WIGGINS 1979: 302.

object to its stand-ins is not that of an aggregate to its parts,[22] he is still attempting to imitate the four-dimensional accounts of fission, fusion, and continuity while foregoing temporal parts. The issue seems to turn on whether the sortal concepts 'man', 'table', etc., which, as has often been pointed out, are bound to conditions which stipulate within reasonable limits what is to count at a time and over time as one and what as many of the kind, are applicable primarily to mereologically constant objects, as Chisholm requires, or to mereologically variable ones, as we believe, and as we contend is the commonsense view. Chisholm's view is forced on him by his theory, and it is not the worse for that. But it has as a consequence that most if not all men use most if not all sortal terms in a philosophically unacceptable way most if not all the time. Since it is this apparently objectionable usage which we inculcate into our children, it seems we must all be living in a kind of linguistic original sin. This view may not be logically inconsistent, but it seems an unacceptably high price to pay for making the world safe for mereological constancy. There are problems raised in this section which we shall take up later, in particular those relating to matter, form, constitution, and the full modal version of mereological essentialism.

5.4 Intermittent Existence and Part-Replacement

There is perhaps a disposition, based on our normal experience, to deny that an object can exist for a while, cease to exist, and then come back into existence again. Even where such ideas take foothold, for example in certain theological contexts, it is common either to attempt a reinterpretation of the idea of going out of existence, or at the very least to hang on to some seed or other substrate whose existence throughout the interval is asserted. Against intermittence, it was stated by Locke 'that one thing cannot have two beginnings'.[23] This looks like what Wittgenstein would have called a grammatical remark about the term 'beginning', but if we look again we see that this term can have both a local and a global meaning. When applied to activities such as walking, talking, singing, climbing, it is true that, globally, there can be only one beginning: we say, for emphasis, 'When did you *first* begin/start climbing?' In the local sense, there can be many

[22] CHISHOLM 1979: 385f.
[23] Locke, *Essay*, book II, ch. 27, § 1; cf. also WIGGINS 1968: 91.

beginnings of an activity: a bird begins to sing anew every morning. Of course, it is a different *episode* which is begun each time of a certain kind of activity, and only the beginning of the first such episode is a global beginning. But this cannot itself be used as an argument for something's never being able to begin existing more than once. Of course existing is not an activity, but there is no indication that 'begin to exist' cannot have a local as well as a global meaning. That something globally begins to exist only once does not mean it cannot come into existence (locally) more than once. It is true that an organism, for instance, cannot have more than one life, and can therefore only have one beginning to its life. But this is because the term 'life' is already global in meaning. There is no such thing as a life which is short of a complete life. But this does not itself entail that the life of an organism must be uninterrupted. It is not absurd, and may even be useful, to consider the possibility of real gaps in the life of an organism.

There is of course always a way out for those who wish to deny intermittence: they can insist that what looks like something's coming into existence for a second or subsequent time is really either one thing existing throughout, but being in some kind of radically altered state for an interval, or else one thing's going out of existence and another, similar thing's coming into existence later. This dual strategy preserves the principle of continuity of existence while providing ways of dealing with putative counter-examples.

Intermittence arose in conjunction with the discussion of tensed sums and products, and will crop up in the next chapter when we turn to superposition (being in the same place at the same time). However, if we approach the question of intermittence undogmatically through concrete examples, we find there is no easy decision on the matter. In the case of artefacts which can be dismantled for storage or service and then reassembled, we have no hesitation in saying that the thing in question exists throughout, even when not ready to perform its characteristic function. Such objects may get scattered in the process however, for example various parts of a car being sent to different workshops for repair, and here we begin to have qualms. Suppose I have a rare vintage car, whose engine must be repaired in Germany while its wheels go to England and the body is repainted in France. Is it normal to regard such a scattered object as an individual car? Another problem is that we are more inclined to accept continuity of existence if the thing actually gets put back together again, but surely whether it does so or not is accidental. Consider a time after dismantling, with the pieces scattered. If they are later reassembled, the thing existed at this time. If they are later lost, or simply not put together again, or cannibalized for other artefacts, we date the demise of the object back to the point of its last dismantling, so it did not exist at this time. Whether a scattered dismantled object exists or not while in that state (so to speak) is not then merely dependent on its present physical state and its previous history, but also on the subsequent history of its parts, which seems wrong.

Now it is certain that persistence conditions for artefacts are vague. We allow them to persist despite part-replacement in repair, but there is no exact limit to when too much has been replaced—or too much replaced *at once*—to allow continuity of existence. Even if we allow perfect retention of parts, however, we can find cases where we want to deny continuity of existence. Suppose we have a machine which is fed some special powder and some water. The machine, when set working, mixes the two, pours the mixture into a mould, bakes it, collecting the steam given off, and ejects a bust of Mozart, which it then crushes to powder, mixes with the extracted water, and goes through the cycle again. We suppose no matter is gained or lost in the process, so each bust is made of exactly the same stuff as each other. Now the pulverization makes it hard to claim identity across the gap, while if the machine produced alternately busts of Mozart and Beethoven it is even harder to claim that one bust of Mozart keeps coming back into existence after a period of radical 'dismantling'. The defender of continuity will therefore claim that each time a new bust is made, despite the total continuity of matter, the matter of one bust is used to make another.

One thing that makes it less plausible to identify across the gap is that the stuff of which the busts are made is mixed around each time, so each minute part of the matter comes to rest at different places in the finished busts each time. We can, however, screw up the tension in the example by taking objects made of a finite number of rigid parts which may be fitted together over and over in exactly the same way, like Chisholm's toy castle. Michael Burke has dealt with such an example.[24] A table made of thirty pieces of wood is dismantled, the pieces used to make a chair, this dismantled, the pieces used again to make a stool and a bird-house, and then finally reassembled in their

[24] BURKE 1980: 391ff.

exact original order to make a table. Burke then claims two things: the table ceases to exist when the pieces are taken apart the first time, and the very same table comes back into existence. The example is beautifully contrived to bring the problem to a head: we have complete material continuity, and reassembly in exactly the same order, to bracket out vagueness and encourage reidentification. At the same time we have excellent reasons for denying that the table exists in the meantime. The continuity theorist is on the spot whichever of his two strategies he employs. However, by making untenable the usual claim that the table exists in a dismantled state, Burke leaves the way open for the continuity theorist to deny identity across the gap. He can claim the case is like that of the busts, despite the sameness of configuration. Each time a new table is brought into being out of the materials of the old. Burke's arguments for identity are thus notably less convincing than those for the table's ceasing to exist. His point is rather that admitting such well-regulated kinds of intermittence (material continuity and structural similarity) brings no harmful ontological consequences, such as being unable to trace the objects concerned across gaps in their lives.

This point seems to me to be well taken, but it is not enough for a continuity theorist, who demands compelling reasons for giving up his basic principle. So the issue looks finely balanced. Continuity theorists have a simple account of existence without gaps. The price they pay for this is some huffing and puffing about dismantled objects where continuity is assumed, or multiplying individual objects where it is not. On the other hand, both views appear to be ontologically benign; each, taken on its own, has intuitions and arguments in its favour, and, if sensibly applied, it does not lead to ontologically bizarre results. The trouble arises through the *opposition* of the two views: each interferes with the smooth working of the other.

This opposition comes to a head in the celebrated problem of the Ship of Theseus,[25] which, we may recall, goes like this. A ship is built and sails the seas. In the course of time, it needs repair, and its parts are successively and gradually replaced over a period, so that at the end of this time none of the parts is original. But a man keeps the original parts and reassembles them in the original order. So now there are two ships, and the question is, which of them is identical with the ship with

[25] For the extensive literature to this problem, cf. GRIFFIN 1977: 177 n.; also WIGGINS 1980: 92 ff., BURKE 1980: 405. For the related Four Worlds Paradox, see SALMON 1982: §28.

which we started? The problem is precisely that there are *competing* claims: one side favours continuity of function despite flux of parts (provided the flux is not too considerable at any refit), while the other side favours material continuity despite the intervening period in which the original parts are not together in the form of a ship. The difficulty is that both sides have complementary points in their favour, so that coming down on one side is unsatisfactory, while the attempt to find an accommodation by relativizing identity, letting each side have its say without interference from the other, by turning a problem into a paradigm, fails to explain why there is a problem in the first place.

5.5 Solution of the Difficulty

It seems to me that there is a way of turning all these features of the opposition to the good without resorting to relative identity. To do so, we must recognize that the sortal concepts associated with terms like 'ship' in everyday life constitute a working compromise between two opposing tendencies. One tendency is to link the identity of a material continuant with the identity of its matter: x is identical with y only if the matter of x is identical with the matter of y. The other tendency is to link the identity of a material continuant with the identity of its form: x is identical with y only if the form of x is identical with the form of y. The opposition between these tendencies give rise to difficulties if they are elevated into philosophical dogmas. In his discussion of individuation, during which he brings up the Ship of Theseus, Hobbes notes that among the opposed parties 'some place *individuity* in the unity of *matter*; others, in the unity of *form*'.[26] Even where sortal terms as normally used tend not to give rise to difficulties, abnormal or artificially concocted cases can activate the tension or potential for ambiguity, because in such cases the working compromise breaks down and we are torn in opposite directions. This is precisely what happens with the Ship of Theseus.

Instead of attempting to dispel the tension, let us simply use it. For each sortal term for material continuants, let us suppose there are two

[26] Hobbes, *De corpore*, part 2, ch. 11, section 7. It should be noted that Hobbes treats the Ship of Theseus problem not, as is now usual, as arising from the conflict between the advocates of material unity and the advocates of formal unity, but rather as an internal difficulty for the latter. He also mentions a third party, those who take every real change to involve an object's going out of existence and another's coming into existence.

sortal terms corresponding to the two extremes. We call these two kinds of sortals *matter-constant* and *form-constant* sortals. So in addition to the sortal 'ship' we suppose there are two other sortal terms, 'matter-constant ship' and 'form-constant ship'.

Before we proceed, a note on the terminology we are using. By 'matter' we mean simply the stuff or makings of something, in its usual, common-sense, relative meaning. The term 'form' is not so easily explained, and differs in what it involves according to the kind of continuant considered. For organisms, the form consists in the characteristic anatomical construction and physiological functioning which together render it a living thing of its kind; for artefacts, on the other hand, the form consists in the properties which render the artefact capable of performing its intended function. 'Form' may also be understood in other ways: for geophysical objects like islands, mountains, and glaciers, the form consists in those physical properties which give the object its characteristic shape and relation to neighbouring material objects. It can be seen that the concept 'form' is heterogeneous: therein consists its summary usefulness.

The Ship of Theseus problem concerns an artefact: for various reasons such problems do not arise so readily among sortals for natural kinds. So a form-constant ship is a material continuant which has the form of a ship (i.e., is so constructed that it can sail and carry people and goods on water), and has this form at all times at which it exists and without interruption. It is of no account whether it is always made of the same matter: the matter can change, as long as the capacity for performing the function is maintained uninterruptedly. A matter-constant ship, on the other hand, is a material continuant which has the form of a ship but which ceases to exist if any matter is added or taken away.

These sortals impose very stringent conditions on the continued existence of objects falling under them. Form-constant objects cannot survive a disruption of continuity of their form: even the shortest intervention which interferes with the capacity to function counts as destroying the artefact. Matter-constant continuants cannot survive the slightest change in their constituent matter. On the other hand, a matter-constant continuant does not require a continuously maintained form. It could survive dismantling and reassembly, provided it is put back together without any matter being lost or gained. If we require that at all times at which it exists, it does have the appropriate form, then it will exist intermittently if wholly or partly dismantled.

Such extreme requirements are unlikely to be realized in practice. For the most part, our interest in artefacts concerns what we can use them for, and here continuity of function is far more important than material continuity. The entrepreneur is in general indifferent as to whether a defective device is repaired or replaced.[27] Interest in material continuity is less frequent, though it does occur, for example among curators of museums and art galleries and keepers of religious relics. In practice, even here a compromise is made in allowing restoration, providing too much of the material substance of an object is not exchanged. Here, as in the case of the entrepreneur, there are no sharp limits to the tolerance. But the tendency to the extremes is there: the curator leans towards the perfectly matter-constant object, while the entrepreneur leans towards the perfectly form-constant object. Usually neither gets his way completely, and if both happened to make claims on one thing, such as a ship, they would normally have to come to a compromise. In certain cases, however, each can have his own way and not at the expense of the other. The Ship of Theseus is just such a case. Here we start with one ship and competing interests, while at the end of the story both the entrepreneur and the curator can be content, each having his ship all to himself. We need to explain why this is neither absurd nor miraculous. The reason is that each was all along claiming a different object from the other, which is why they can both end up satisfied, but initially both objects claimed the same matter, which is why there was a conflict of interests to begin with.

The entrepreneur has what he wants because he has a ship out sailing the seas, except for occasional refits in port. His is an (almost) form-constant ship, and he keeps the same one throughout. The curator has what he wants because, as the story goes, he gets back all the original ship parts reassembled in the original ship shape. He gets a matter-constant ship, the same one he started out with. Each is right that he beheld and went on board his own ship to start with, and yet each was also right to regard the other as a rival, since every part of the matter-constant ship, including the whole, was part of the form-constant one, and vice versa. The matter-constant ship and the form-constant ship initially *coincide*, but later do not, since one is in a

[27] Cf. WIGGINS 1980: 98: 'for many practical purposes, we normally do not care very much about the difference between artifact survival and artifact replacement. (A negligence that in no way undermines the real distinction between these.)' We agree with *both* parts of this.

museum while the other is out sailing. We had two ships all along, but not in the same sense of 'ship'.

The solution depends on two controversial ontological theses: that distinct continuants may have the same parts at the same time (may coincide), and that continuants may exist intermittently. In this case, the matter-constant ship went out of existence at the first refit, while the form-constant ship carried on. From this point on their histories diverge, which is why they are clearly distinct. The reassembly of the original parts in the original order constitutes the *rebirth* of the matter-constant ship. But this reassembly need differ qualitatively in no way from the building of a new ship—and indeed, it *is* simultaneously the building of a new ship and the rebuilding of an old one. For the *reassembly* of a matter-constant ship is frequently the first-time assembly of a new form-constant ship. A shipwright cannot build one without the other, since they coincide. Here our definition of coincidence proves its worth. For the shipwright does not have two jobs to do. He builds both a matter-constant and a form-constant ship in the same operation, since *every part of the one is a part of the other*. Why then, when the Ship of Theseus is reassembled, is there suddenly a new form-constant ship in addition to the one still out on the seven seas? Because the whole saga can start again from the beginning. The newly reassembled ship could (probably against the curator's wishes) set off to sea, need repairs and refits, and the old parts be reassembled yet again. We could then have three or (if something similar happened elsewhere to the form-constant ship which starred in the original controversy) four ships. In principle, the process could be continued indefinitely. It is not a miracle—no law of nature is contravened in the process. The only problem is that of finding an unobjectionable description of what goes on.

It is important to see not only that relative identity is not used, but also that we do not have fission of ships. Of the later ships deriving from the originals, at most one is identical with the original matter-constant ship, and at most one is identical with the original form-constant ship. At each new refit, a new matter-constant ship comes into existence, though most of these are not resurrected. And at each reassembly, a new form-constant ship comes into existence. There are indeed lines of descent, but no ship (of either sort) splits to become two ships. Also, at no time at which we have a (complete, not dismantled) ship in view are we confronted with something which is *only* a matter-constant ship (for instance in the museum) or *only* a form-constant ship (at sea). In previous discussions of the problem, philosophers have concentrated on the pseudo-question of which is the 'real' ship, so that, even while recognizing divergent interests and perhaps even different senses of 'ship', they have not recognized that the two kinds of 'ship' coincide, so the problem is iterable.

Because the Ship of Theseus problem uncovers the latent ambiguity in the term 'ship', it is a benign *antinomy* in the sense of Quine,[28] showing the need for basic conceptual reform. There is a similar precedent in the case of the term 'word', which needs to be made precise for certain purposes either as 'type-word' or as 'token-word'. For most purposes the distinction is unnecessary, but there is a pressure towards bifurcation which could lead to conflicts and taking sides just as in the ship case. When the bifurcation is recognized, significant progress can be made in clearing up muddles by using the more differentiated terms. Of course, this is only a parallel: the type/token distinction is not the same as the form-constant/matter-constant distinction.

We claim that this solution of the problem not only relieves conceptual cramp: it also explains why each side of the controversy has something in its favour, and which points these are likely to be. But it may be objected that the price is too high: we practically double the population of artefacts at a stroke, and this offends against Ockham's Razor. Of course the world does not *look* any the more crowded, because of coincidence, but is the distinction not still ontologically excessive? To this it may be answered, first, that although our theory multiplies entities, it cannot be accused of doing so beyond necessity, since it solves problems which more economical theories could not. And secondly, if the supporters of material constancy have been battling against the monopoly of mercantile conceptions—for millennia, if Hobbes is to be believed—then we have not suddenly produced new objects from nowhere, but have simply given their due to objects which have been recognized by some people for a long time but which the weight of practical interest encourages us to overlook. Another advantage of the solution is that it accounts for the attraction of 'best-alternative' theories as to which is the real Ship of Theseus. According to such theories, the decision as to which is the real ship depends on circumstance: if there is only one candidate, whether it be the merchant's or the curator's choice, that is the ship,

[28] QUINE 1976*b*: 5.

and if there is more than candidate after a certain time, then that one is taken to be identical with the original ship whose claims to be its successor are the best. If, as we have suggested, the term 'ship' possesses in real life a certain latitude, then it is natural that in identifying ships through time we use this latitude and take the line of least resistance. That on occasion there may be more than one candidate with equally good claims is no worry to us, since we are not committed to the view that the common concept of ship should be able to resolve all the problem cases. The 'best-candidate' theorist recognizes the flexibility of the common concept but gets into difficulty because it is again assumed that there is a uniquely correct answer to the question 'Which is the *real* ship?', so the fact that there are apparently two real ships is disturbing.[29]

5.6 Corroboration Found in an English Idiom

We have accepted that practical interests push usage of sortals for artefacts in the direction of form constancy, and have had to appeal to slightly quirky interests like those of curators and collectors to show that terms may tend in a matter-constant direction as well. But in fact we do not need to contrive examples, since there are a few lexical items of English and other languages which serve to designate a particular kind of object in a particular configuration, and support a matter-constant sense, where, importantly, the sense of 'matter' involved is relative.

The clearest example is 'fist'. The *OED* defines a fist as 'the hand clenched or closed tightly, with the fingers doubled into the palm, *esp.* for the purpose of striking a blow'. We abbreviate this as 'fist $=_{df}$ clenched hand'. Similar definitions can be given for configurations of hair, such as buns and pigtails. So every normal reader carries around with him or her the equipment for producing and destroying two intermittently existing continuants. All he need do is clench and unclench his hands. When the hand is unclenched, the fist ceases to exist, and when it is clenched again, the same fist comes back into existence. What is decisive for fist identity is merely the continuity of the hand. There are many makings of fists, but no one *has* more than

[29] See WIGGINS 1980: 206–8, NOONAN 1985*a*, 1985*b*, who argues strongly against 'best-candidate' theories, and GARRETT 1985, who attempts to rescue them by denying the transitivity of identity.

two fists. So 'fist' has an overwhelmingly matter-constant sense, and yet the matter—the hand—need not be mereologically constant, and usually is not. The hand itself has as its matter flesh, blood, bone, etc., and this can be in flux without in the slightest impugning its identity or that of the fist it can form.

The example is simple because it deals with form as configuration, but it is generalizable to form as function in the case of artefacts. If we take a multi-purpose object like a sofa-bed or a Swiss pocket-knife, we see that it can satisfy only (we suppose) one of its possible uses at once. When we fold the sofa out, it ceases to exist, and the bed comes into existence, and when we fold the bed up the reverse happens. The example is mentioned by Burke,[30] who denies the analysis we have given, however, on the grounds that an object's sort is a function of its qualities, and coincident objects must have the same qualities. Even if we accept this, we can argue that a folded up sofa-bed has different qualities from a folded out one, so it can be alternately a sofa and a bed. But Burke's assumption that an object's sort is a function of its qualities is independently wrong, especially if we look at objects in use in a community. There are numerous examples of pairs of objects which can be physico-chemically exactly alike while belonging to different sorts: holy water and normal water, genuine banknotes and perfectly forged banknotes, meteorites and other pieces of rock, original Celtic brooches and replicas, wedding-rings and other rings, summit crosses and other crosses, maybe also persons and their bodies. Of course each object in the pair falls under a higher sortal, but it would be silly to deny this.

5.7 Further Cases of Intermittence, and Conclusion

As we have already seen in the case of the fist, intermittence can arise otherwise than through dismantling and part-replacement, which is typical for artefacts. We can find intermittence among objects which are not rudely physical, but institutional or higher-order. For instance, the state of Austria ceased to exist in 1938 upon the *Anschluss*, but came back into existence in 1945. In this case the foundations for identity across the gap were the geographical and

[30] BURKE 1980: 394ff. We are of course committed to denying Burke's basic thesis that 'different material objects cannot be simultaneously embodied within just the same matter' (391). We are nevertheless much indebted to his discussion.

demographic ones, coupled with substantially the same constitution in the Second Republic as in the First. However, if we could show that intermittence could occur not only among artefacts and higher-order objects, but also among natural things, then we should have given it a secure place on the ontological map.

We should naturally expect intermittence among organisms to be at the very least exceedingly rare. It is normally physically imperative for organisms to maintain and repair themselves at a rapid rate, so that it is normally impossible for them simply to 'stand idle' in the way a machine can. There are states of organisms which seem to approach such conditions, such as hibernation of mammals or perhaps certain yogic states. But in neither case do we get any interest in conceiving matter-constant organisms. While a mass of matter briefly in the form of a man is certainly something rather than nothing, there is no reason in normal circumstances to think of calling it a matter-constant man.

However, if the state of medical technology were to advance in such a way that we could 'freeze' the normal metabolic change of parts in a sort of artificial hibernation, we might want to think again. But for organisms it is not mereological considerations which are uppermost. The form of an organism is at least as much a matter of physiology as of anatomy. The continued existence of an organism depends not primarily on its structure but on the continuation of characteristic processes within it, which, admittedly, require an anatomical basis and which tend to preserve anatomy. The mere cessation of such processes, or at least sufficiently many of the most vital ones, constitutes the going out of existence (death) of the organism, no matter how well preserved its parts are. The difference between a living organism and a corpse consists in some cases (though not in all) almost exclusively in the absence of characteristic inner processes. If it were possible to interrupt the activities, preserve the structure of the underlying object, and restart the activities without impairment later, then, to the extent that we make the occurrence of the activities an essential characteristic of the organism, we have a reason to say that it ceased to exist and then came back to life again. To identify across the gap we require of course at least substantial bodily continuity.

There is no reason to think that such a thing is impossible, and indeed it may already have taken place. In certain forms of open-heart surgery the body temperature is artificially lowered by heart–lung machine until it is sufficiently low for atrophy not to take place for a period during which the machine is switched off, there is no blood-flow, and the brain shows a completely flat EEG. This state is sometimes popularly described as a 'death-like sleep'. But of course it is not sleep, since in sleep there are characteristic forms of brain activity. If we refuse to call it death, that is because we reserve this fateful word for a global use, signifying the last or irreversible going out of existence. In such cases the patient comes round after the operation without loss of memory of what went before. So we satisfy both bodily and memory requirements for identity of the person before and after the operation. But there appears to me no reason to deny that the person *does not exist* during the operation. The surgeons carry out their crucial moves on something, a body, which falls short of being a person or even a living human being; but provided they do their work well, the basis for continued life is not impaired—indeed, it ought to be enhanced—and the person literally comes back to life after the operation. There is *this* much to Locke's memory-connectability theory of personal identity, that were we to find after the operation an insuperable amnesia and loss of skills, but an unimpaired general capacity for learning, then it would be unjust to punish the person existing after the operation for crimes committed by the person before, so by Locke's 'forensic' view the two are different persons, even if they successively claim the same human body.

It may be objected[31] that since the concept *person* has as its point or *raison d'être* both the provision of bearers of rights and the provision of substrata of changes governed by psychological laws applicable to rational beings, all we require is the continued *capacity* to generate the characteristic processes demanded, rather than the actual activation of these capacities. A person is still a person while asleep, so why is not the person still there throughout the operation, if the capacities concerned are physically realized in human anatomy? To this we reply that, while the preservation of the physical substratum is essential, it is not sufficient to guarantee the continued existence of a person, otherwise an intact corpse would be a person. Some of the physical *states* of the organism must be preserved in order to serve as the foundation for the capacities which are later exercised. The person must, so to speak, be still 'ticking over'. If this is not secured, then we have no person, and indeed no human being. But these states may only be realized in a certain kind of body which itself has other

[31] And has been, by Doepke in correspondence. I am most grateful for his objections, although I could not go along with them.

characteristics which are required if the states characteristic of a person are to be maintained. In keeping the body in good shape, the surgeons do not preserve the person throughout, nor perhaps even the immediate foundation for the person (the human being), but they do preserve the foundation for the foundation, and that is why the person can come back to life.

There are certain kinds of electronic typewriter which can store text in a memory. The machine can be switched off and still retain what is stored in its memory, provided a small current flows through it. That is like a person asleep. Normally a passage of current is required to prevent the information from being lost. If we could freeze the information in the store to allow the current to be switched off without loss of information, that would be like the body on the operating table. In no case is there any change of structure. What is at issue is what states are retained to allow the machine to take up from where it left off. Normally, if we switch off the current, the memory is erased. That is like a person dying. But if we preserve the capacity for remembering in the absence of the usual renewing processes, we have done enough. That is like a person going out of existence and coming back into existence again.

Our final theory turns out to be not so different from that of Chisholm except that we do not distinguish matter constancy as 'strict and philosophical' as against 'loose and popular' form constancy. We have the advantage over Chisholm of making clear how the actual use of 'ship' ties in with its possible use to mean 'matter-constant ship'; as an extension of latent tendencies, without having to claim that the normal use of sortal nouns is fundamentally in error. We also do not need an absolute conception of matter, but only an uncontroversial relative one. And, finally, we are not committed to mereological essentialism.

On the other hand in our striving to gain respectability for intermittence of existence, we should not wish to do down the claims of continuity. The priority in deciding what kind of thing a continuant is usually goes to the form-constant end of the spectrum. A matter-constant ship is after all a matter-constant *ship*, i.e. something in a shape and condition suitable for use as a ship. And in each case, intermittently existing objects presuppose some continuously existing material substrate (mediate or immediate) whose continued existence allows us to identify across the gap. There could be intermittently existing objects whose fundaments are other intermittently existing objects, such as the elaborate Austrian peasant hair-style made by intertwining two pigtails. But such a chain of dependencies sooner or later (usually sooner) comes back to continually existing objects.

Even so, material constancy is not the same as material continuity. An artefact is matter-constant if, at any time at which it exists, it must have the same matter; but it need not exist throughout, nor (like the hairpiece) need its immediate matter, although *some* matter must exist throughout. Despite the stringent requirements on matter-constant objects, it should be noted that they are not the same as those of mereological constancy, since one and the same matter-constant object may at different times coincide with different form-constant objects even if it does not change an atom. And in any case, matter-constant objects like fists clearly do change their minute parts. But this brings us to consider matter and constitution in general, which is the subject of the next chapter.

6 Superposition, Composition, and Matter

Rejection of the antisymmetry of part–whole allows us to recognize different objects having the same parts, which we exploited in the previous chapter. Objects with the same parts are necessarily in the same place at all times at which they have the same parts. This leads to the more general question as to how more than one object can be in the same place at the same time. We defend this idea against objections, and consider the more difficult issue of the conditions under which such superposition is possible. It turns out that this frequently happens when one object is the matter of, composes, or constitutes another, and this in turn leads us to a general consideration of matter and its likely mereological properties. As in the previous chapter, it is impossible to isolate mereological from more general ontological issues.

6.1 Superposition and Coincidence

In the previous chapter we purloined the word 'coincident' for the semi-technical concept of community of parts. We defined it in terms of mutual inclusion:

CTD5 $a <>_t b \equiv a <_t b \wedge b <_t a$

but given the transitivity of part–whole, this amounts to a and b having all their parts in common:

CTT22 $a <>_t b \equiv \mathrm{Ex}_t a \wedge \mathrm{Ex}_t b \wedge \forall x \ulcorner x <_t a \equiv x <_t b \urcorner$

In extensional mereology, coincidence (as a timeless predicate) entails identity, but we now have enough reasons to think this principle cannot be applied in the tensed case. There is no reason to think a matter-constant ship has parts which its corresponding form-constant one does not, or vice versa. Similarly, even if Tibbles loses her tail in an accident, so that she does not coincide throughout with Tib + Tail, she coincides with this sum first and then later with Tib alone. The mere fact that she could survive this mutilation, whereas the sum cannot, is enough to distinguish them, even if no such mutilation ever occurs. A less controversial example, which involves neither appeal to special philosophical theories nor to technical mereological concepts, is that of a person and her body. A person cannot be identical with her body, because the person may die and the body be left. But during her lifetime the person has no additional *parts* that the body fails to have, in particular no immaterial parts. So they must coincide.[1]

We may also have coincidence between groups. It may happen that for a period two distinct clubs or committees have the same membership. Where '<' is interpreted as the subclass relation, they then coincide. However we shall find that this interpretation for groups (paralleling that of Chapter 4 for classes) may not be the most natural, so that sameness of membership may not suffice to guarantee sameness of parts in the most natural sense of 'part'.

The term 'coincidence' is often employed for the concept of two things' being in the same place at the same time.[2] We shall instead use the term 'superposition' for this concept: the word is used in both physics and geometry for the idea of really or ideally bringing two distinct figures or phenomena into spatial coincidence ('Deckung'). Objects which coincide in the sense of mutual containment are obviously superposed. The temporary indiscernibility of coincident pairs like a person and that person's body, or a matter-constant and a form-constant ship, rest on this.

There are certain cases where objects are superposed which for categorial reasons cannot coincide. A continuant and an occurrent involving all of it occupy the same spatial region for a while, but clearly, since they belong to different categories, cannot have a common part, and equally clearly they do not *compete* for this region. A concrete collection or class of coexisting continuants occupy the same region as their mereological sum if they have one. So Tib and Tail occupy (between them) the same region as Tib + Tail and Tibbles, but the class {Tib, Tail} has (in the sense of 'part' as subclass) only three parts, whereas Tibbles and the sum have many more. It might be questioned whether a plurality can occupy space, but this is an unobjectionable aspect of taking its concreteness seriously, and leads to no contradictions. The connected rabbit parts form a different class from the rabbits, but the two classes occupy the same spatial region, and again they cannot possibly compete for the space they occupy. Similarly, the gold in a ring and the ring are not identical (since they

[1] I here go back on critical remarks made in SIMONS 1981*d*: 181 about WIGGINS 1980: 164. I was previously mixing up the notions *body* and *corpse*, which is easy enough to do, since 'body' sometimes does simply mean the same as 'corpse'—but not always.

[2] It is so used e.g. by Wiggins and Doepke. Burke uses 'cohabitation'.

have different life histories), and in fact they are not coincident either. They are clearly superposed, and it appears to conform to usage to say that every part (sub-portion) of the gold in the ring is part of the ring, but it is not obviously usual to say that a proper or improper part of the ring is part of the gold in the ring. Theories are not obliged to conform to usage, but we shall see there are systematic reasons to deny that an individual can be part of the mass of stuff making it up. Taking this for the moment as accepted, the ring and its gold are, as individual and mass, superposed but not coincident. Similar remarks apply to the ring, or the gold, and the gold atoms making up the mass of gold (and hence making up the ring).

There is no small resistance in the philosophical community to the thesis that different objects can be in the same place at the same time, a thesis supported in recent years particularly by David Wiggins.[3] The strategies employed to avoid this undesirable position have been clearly charted by Frederick Doepke,[4] who also ably demolishes the various objections to superposition. The following discussion is largely based on that of Doepke. The strategies he distinguishes are:

(1) The One–Many View
(2) The Relative Identity View
(3) The Dichronic View
(4) The Reductivist View

The One–Many View disallows (quite properly) identity between a plurality of objects, such as a number of wooden boards, and a single object, such as a ship, made up of them. But not all cases of ostensible superposition can be dealt with by finding one object to be one and the other many, as testified by the person/body case. Cases of the sort exemplified by the ring and the gold are, Doepke suggests, perhaps similar. We agree: here we have neither one–one nor one–many superposition but rather one–much superposition. However, both the one–many and the one–much cases are cases of *superposition*, only not the superposition of two individuals. Such is the extent to which metaphysics has focused on individuals and ignored masses and pluralities that only the one–one case is found threatening. But in the broad sense of 'object' in which either a class or a mass is an object, i.e. (in analogous senses) *one* object, the superposition is in every case that of two objects.

[3] WIGGINS 1967, 1968, 1980. [4] DOEPKE 1982.

The Relative Identity View comes in two versions, a temporal and a sortal version. The sortal theory of relative identity, known as Thesis **R**, says that for sortals *F* and *G* it is possible to find *a* and *b* such that *a* and *b* are both *F*s and *G*s, *a* is the same *F* as *b* but not the same *G*. This thesis is associated particularly with Geach, and receives its most extensive defence from Nicholas Griffin.[5] Wiggins has forcefully argued[6] that **R** violates Leibniz' Law, and since the latter is necessarily true, **R** is necessarily false. Both Wiggins and Doepke point out that Griffin has no acceptable substitute for Leibniz's Law;[7] as Doepke points out, 'relative identity' is just a misnomer for certain kinds of resemblance.[8]

The more interesting relative identity view is one which Doepke has found in unpublished work of Paul Grice and George Myro.[9] They challenge Wiggins's assumption that things which are ever different are always different, pointing out that Wiggins's arguments for the distinctness of superposed objects rest on finding properties in which they differ at times other than those at which they are superposed. This remark applies also to the treatment of ships in the previous chapter. So they argue that identity is relative to time: objects may be identical at one time and distinct at another. This notion is logically captured by the schema

TI $a =_t b \equiv \forall F \ulcorner F_t a \equiv F_t b \urcorner$

where the quantifier ranges only over properties whose instantiation does not entail the instantiation of any property at any other time (which rules out properties like being two years old, being ex-president of the USA, or being a bride-to-be) Doepke's argument against this is essentially the same as that against sortal-relative identity. Despite its interest, this relation—we may call it *temporary indiscernibility*—is characterized by a *restriction* of Leibniz's Law. If

[5] GRIFFIN 1977. For references to Geach and other supporters see this and also WIGGINS 1980: 16n.

[6] WIGGINS 1967, 1980.

[7] Cf. GRIFFIN 1977: 140, WIGGINS 1980: 193, DOEPKE 1982: 50.

[8] Doepke's point is confirmed by examining Griffin's examples, e.g. (GRIFFIN 1977: 205), 'Bill's car is the same colour as Tom's' is taken as a relative identity statement. Griffin argues that cars may fall under the predicate 'colour' without being colours (206), but his defence is void for want of exactness: something may fall under the predicate 'ζ is a colour' (and then it is not a car) or it may fall under either 'ζ is colour*ed*' or 'ζ *has a colour*' (and then it may be a car). Griffin even takes it as a bonus point (211) that relative identity statements include common-property statements.

[9] Cf. the reference in DOEPKE 1982: 47.

we dignify this kind of resemblance with the name 'identity', then we might as well take any other resemblance as a kind of identity, such as the 'surface identity' of a body with its surface, since they share numerous ('superficial') properties. Temporary indiscernibility turns out to be important in the theory of constitution discussed below. We already presupposed the unacceptability of temporary identity in setting up the system CT in the previous chapter, and Doepke's argument against Grice and Myro vindicates this.

The Dichronic View rejects the assumption that the superposed objects exist at the same time: it claims that, for example, gold is *turned into* a ring, which then (for example, when smashed or melted down) turns into—i.e. is replaced by—gold. But gold and ring exist at different times. Similarly, a person does not coincide with her body, but turns into a body—i.e. a corpse—at death.[10] The Dichronic View rejects the idea that in substantial changes (comings into and goings out of existence) there is a substratum surviving the change: it views such changes as the replacement of one object by another. But the view is less well placed than the one we shall advocate to explain the way many properties are carried across from one object to the other. This continuity requires no special explanation if a substratum having these properties exists throughout the change.

The Reductivist View regards as real only the ultimate constituents of which continuants are made, everything else being a logical construction from such ultimate reals. Such views usually involve a considerable revisionary element, eliminating from the chosen canonical language terms referring to and predicates true of such constructions, which turn out to be most or all of the objects with which we are familiar. If natural language is tolerated, this is usually for pragmatic reasons, it being humanly impossible to dispense with the benign fictions by which we live. While this attitude rightly emphasizes the importance of material constitution, it overlooks the fact that parts are not always ontologically prior to their wholes. A whole put together out of independently pre-existing parts, and continuing to exist by mere default of anything happening to destroy it, such as a pile of stones, is such an ontologically posterior whole. But interesting integrated wholes, like organisms, possess properties and operate according to laws which are relatively independent of the

[10] This is what I should have said in SIMONS 1981*d* (cf. note 1 above), though I never embraced the Dichronic View as a general position.

particular material constituents happening to make them up at a particular time. This is seen by the fact that they survive, and sustain these properties and operations, despite flux in their parts. We shall return in Part III to discuss the implications of this view, noting here merely that this is also Doepke's reason for rejecting reductivism: the existence of constituted objects which retain their properties through flux in parts (a flux which is indeed *selectively regulated* in the case of organisms, where some potential constituents are absorbed, others modified and absorbed, yet others rejected) makes it unnecessary to explain specially *why* just these successive consignments of chemicals take on certain characteristics, such as having a memory.[11]

Having fended off objections to superposition, we should now consider more closely what it consists in and involves. We must distinguish between a loose and a strict application of 'being in the same place at the same time'. An object's position may be specified with greater or lesser accuracy according to purpose. For some purposes, it is acceptable to say that two people are in the same place at once, for instance if they are in the same room or the same town. But this is hardly a case of superposition. What we mean is that no matter how exactly we specify the position of an object, it is the same as that of another. Let us call any portion of space containing an object a *container* for it, and the minimum container its *receptacle*.[12] So a sponge and the water held in it are only rudely, not properly superposed. The exact receptacles of water and sponge are highly intertwined, but either completely disjoint or at least almost completely disjoint. Only when we round the complicatedly shaped receptacles off to get a simply shaped container for each do we find that these containers coincide. (It is always possible to find a common container for coexistent objects, but not usually by rounding off, as here.) Where we hear of apparently superposed objects (which may be fluid as well as solid), we usually invoke our knowledge of the gappy nature of matter to avoid describing these as cases of genuine superposition, either by assimilating them to the sponge/water case, or by treating them like a box of jumbled black and white chess-men.

[11] DOEPKE 1982: 58 f. A similar kind of argument is used in CHISHOLM 1976 for the strict and philosophical identity of the self over time, as contrasted with the mere loose and popular identity of the body. As far as bodies go, Chisholm is a reductivist.

[12] SHARVY 1983*a*: 446. Sharvy took the term from CARTWRIGHT 1975: 153, but Cartwright uses it for sets of points, whereas we, like Sharvy, take 'space' to be a mass term, and do not presuppose the existence of points.

This applies even where, as in the case of solutions and alloys, the molecules of the constituents get intertwined. For practical purposes of course, we do not distinguish the spaces occupied by different components in a mixture, for example of gases, but simply take a small common container. But there is stronger than purely pragmatic resistance to the idea that two objects can be superposed and yet have *no* part in common. Wiggins expresses this in the principle[13]

> A and a proper part or constituent B of a third thing C, where $A \neq C$ and $A \neq B$, and where no part or constituent of A is any part or constituent of B or of C, cannot completely occupy exactly the same volume at exactly the same time.

Leaving Wiggins's reasons aside for a moment, let us assess what this claim amounts to. We let '$r_t a$' stand for the receptacle of a at t, allowing space to be subject to the tenseless operators and predicates of extensional mereology: we assume fixed a reference frame which determines the identity of tracts of space. On this application, we can assume without worry all the axioms of full extensional mereology: neither the sum axiom nor the strong supplementation principle is controversial. The implication

CTA11 $a <_t b \supset r_t a < r_t b$

may be taken as axiomatic. We define four purely spatial relations on continuants in terms of the mereological relations of their receptacles:

CTD19 $a \text{ in}_t\ b \equiv r_t a < r_t b$

CTD20 $a \text{ sup}_t\ b \equiv r_t a = r_t b$

CTD21 $a \text{ ex}_t\ b \equiv r_t a \wr r_t b$

CTD22 $a \text{ ov}_t\ b \equiv r_t a \circ r_t b$

reading the predicates respectively as 'is inside', 'is superposed with', 'is outside', and 'overlaps' (in each case 'at t'.) If a does not exist at t, then $r_t a$ does not exist, so a continuant can only enter into these spatial relations at times at which it exists.

Wiggins's claim (ignoring the third object) is

$$a \neq b \wedge a\ {}_2\wr_t\ b \supset \sim (a \text{ sup}_t\ b)$$

which may be simplified and contraposed to give

WP $a \text{ sup}_t\ b \supset a \circ_t b$

[13] WIGGINS 1968: 94.

where the abbreviation stands for 'Wiggins's Principle'. This claim does not appear especially startling. Indeed, it appears as though it can be strengthened. If we assume, in addition to the principles given above, a strong supplementation principle relativized to times

SSP $\forall x \ulcorner x <_t a \supset x \circ_t b \urcorner \supset a <_t b$

and the plausible principle

PP $a \text{ in}_t b \supset \exists x \ulcorner x <_t b \wedge a \text{ sup}_t x \urcorner$

(if *a* is inside *b* at *t*, then *a* is superposed at *t* with a part of *b*), then we can prove the stronger *coincidence principle*

CP $a \text{ sup}_t b \supset a <>_t b$

which, since its converse is a theorem, tells us that superposition and coincidence amount to the same thing. We sketch the proof:

(1)	$a \text{ sup}_t b$	assumption
(2)	$\sim (a <_t b)$	assumption
(3)	$\exists x \ulcorner x <_t a \wedge x \,_2\wr_t\, b \urcorner$	2, SSP contraposed
(4)	$c <_t a \wedge c \,_2\wr_t\, b$	3, assumption
(5)	$c \text{ in}_t a$	4, CTA11, df. in
(6)	$c \text{ in}_t b$	1, 5, df. sup
(7)	$\exists x \ulcorner x <_t b \wedge c \text{ sup}_t x \urcorner$	6, PP
(8)	$d <_t b \wedge c \text{ sup}_t d$	7, assumption
(9)	$c \circ_t d$	8, WP
(10)	$c \circ_t b$	8, 9

(4) and (10) are in contradiction, whence

(11) $a <_t b$

and by parity

(12) $b <_t a$

If the coincidence principle were correct, it would establish a *very* close conceptual connection between mereological relations and spatial relations among continuants. At all events, we can agree with Wiggins that 'space can be mapped only by reference to its occupants',[14] so the conceptual value of concepts of part–whole relations among continuants would consist in good measure in their being necessary for the development of concepts of spatial relations.

The coincidence principle is neat and the simplifications it affords tempting. Nevertheless, these simplifications are bought at a price,

[14] Ibid: 93.

and on balance I think it should be rejected. If so, which premisses of the above argument are to be rejected: WP, PP, or SSP? PP seems to me to be unobjectionable. My own choice would be to reject SSP, a choice prefigured in Part I. Before coming to this, it is worth testing Wiggins's Principle WP against a hypothetical counter-example due to Sharvy.

6.2 Mixtures

The term 'mixture' is used in everyday life for any object (usually a mass or plurality) consisting of two or more kinds of object such that the particles or members of the kinds are more or less evenly distributed among one another. The word normally connotes formation by mixing, the mechanical interspersion of small parts, whence it has acquired a special sense in chemistry which differentiates mixtures from compounds and solutions, suggesting that in mixtures the ingredients retain most of the most important characteristics they have in isolation, that the relation between the particles or members is one of mere spatial proximity rather than one involving important structural connections. Since we are interested only in the spatial relations of the parts, we use the term 'mixture' liberally to cover *any* object, including compounds, solutions, alloys, and amalgams, where two or more kinds of thing or stuff are closely interspersed, irrespective of issues of other relationships between proximate parts or of the emergence of new properties.

It is to be expected that a mixture of two or more substances falls under a different noun from either of its constituents. A mixture of black coffee and milk is neither black coffee nor milk. In Vienna, where such mixtures are taken more seriously than elsewhere, there are three different terms used of them according to the proportions of the mix. But while the milk *in* such a mixture is still milk, the coffee in it is no longer black coffee, since that is by definition (liquid) coffee unmixed with milk or cream. The English—but not the Viennese—equivocate, and call the mixture 'coffee' as well. Similar uncertainties attend the use of terms for various other solutions and mixtures. Is salt water water which has salt dissolved in it, or is it the solution of salt and water? When it matters (for medical purposes, for instance), we distinguish the two. When there is a point in tracing what happens to a substance in a mixture, we find a word which applies to it irrespective of whether it is mixed or not; it is at this level that names of chemical substances have their application. 'Water' is a suitable chemical term, but 'dirty water' is not. Water remains water when mixed with dirt, in whatever proportions. But if a cupful of dirty water is mixed evenly with a ton of earth, no dirty water remains, and the same goes if we mix it evenly with a lake of clean water. Similarly when there is a point in tracing the constituents of chemical compounds, no matter how compounded, we arrive at names of the chemical elements. And so on down.

A number of part-mereological, part-spatial concepts apply to mixtures. One is the relative *homogeneity* of the mix, having to do with the evenness of distribution of the various ingredients. Another concerns the relative *fineness* of the mix. For instance, a salad mixed from 1 cm diced cubes of apple and cheese is half as fine as one where the cubes are of side 5 mm. The pieces 'get closer together', in the sense that in a homogeneous mixture the average distance between the centre of a cheese cube and that of the nearest apple cube (or vice versa) is half as great. These ideas are formalized together by Sharvy as *relative homogeneity*. Since the ideal or perfect mixture is best conceived in terms of masses, we henceforth ignore pluralities. Let *m* be some mass, and let *s* be a class of parts of this mass (these are all masses according to the principles of Chapter 4). Then *s* is a *partition* of *m* iff members of *s* are discrete and their sum is *m*; in the terms of Chapter 4 (dropping 'M' subscripts):

$$s \operatorname{ptn} m \equiv \operatorname{dscr}(s) \wedge \operatorname{Sm}[s] = m$$

A mixture has a *natural partition* into its ingredients. We now give Sharvy's important definition:[15]

> A partition *s* of a mass *m* is δ-homogeneous iff every sphere of diameter greater than δ lying wholly within *r*(*m*) [the receptacle of *m*; we ignore the time-factor] overlaps the receptacle of each member of *s*.

The success we have had in explaining away apparent superposition among the ingredients in mixtures suggests a general conjecture: no mixture is indefinitely fine: all are grainy. In terms of Sharvy's definition, for every mixture there is some sufficiently small, but still positive δ for which the mixture is not δ-homogeneous.

In his discussion of true mixtures,[16] Aristotle demands that a mixture in the strict sense be *homeomerous* or like-parted. He instances

[15] SHARVY 1983*a*: 446. The definition could no doubt be extended to various other kinds of topological space if need be.

[16] Especially *De Generatione et Corruptione*, 327–8. Aristotle's *mixis* is more like a chemical combination than what chemists call a 'mixture' and what he calls a 'synthesis', since the properties of the mixture are not the mere resultant of the properties of the ingredients, in the sense of not being a mere weighted average.

water, which he wrongly conceived as a continuous indefinitely divisible element. In Sharvy's terms a mixture is homeomerous iff it is zero-homogeneous with respect to its natural partition. As he then points out, if there were such a thing as a homeomerous mixture, the receptacles of the ingredients would coincide. We can prove this as follows. First, we assume that space is indefinitely divisible without points, i.e. we treat it as atomless. In these circumstances *spheres* form a base for spatial *regions*, that is, following SF11–12 of Chapter 1:

$$\forall x \ulcorner \text{reg}x \supset \exists y \ulcorner \text{sph}y \wedge y < x \urcorner\urcorner$$

$$\forall xy \ulcorner \text{reg}x \wedge \text{reg}y \supset \forall z \ulcorner \text{sph}z \supset (z < x \equiv z < y)\urcorner \supset x = y$$

From these, along with Strong Supplementation (which is valid for regions), it follows that

$$\forall xy \ulcorner \text{reg}x \wedge \text{reg}y \supset \forall z \ulcorner \text{sph}z \supset. z < x \supset z \circ y \urcorner \supset x < y \urcorner$$

Now let s be the natural partition of a homeomerous mixture m. Then the definition of 'homeomerous' tells us:

$$\forall x \ulcorner \text{sph}x \wedge x < r(m) \supset \forall a \ulcorner a \varepsilon s \supset x \circ r(a)\urcorner\urcorner$$

from which it follows by distribution and permutation that

$$\forall a \ulcorner a \varepsilon s \supset \forall x \ulcorner \text{sph}x \supset . x < r(m) \supset x \circ r(a)\urcorner\urcorner$$

whence by Strong Supplementation and the formula above,

$$\forall a \ulcorner a \varepsilon s \supset r(m) < r(a)\urcorner$$

and since $a < m$, $r(a) < r(m)$, so we have:

$$\forall a \ulcorner a \varepsilon s \supset r(a) = r(m)\urcorner$$

But s are discrete, so if $a \varepsilon s$, $b \varepsilon s$, $a \neq b$, we have

$$a \,\text{sup}\, b \wedge a \wr b$$

a counter-example to Wiggins's Principle.

What are we to make of this? If Wiggins's Principle is 'a kind of necessary truth',[17] then the apparent consistency of the idea of a perfect mixture tells against him. Wiggins's argument for his principle runs:[18]

Suppose *A* and *B* were distinct and in the same place at the same time. Then they could not have been distinguished by place. But then they would have had to be distinguished by their properties. But no volume or area of space can be qualified simultaneously by distinct predicates in any range (color, shape, texture and so forth).

[17] WIGGINS 1968: 93. [18] Ibid: 94.

This last may be true, but it does not tell against the possibility of a perfect mixture, since the qualities of a mixture need not be those of its ingredients in isolation, as is demonstrated by the imperfect mixtures of everyday. The properties of a mixture may be expected to be some kind of resultant of the properties (though not necessarily of the perceptual qualities) of the ingredients. That we have become accustomed to thinking of such properties as resulting from the structural relations of particles in combination does not mean that an explanation other than one using corpuscular notions would have been unintelligible from the start. If a given mass of matter occupied the whole of a given region, but occupation were a matter of degree rather than an all-or-nothing affair, then it would be possible for different masses of matter to occupy the same region, i.e. be superposed, although their occupation might have different intensity distributions. In a way, this is more like the notion of a field than that of the classical impenetrable particle, and it could well be the right way to view matter. If that were so, then Wiggins's Principle would be wrong, and at any rate we must therefore doubt its necessity.

6.3 Can Things of a Kind Be Superposed?

Wiggins emphatically supports the following weaker principle due to Locke:[19]

LP No two things *of the same kind* (that is, no two things which satisfy the same sortal or substance concept) can occupy exactly the same volume at exactly the same time.

Superposition as such does not infringe this, since it says merely that different *material objects* may be superposed, but *material object* is not what Wiggins calls a sortal or substance concept, since it is the fact of its falling under such a concept which enables us to trace, individuate, and distinguish an object, whereas the examples of the previous chapter show that this could not be accomplished by the concept *material object.*

What is at stake is that if we allow distinct continuants of a kind to be superposed, then we do not have determinate means for tracing things of the kind in question through time, since they become

[19] Ibid: 93. Locke's classic formulation runs: 'For we never finding, nor conceiving it possible, that two things of the same kind should exist in the same place at the same time, we rightly conclude, that whatever exists any where at any time, excludes all of the same kind' (*Essay* II, ch. 27, § 1). On the answer to this by Leibniz, which is essentially also ours, cf. note 21 below.

temporarily indiscernible from one another upon their superposition. If two αs approach and then coalesce, and then two αs head off again in different directions, then we have in general no way to say which, if any, of the later two is to be identified with which of the former two. An *epistemically* similar problem is provided by the coming together of two swarms of bees. We do not in general know whether the two swarms after the separation are the same two as the two which came together, let alone which is which. In part this is a terminological problem of how much transfer of membership impugns swarm identity, but even if we were totally strict and insisted that not a single bee changes allegiances, we still have an epistemic problem of knowing which swarm is which. In principle this could be done by tracing exactly the paths of each single bee. Swarms of bees are gappy, and distinct bees are discrete, so there is no ontological problem.

There are apparent cases where entities of a kind can be superposed and yet where we can trace their paths through the point or period of superposition and out on the other side. Examples include moving shadows on a wall or spots of light from different lamps, wave-fronts moving parallel to one another with non-zero relative velocity, and, to take a fictional example due to Shorter,[20] clouds of water vapour set up by 'cloud projectors'. In each case we have a means of identifying objects across the superposition. In the case of the waves we assume continuity of wave velocity, while in the other cases we can trace the projected whatevers back along different causal paths.[21]

[20] SHORTER 1977: 402ff.

[21] Cf. Leibniz's reply to Locke: 'The method which you seem to be offering here as the only one for distinguishing things of the same kind, is founded on the assumption that interpenetration is contrary to nature. This is a reasonable assumption; but experience shows that we are not bound to it when it comes to distinguishing things. For instance, we find that two shadows or two rays of light interpenetrate, and we could devise an imaginary world in which bodies did the same. Yet we can still distinguish one ray from the other just by the direction of their paths, even when they intersect' (*New Essays* II, ch. 27, § 1). Note that there are two senses of 'shadow'. In one sense, a shadow is three-dimensional, extending from an opaque object away from a light source, e.g. the cone-shaped shadow cast by the earth away from the sun. In the other sense a shadow is a two-dimensional figure obtained when a reflective surface is placed partly within a three-dimensional shadow, and is its partial projection on this surface. While, as Leibniz's discussion makes clear, he is thinking of three-dimensional shadows (since a light ray is not two-dimensional, and the examples are obviously alike), it is only in the two-dimensional sense that shadows can completely coincide rather than merely overlap. SHARVY 1983*a*: 451ff. discusses what he calls a 'projection model' of homeomerous mixtures, recalling Plato's discussion of shadows in the cave in the *Republic*. In practice this model tells us little about homeomerity: he admits it is 'purely geometric'. But Leibniz is also right that *parts* of different three-dimensional shadows may be superposed.

Now one reason which Wiggins offers for believing the principle is this:[22]

Space can be mapped only by reference to its occupants, and spatial facts are conceptually dependent on the existence of facts about particulars and the identities of particulars. If space is to be mapped by reference to persisting particulars, then the nonidentity of particulars *A* and *B*, both of kind *f*, must be sufficient to establish that the place of *A* at $t_1 \neq$ the place of *B* at t_1.

We may agree about the mapping of space by reference to particulars in general, and yet accept cases where objects of the same kind coincide, since all that the requirement of mapping demands is that *some* continuants be unable to coincide with others of their kind.[23] Exceptions, provided they are recognizable as such, and perhaps provided they are somewhat in the minority (and both of these conditions appear to be fulfilled in fact), are tolerable.

One reaction to the apparent exceptions is to hold that they do not constitute exceptions since the objects in questions (waves, shadows, clouds) are not material objects or substances. The only example where this may clearly be doubted is fictional. Indeed, it may be true that shadows and waves are not substances so much as accidents or disturbances, but this strategy is unconvincing. It is begging the question to say that if two things of a kind can be superposed, they cannot be substances. What the cases of projections and waves in general suggest is that Wiggins's anxiety that we might lose track of objects of a kind if they become superposed may be allayed if we can find tracing principles which are strong enough to tide us over temporary indiscernibility, such as distinct causes or continuity of motion. Only if everything by which we can trace a continuant is so to speak packed into its receptacle is Wiggins's fear justified, and in such cases his principle appears good. Since such cases seem to form the majority, we have no trouble in mapping space.

A sortal noun for a kind of continuant tells us, *inter alia*, under what conditions the object continues to exist and under what conditions it ceases to exist. Call these the *persistence conditions* for the kind of object in question. That different objects may be superposed follows from the fact that a single parcel of matter may be in such a state that it happens simultaneously to fulfil several sets of persistence conditions

[22] WIGGINS 1968: 93.

[23] Cf. SHORTER 1977: 406: 'there will still be plenty of non-coinciding objects around to make use of'.

at once. But apart from the cloud example, we have yet to come across a clear case of material objects of the same kind which may coincide.

At one time I thought I had found a counter-example to LP among material continuants.[24] After a number of criticisms, I am now persuaded what I had was not a counter-example.[25] The reasons why it is not will help us to shed more light on LP, and suggest why it is valid, if it is valid. Here is the example.

Suppose we have some granular material which we can manipulate grain by grain. In particular, we can put grains together to form larger or smaller heaps of the material. Call an object an *n*-heap if it is maximal (i.e. a whole heap) and contains at least *n* grains of the material. Then if *m* is greater than *n*, an *m*-heap is also an *n*-heap. Suppose we have, say, a 10-heap consisting of just ten grains, and then add further grains one by one. We successively bring an 11-heap, a 12-heap, and so on into existence, but by definition these are all also 10-heaps. Since they come into existence at different times, they must all be distinct 10-heaps. Hence an arbitrary number of 10-heaps may coincide (and hence be superposed).

The problem with the example is that, as defined, the terms '*n*-heap' give only necessary, but not sufficient conditions enabling us to trace an object falling under them. The term '10-heap' *alone* does not tell us under what conditions a 10-heap comes into or goes out of existence. We can see this by considering a 10-heap consisting of 20 grains. If we take two grains away simultaneously, especially if these are not the last two added, *which* of the eleven coincident 10-heaps supposed to be there are destroyed? That we are unable to give an answer shows that the terms '*n*-heap' do not impart identity and persistence conditions.

We may grant that there are descriptive terms which do not impart such conditions: a correct ostensive use of 'that red object', for instance, does not tell us *what* we are referring to, since objects with different persistence conditions may co-occupy the place indicated. The terms '*n*-heap' appear to be like this, meaning something like 'heap containing at least *n* grains', but from this description alone we do not know what sort of heap is referred to. Why, then, should it be necessary for sortal terms to impart necessary and sufficient identity and persistence conditions? Doepke has argued that it is precisely *because* substance sortals function to tell us which object we are

[24] See SIMONS 1985*b*.

[25] See NOONAN 1986, HARRIS 1986, DOEPKE 1986. The reader for the Press also helped to persuade me of the error of my ways. For the recantation, see SIMONS 1986*b*.

considering that we cannot be left in the dark.[26] Sortal terms are general, so to pick out objects falling under them we also have to give something besides the term: Doepke plausibly suggests the place. If term and place together do not pick out precisely one object, how can the term be a sortal? Let us suppose for the moment that nothing else is required; we come back to possible third factors below.

With this in mind, let us reconsider the heap example. Can it be reconstructed using genuine sortal terms? It seems not. Consider the new terms '*n*-Heap', where an *n*-Heap is something which comes into existence by bringing at least *n* grains together where before there were not at least *n* together, and which goes out of existence again only when less than *n* grains remain. Under these conditions we cannot get more than one 10-Heap to coincide by adding grains, since once we have formed a 10-Heap, it grows and remains the only 10-Heap until the number of grains falls back below ten again, when it ceases to exist. We *can* arrange for arbitrarily many material objects to coincide, but they belong to different sorts (10-Heap, 11-Heap, and so on), and so do not constitute a counter-example to LP.

The fact that one proposal fails does not suffice to show that either LP or the stronger thesis that more than one thing of a kind can coincide is correct: perhaps heaps are not a suitable example. We need to consider the problem generally.

Suppose a sortal term for *F*s gives us necessary and sufficient conditions for the identity of *F*s, and suppose two distinct *F*s, F_1 and F_2, coincide. There are two cases to consider. Either the coincidence is permanent (i.e. F_1 and F_2 coincide at all times at which either exists), or else it is temporary (i.e. there is a time at which either only one of F_1 and F_2 exists, or else both exist but are not coincident.) Suppose we rule out temporary coincidence for the moment. That leaves permanent coincidence. Now it is clear that F_1 and F_2 cannot be distinguished in the actual world by any of their observable properties at any time during their lives. Hence they can at best be distinguished by some modal property. Putting it in terms of possible worlds, we may say that there is a world in which F_1 and F_2 are non-modally distinguishable. Again, we consider cases. One possibility is that only one of F_1, F_2 exists in the world in question. Another is that both exist. Then there are two cases to consider. Either F_1 and F_2 coincide in this world or they do not. If their coincidence is permanent, we cannot

[26] DOEPKE 1986.

distinguish them non-modally as we assumed. Hence they cannot coincide permanently. And now we face the problem of cross-world identity. *Which* of the two *F*s is F_1 and which is F_2? We can distinguish them by their properties within this non-actual world, but on what grounds do we identify one of them with one of the two coinciding *F*s in the actual world, and the other with the other? It seems that we should have some ground for identification: a mere stipulation or convention is insufficient.[27] If we then rule out all cases in which F_1 and F_2 exist together in the same world, we are left only with the case where one of them exists and the other does not. But then, *given* that the two are non-modally indistinguishable in the actual world, we have no grounds for thinking there are *two Fs* at all.

So let us come back to the case of temporary coincidence in the actual world. Here the assumption is that the life histories of the two *F*s diverge mereologically at some time either before or after the period of coincidence. One problem here is that of fending off the argument that what happens is not that two *F*s come to coincide but that rather two *F*s join together (fusion) or one splits apart (fission). Our normal way of dealing with cases of fission and fusion is to assert that they involve *replacement* of one *F* by two *F*s, or vice versa, so there is no question of coincidence of distinct *F*s. We saw above in Shorter's cloud example that there are nevertheless cases where we appear to have reasons for supposing that material objects may survive coincidence, converging and then diverging again. It does not appear absurd to claim that this is simply fusion followed by fission, understood as before as involving replacement, so that strictly speaking the cloud projected by one of the machines before the coincidence is distinct from that projected by the same machine after the coincidence. Another suggestion is that we may indeed identify the cloud before with the cloud after, but that this cloud exists intermittently, ceasing to exist at that moment at which the two clouds supposedly coincide. However, we have reasons to suppose that neither of these positions gives the best description of the case. Firstly, against the fusion/fission account, the persistence of the clouds depends on the persistence of the causal conditions for their production in the form of the functioning cloud projectors. We individuate the clouds via the projectors: here we have a third factor

[27] For arguments against ungrounded cross-world identities, see FORBES 1985: 126 ff.

besides term and place allowing us to trace objects across coincidence, and since the same projectors are working uninterruptedly, we have good reason to identify across the coincidence. Against saying the clouds exist intermittently, we have the fact that either projector would still have produced its cloud at that place and time even if the other had not been working; the identity of either cloud with a cloud at the point of coincidence would then depend on the latter's merely external relation to the other projector, in violation of a plausible requirement which Wiggins calls the '*Only a and b*' condition.[28] This also tells against the fusion/fission analysis, since whether the later cloud projected by one projector is identical with the earlier one projected by the same projector would depend on an external relation to the second projector.

One statement of the *Only a and b* principle by Wiggins is that it is the 'requirement that what makes the identity of *a* and *b* should be independent of objects distinct from *a* and *b*'.[29] Thus formulated, this appears to be already violated by the clouds example, since whether we have the same cloud depends on whether its projector continues to function, and so refers us to the projector. Similarly, the identity of dependent objects like headaches refers back (in part) to the identity of their substrates. So perhaps we should add to the principle 'apart from objects upon whose existence the existence of *a* and *b* depends'. However, this suggests a short way with the clouds example: projected clouds of this kind are not substances at all, but mere disturbances like shadows or waves, whose existence and identity depends on other things, so it is not surprising that they fail to fulfil all we require of substances. This reasoning would rule out by extension all cases where identity across coincidence is carried by a third factor. If substances are objects for whose identity the *Only a and b* rule applies *as stated*, then Doepke's argument against superposition applies. The only residual doubt is whether this is not too stipulative regarding 'substance'. In view of the naturalness of the distinction between objects which are and those which are not existentially dependent on others (to be discussed below in Chapter 8), I think it is not too stipulative, and from this, together with the frankly fictional nature of

[28] WIGGINS 1980: 96.

[29] Ibid: 233. The formulation of NOONAN 1986 of the 'Only X and Y' principle is somewhat different. Both Wiggins and Noonan use the principle to rule out 'best candidate' theories of identity.

the clouds example, I conclude we have no good reason to deny Locke's principle LP *for substances.*

6.4 Composition

In discussing objections to superposition, we noted that superposed objects may share many properties. If they share all time-blinkered properties, those whose instantiation entails nothing about properties had at other times, we called them 'temporarily indiscernible'. Burke spoke of objects sharing all their qualities; temporary indiscernibility would seem to imply qualitative indiscernibility, but not vice versa, since the proverbial two peas in a pod might be qualitatively indiscernible. Superposition of objects goes a good way towards explaining their apparent likeness, and Doepke used this to turn arguments which found superposition wasteful of objects.

Some superposed continuants are coincident, that is, they share all their parts. This makes it even easier to explain community of properties, since many physical properties, such as weight, shape, size, colour, and volume, are determined by the nature and distribution of a continuant's parts. Clearly coincidence is sufficient for superposition. But is it necessary? The coincidence principle

CP $a \operatorname{sup}_t b \supset a <>_t b$

says precisely that it is. But Doepke contends that CP is false, because there are cases in which it fails:[30]

> Consider you and the collection of atoms of which you are now composed. Appealing to intuition, I suggest that your heart is a part of you but not a part of this collection of atoms. Similarly Theseus' Ship, but not the wood of the ship, is composed of boards. Though every part of the collection of atoms is a part of you and every part of the wood is a part of the ship, you and the ship have 'additional' parts not shared by the collection of atoms and the wood.

If Caesar (C) and the collection of atoms or mass of matter m are coincident at time t, i.e. $C <>_t m$, then Caesar's heart h is part of m at t. Doepke appeals to intuition to deny this. What is the support for this intuition? Some informants I questioned agreed with Doepke, others said that it was merely odd or unusual, but not false, to say that $h <_t m$. We need reasons. At first sight, considerations of simplicity are in favour of CP. If coincidence is both necessary and sufficient for

[30] DOEPKE 1982: 51.

superposition, the connections between mereological and spatial considerations for continuants are very straightforward. Mereology reveals part of its importance in connection with spatial matters. One way to tell for sure that one spatial extent is part of another is to find a continuant occupying each such that the one continuant is part of the other. Conversely, if s_1 and s_2 are regions such that $s_1 < s_2$, then for any x, y, and t such that $s_1 = r_t x$ and $s_2 = r_t y$, we must have $x <_t y$. This latter is what must go if CP is not true.

Doepke emphasizes[31] the need for an account of a concept to bring out the *point* of applying it. If the coincidence principle is correct, then it is clear that one thing the concept of *part* does is to account for things being wholly or partially in the same place at the same time. More radically perhaps, without a concept of *part* applying to continuants we should have no concept of spatial containment, perhaps no concept of space at all. However, this important role for mereology may be maintained equally well but more indirectly if we abandon the coincidence principle in favour of a weaker position involving a more complex notion which Doepke calls *complete-composition*, and which he characterizes recursively as follows:[32]

> *y* is completely-composed of *x* at time *t* iff (def.) at time *t*, *x* has parts & every part of *x* is a part of *y* & any part of *y* which is not a part of *x* is completely-composed of parts of *x*

We may symbolize this as

CTD23 $$y \,\mathrm{cc}_t\, x \equiv \exists z \ulcorner z <_t x \urcorner \wedge \forall z \ulcorner z <_t x \supset z <_t y \urcorner \wedge \forall z \ulcorner z <_t y \wedge \sim(z <_t x) \supset \exists a \ulcorner \forall w \ulcorner w \,\varepsilon\, a \supset w <_t x \urcorner \supset z \,\mathrm{cc}_t\, a \urcorner\urcorner$$

This may be simplified by introducing a Leśniewskian functor 'pt$_t$' defined

CTD24 $$a \,\varepsilon\, \mathrm{pt}_t[b] \equiv a <_t b$$

to give

CTT23 $$y \,\mathrm{cc}_t\, x \equiv x <_t y \wedge \forall z \ulcorner z <_t y \wedge \sim(z <_t x) \supset \exists a \ulcorner a \sqsubset \mathrm{pt}_t[x] \wedge z \,\mathrm{cc}_t\, a \urcorner\urcorner$$

If x and y coincide at t, then they are completely-composed of one another, since the second conjunct is automatically true.

Several points about this definition merit comment. In the first

[31] Ibid: 54, taking his point from Dummett. [32] Ibid: 52.

place, while it has the form of a recursive definition, it can only be a proper recursive definition if there is a beginning to the recursion, where one thing is completely-composed of another in a sense which does not further bring in complete-composition on the right-hand side, which means coincidence. The definition as such does not guarantee such a level: it would be possible to find an infinite descending chain of objects, each completely-composed out of the ones below. Maybe this is a disadvantage which should be ironed out in the definition, or perhaps it is better left as a further postulate that bedrock is reached. The point of the definition is to provide a weaker sufficient condition for superposition than coincidence. This it can be proved formally to do, provided we make a couple of plausible additional assumptions. The first is that the receptacle of a plurality of spatial continuants is the sum of the receptacles of its members:

CTA12 $r_t a \approx \sigma s \ulcorner \exists x \ulcorner x \varepsilon a \wedge s = r_t x \urcorner\urcorner$

from which follows

CTT24 $\exists x \ulcorner x \varepsilon a \urcorner \wedge \forall x \ulcorner x \varepsilon a \supset x \,\mathrm{in}_t\, y \urcorner \supset a \,\mathrm{in}_t\, y$

Secondly, we require that every part of the receptacle of an object contains some part of the object:

CTA13 $\forall s \ulcorner s < r_t x \supset \exists y \ulcorner y <_t x \wedge r_t y < s \urcorner\urcorner$.

We now prove

CTT25 $y \,\mathrm{cc}_t\, x \supset y \,\mathrm{sup}_t\, x$

Proof (1)	$y \,\mathrm{cc}_t\, x$	assumption
(2)	$x <_t y$	1, CTD23
(3)	$\forall z \ulcorner z <_t y \wedge \sim (z <_t x) \supset \exists a \ulcorner a \sqsubset \mathrm{pt}_t[x] \wedge z \,\mathrm{cc}_t\, a \urcorner\urcorner$	1, CTT23
(4)	$x \,\mathrm{in}_t\, y$	2, CTA11, CTD19
(5)	$\sim (y \,\mathrm{in}_t\, x)$	assumption for *reductio*
(6)	$\sim (r_t y < r_t x)$	5, CTD19
(7)	$\exists s$ $s < r_t y \wedge s \wr r_t x$	6, SSP for regions
(8)	$\exists z$ $z <_t y \wedge r_t z < s$	7, CTA13
(9)	$r_t z \wr r_t x$	7, 8
(10)	$z \,\mathrm{ex}_t\, x$	9, CTD21
(11)	$\sim (z <_t x)$	(otherwise contradicts 10)
(12)	$\exists a$ $a \sqsubset \mathrm{pt}_t[x] \wedge z \,\mathrm{cc}_t\, a$	8, 11, 3
(13)	$\forall w \ulcorner w \varepsilon a \supset w \,\mathrm{in}_t\, r_t x \urcorner$	12, CTD19, CTD24

(14) $a \text{ in}_t x$ — 12, 13, CTT24
(15) $a \text{ in}_t z$ — 12, as line 4
(16) $x \text{ ov}_t z$ — 14, 15, CTD19, CTD22

Lines (10) and (16) are in contradiction, whence

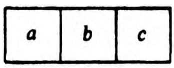
(17) $y \text{ in}_t x$
(18) $y \text{ cc}_t x \supset y \text{ sup}_t x$ — 1, 4, 18, CTD20

Neither of the additional assumptions seems in any way impeachable.

If complete-composition is a sufficient condition for superposition, is it necessary? Not if Wiggins's Principle can be falsified, for if two objects have no common parts, neither can be completely-composed of the other. Doepke holds, on the other hand,[33]

> that in all cases of spatially coinciding objects which share the whole variety of properties we have discussed, the reason they are so alike is the fact that one object is completely-composed of the other.

As we mentioned when discussing mixtures, the properties of a perfect mixture are unlikely to be directly inherited from those of their ingredients, but rather to be some kind of compromise between them; so this apparent counter-example would not perhaps worry Doepke. On the other hand, the literal formulation still seems to be too strong, since we can construct a simple finite model satisfying plausible part–whole principles for which his statement does not hold. Let *a*, *b*, and *c* be three adjacent squares (below), and consider the pluralities



$\{a+b, c\}$, $\{a, b+c\}$. These are superposed, but neither is completely-composed of the other; $a+b$ is not part of $\{a, b+c\}$ and $b+c$ is not part of $\{a+b, c\}$, where of course we must understand the part–whole predicate with a class on the right-hand side as the class-inclusion predicate, as explained in Chapter 4. Rather, every member of each of the two classes is completely-composed of members of $\{a, b, c\}$, and this is a more complicated relationship than the one Doepke considers.

This brings us to one of the most salient and difficult characteristics of Doepke's account of complete-composition. As the recursive clause makes clear, something may be completely-composed of a plurality of objects; Doepke expressly allows this to accommodate the one–many view of superposition. To the extent that the technical concept of

[33] Ibid.

complete-composition is intended as an approximation to the common or garden concept of composition, this is to be expected and welcomed. Before considering Doepke's notion more fully, let us consider the possibilities for the common notion first, indulging in a little linguistic phenomenology. The possibilities are given in Table 6.1.

TABLE 6.1. *Possibilities of Composition*

Category can be composed/made of	Category	Example
individual	individuals	wall/stones
individual	mass	sweater/sm wool
individual	individual + mass	toffee-apple/ an apple + sm toffee
individual	individuals + mass	fruit-cake/currants + sm dough
mass	individuals	gold/gold atoms
mass	masses	dough/flour + water
mass	mass(es) + individuals	blood/plasma + blood cells
collection	individuals	pack/wolves
collection	mass	snowballs/sm snow
collection	individuals + mass	toffee-apples/apples + sm toffee

In addition to these clearly acceptable cases, there are various borderline ones:

mass	individual	?sm gold/one gold atom
mass	individual + mass(es)	?sm martini/sm gin + sm vermouth + one olive
collection	individual	?Committee/one person

All of these arise from the fact that instead of several individuals, we have only one. All seem to be worth accepting for the sake of tidiness. It jars to speak of something as composed of one individual, but it would be unreasonable, for instance, to allow a thousand, ten, or two gold atoms to comprise sm gold, and yet deny this to the smallest naturally occurring quantity of gold, namely a single atom.

Continuing the possibilities, we have

collection	collections	??league of clubs/ several clubs

This problem was discussed in Chapter 4. Note that where we have mere plural reference, as 'these wolves', 'the ingredients of this cake', 'these clubs' we cannot speak of the wolves being *composed* of the wolves etc. In this case we have identity rather than composition. Finally we have

individual	individual	*this chair/ this box

If 'this chair is made of a box' means that a box was dismantled to make the chair, then we do not have a genuine case of composition, since the chair and the box do not exist at the same time. This applies even if the chair is itself dismantled and the same box reassembled, as argued in the previous chapter: here we have intermittently existing objects, but neither is composed of the other, though both are composed (alternately) of the same wood. If, on the other hand, the box is intact when a chair, either they are identical or else, more plausibly, the relationship between box and chair is that one is *serving as* or *playing the role of* the other.[34] This relationship is important, since it is of the same type as that obtaining between hand and fist (hand/hand in the shape of a fist) or wood and matter-constant ship. But it is not a relation of composition. Note, however, that we can say the box *constitutes* a chair, so 'constitutes' is in this respect more general than 'composes'.

It emerges from this investigation that, apart from the last case, the 'is made of' relation, the relation of composition, is categorially more flexible than the rather tamed variety we took in Chapter 4. What happens if we turn it round and ask after the relation of something to what helps to make it up? In many cases it is natural to speak here of parts. A stone is part of a wall, the toffee is part of the toffee-apple, and so on. Where we have a plurality, the currants *are* part of the cake, the blood cells are part of the blood. Only where something consists *solely* of certain individuals does the word 'part' jar, simply because in natural language it means 'proper part'. The wolves are not *part* of the pack, but *all* of it. We have long been used to riding over this minor disagreement with usage. But because the wolves are all of the pack (i.e. none are missing) we are not entitled to say they *are* (i.e. are

[34] Something similar is mentioned by WIGGINS 1980: 43, concerning 'That piece of sugar can be your queen [at chess] while I glue the head of the queen back on.'

identical with) the pack, because 'the wolves' is a simple plural term referring to just these animals, whereas 'the pack' refers to a group, and the group and plurality, while they here coincide in membership, have different identity conditions. The wolves are the matter of the pack. Apart from these cases, there are some where 'part' cannot be used:

*This snow is part of these snowballs
*This toffee is part of these toffee-apples
*These apples are part of these toffee-apples

The reason is in each case the same: where '*b*' is plural, 'part of *b*' means 'some of *b*', referring to a subclass of those referred to by '*b*', and sm snow is not sm snowballs, nor are (mere) apples or toffee sm toffee apples. For the same reason, since for plurals 'some' connotes more than one, we should describe a single wolf as *one of* or *a member of* a pack, rather than part of the pack. But this latter is a mere borderline case to be overlooked on systematization.

It will be apparent that, with these exceptions, the term 'part' is in general more unruly than allowed in Chapter 4. We have individuals as part of masses and vice versa. In particular cases plural and mass reference appear virtually interchangeable: 'this gold' and 'these gold atoms' do differ grammatically, but it is difficult to resist the temptation to *identify* the mass with the plurality, and there are in any case all sorts of parallels between mass and plural reference.[35] Nevertheless the two cannot be the same, since whatever is part of the gold atoms (i.e. some of the gold atoms) is (predicatively) gold, whereas not every part of the gold is gold—for example, the neutrons in it are not. Mass terms and plural terms differ principally in the *indifference* of mass terms to matters of division. A mass term can be used irrespective of how, indeed whether, the denotatum comes parcelled in units. In Chapter 4 we sealed the three senses of 'part' off from one another, and it is now clear that this was an over-simplification. But for plurals, 'part of' still is confined to the subclass relation.

If we look at the possible forms of expression used when *a* is made up, among other things, of *b*, we find four cases:

(1) *b* is part of *a*
(2) *b* is a part of *a*

[35] LAYCOCK 1972 regards plurals as mass terms. Cf. GRIFFIN 1977: § 2.4 and p. 61.

(3) *b* are part of *a*
(4) *b* are parts of *a*

Case (1) applies when *b* is a mass or an individual, case (2) only when *b* is an individual, and case (3) when *b* is a plurality, for example, the currants are part of the fruit-cake. We should not say they are parts of the cake, just as we say a single currant is part, but not *a* part of the cake. As Sharvy has pointed out[36] 'are part of' is the plural of 'is part of' whereas 'are parts of' is the plural of 'is a part of'. We say (4) only where each one of *b* is *a* part of *a*, for example, the crankshaft, clutch, and gearbox are parts of a car. The fact that we can use (1) even for singular *count* nouns (one's nose is part of one's face, the seat is part of the chair, etc.) suggests that, among individuals, there is some difference between the 'is part of' and the 'is a part of' relations, as these find expression in ordinary language. And indeed there is a difference: whatever is a part of something is also part of it, but not vice versa. The front half of a car, forward of some imaginary plane, is part of, but not a part of, the car. To say that something is a part is to say that it is something which is part and has some further additional properties. It is not altogether easy to say what this 'extra' consists in, especially as we also use 'is *a* part' emphatically, meaning 'is one part (among others)', even for part in the weaker sense. For artefacts, something is a part when it is a *component* of a thing, a piece typically existing as a unit before the whole artefact is assembled, capable of unitary replacement, capable of surviving dismemberment of the whole, perhaps in addition fulfilling a unitary function. For things other than artefacts this meaning is inapplicable, and if something is a part it may be because it is a salient or prominently delimited part, or because it is functionally unitary, like an organ. But 'part' does not normally mean 'component' where the whole is not created by assembly, or at least—as in modern 'spare-part' surgery—where it is not unitarily replaceable. Since, then, a component or unitary part is part of something satisfying certain further conditions, we feel entitled to follow Sharvy[37] in saying that mereology (part–whole theory) is as such concerned with the wider or weaker notion of 'part'. The boundaries of the stronger concept (or concepts?) are hard to draw, and we must bring in considerations from outside mereology. Nevertheless, it is important to attempt to outline what the more

[36] SHARVY 1980: 619. [37] Ibid., and SHARVY 1983*a*, 1983*b*.

restricted concept involves, and we shall work towards such an attempt in Part III below. It should not be thought that it is solely interference from the stronger notion of part such that . . . which undermines the postulation of arbitrary sums in extensional mereology. Even with the weaker notion, there is no guarantee that every plurality of individuals has an individual which is their sum. The sum axiom still fits classes and masses far better than individuals, as was argued in Chapter 4.

The untidiness of natural language in its use of 'part' is perhaps one of the chief reasons why mereologists have preferred to investigate formal systems with nice algebraic properties rather than get out and mix it with reality in all its messiness. Perhaps the single most daunting methodological problem this book faces is that of bringing formal considerations into some kind of meaningful contact with the plethora of phenomena found in the wild.

With these complications in mind, let us return to Doepke's definition of complete-composition, on the assumption that he intends to use it to approximate the plain notion of composition, with some harmless and profitable exceptions. Clearly it must *include* all cases of common composition, otherwise some cases of superposition would slip through his grasp. It is now clear that the formulation he gives, although intended to cover the one–many case, needs adapting according to whether we have a collection, mass, or individual, and according to the combination of constituents. We give two samples: the other cases can easily be constructed by analogy with these.

Sample 1: Individual (*a*) composed of individuals (*b*) and mass (*c*)

> *a* is completely composed of *b* and *c* at *t* iff every one of *b* is part of *a* at *t*, *c* is part of *a* at *t*, and every part of *a* which is neither one of *b* nor part of *c* at *t* is completely composed either of some of *b*, or part of *c*, or both some of *b* and part of *c* at *t*

Sample 2: Collection (*b*) composed of mass (*c*)

> *b* is completely composed of *c* at *t* iff every part of *c* overlaps at least one of *b* at *t* and every one of *b* is completely composed of part of *c* at *t*

For instance, every snowball is completely composed of sm snow; it seems as if we have not only superposition but coincidence. The case of the squares—collection composed of individuals—is actually more complicated than the wolf-pack case.

From all these cases it can be seen that Doepke would not envisage the falsification of Wiggins's Principle: if *a* and *b* are superposed, they must have some common part; indeed, both must be completely-composed of some third *c*.

Rather than speculate whether every possible case of superposition is covered by one of the many possible versions of Doepke's idea, we should draw attention to a problematic feature of the definition, although it is not theoretically crippling, as it can be easily amended. Take the simple case of a wall being completely-composed of stones—suppose it is a mortarless or drystone wall such as are found in northern England—then every stone is part of the wall, but the other part of the definition would need to run, 'every part of the wall which is not a stone or perhaps several stones is completely-composed of some of the stones'. In other words, the term 'part' rules out proper parts of stones as parts of the wall, which is to say that Doepke is really using 'part' to mean 'a part', i.e. 'component part'. But there is no guarantee that using this narrower concept of part will yield the required result concerning superposition.[38] If, however, we insert the wider meaning of 'part' we must modify the definition, for example in this case to:

> Every stone is part of the wall, and every part of the wall which is not a stone (or perhaps several stones) is completely-composed of one or more parts (proper or improper) of one or more stones

6.5 Constitution

All of this complication is necessary if we are to capture all the varieties of composition, and it seems to me that, suitably amended as outlined, Doepke's suggestion is along the right lines. However, when returning to superposition, we are still no clearer whether life can be simplified by letting superposition entail coincidence in cases other than those involving pluralities. Composition, unlike complete-composition, is asymmetric and transitive: if *a* is made up of *b*, and *b* of *c*, then *a* is made up of *c*, and if *a* is made up of *b*, then *b* is *not* made up of *a*. This is so even if the two are coincident, as for instance are the snowball and the snow which makes it up; we cannot say the snow is

[38] This interpretation of 'part' can be inferred from Doepke's examples, but he has in any case confirmed in correspondence that a wider notion of 'part' is needed, without giving way on the Coincidence Principle.

made up of the snowball. Composition therefore involves more than mereological considerations, since otherwise snow and snowball would be composed of one another (as indeed they are completely-composed of one another.) Nor is it necessarily the case that an object composed of one or more others came into being by putting these together in some way. That may work well enough for many artefacts, but it does not work for organisms, as they are in natural flux, and what composed an object at its beginning may have little or nothing in common with what composes it later. In any case organisms can come into being by fission, like amoebae.

The allusion to coming to be is helpful in directing attention to its opposite, ceasing to be. If *a* is composed of *b*, then one way in which *a* may cease to be is if *b* changes in a certain way, without itself ceasing to be (there will in general be other ways in which *a* may cease to be as well). If the snow scatters, the snowball ceases to exist, since a snowball is made of snow compressed together. If the wolves disperse completely, the pack ceases to exist, although every single wolf may survive. On the other hand if the snow melts, the snowball ceases to exist, and if all the wolves die, the pack too goes out of existence. In Aristotelian terms, the snow (wolves) is (are) *material cause* of the snowball (pack). Once again, Doepke has come up with a suitable definition:[39]

> x constitutes y at time t iff x could be a substratum of y's destruction.

That is to say, at any time at or after t, y could cease to exist by virtue of a change in x which x survives. The snow then constitutes the snowball, the wolves the pack, and so on for other cases of composition.

Composition entails constitution, but does the converse hold? A hand constitutes a fist in virtue of of being clenched, but it is not obvious that it *composes* a fist, and certainly a fist is not composed of a hand plus some additional part—there is no such part. Doepke argues that the body/person case is the same: a person is a body with certain anatomical configurations. We argued in the previous chapter that this is false, however. But the existence of a body is a necessary condition for the existence of a person, so it is clear that the body constitutes the person, and the extra required is the presence of the

[39] DOEPKE 1982: 54.

right sort of processes in the body. If these stop, this is a change in the body, which may nevertheless survive it, whereas the cessation of the processes is the death of the person. When *x* constitutes *y*, there are certain properties of *x* which are *accidental* to *x*, but *essential* to *y*. It is essential to a person that he or she have certain processes going on in his/her body, but accidental to the body that these processes be going on in it. Where the essential properties concern the type and disposition of parts, this is often a case of composition, but in other cases, such as that of body/person, it is not. The distinction between accidental and essential parts, as well as the question of that in virtue of which something composed or constituted is an integral object (in Aristotle's terms, the question of form) will occupy us further in Part III. Here we are only concerned with the question of matter.

The definition of constitution is in need of a small amendment. Suppose a wall is composed of stones. Then they also constitute it—one way to destroy the wall is to take it apart and scatter the stones. Now the wall may survive the loss of the odd stone here or there. Suppose one of the stones is pulverized; then the wall continues to exist, provided the loss of this stone does not bring it to collapse, but the collection of stones of which the wall was made no longer exists, since a member has ceased to exist, and the collection was a (plural) sum in the sense of SUM. It would therefore appear that the wall can survive a certain kind of change which constitutes the destruction of the collection of stones, and therefore that the wall constitutes the stones! Similar remarks apply to anything which can survive the loss of some of its parts. The snowball can be diminished without going out of existence by the melting of some of its snow, and so on. In such cases terms like 'the stones composing the wall' or 'the snow constituting the snowball' are seen to be temporally variable designators. We surely want to rule out mutual constitution. The difficulty is easily removed by making the following minor alteration to the definition:

x constitutes *y* iff *x* could be a substratum of *y*'s *total* destruction

What is meant by 'total destruction' varies according to context. We describe a wall as partly destroyed or demolished when some of its components or parts are no longer in wall configuration but others are, and totally destroyed when none are left in the right configuration. To totally demolish a stone wall it suffices to take each stone apart from all the others and lay them flat on the ground, or to knock the whole lot over. There is no need to go to the lengths of pulverizing each stone. Total destruction of an object does not usually require total destruction of every part of it. The aim of introducing the idea of total destruction is that it frequently makes sense to distinguish total from partial destruction. A mass of snow may be totally destroyed by melting it *all*, a number of wolves totally destroyed by killing all of them. Where it makes little or no sense to distinguish total from partial destruction, one may take the adjective 'total' in the definition to have zero modifying force.

Returning to the problematical examples, we see that this amendment takes care of them. A wall may survive partial destruction of the stones making it up, but not their total destruction, whereas a wall may be totally destroyed and its stones remain unscathed. Of course, an object which lives through flux in parts may continue to exist although all its previous components are destroyed, so long as its present components remain reasonably intact. But a constituted object may in general be destroyed with little or no destruction of its components, so the asymmetry of constitution is guaranteed. The point demonstrates another aspect of the practical and theoretical importance of objects in flux: theoretically, they present a special case; practically, an object in flux has a further kind of survival potential.

6.6 The Problem of Matter

The material cause of an object is its proximate or immediate matter; this may itself be constituted or composed of something else in turn, which is then relatively more remote matter of the original (both composition and constitution being transitive). We are familiar with the chains this can give rise to: organism—organs—cells—molecules—atoms—sub-atomic particles, and so on. The question naturally arises whether any or every such chain has an end: is there something which we could call ultimate matter, which would constitute everything material and not itself be constituted of anything further? This is the most ancient of philosophical questions, and it is usually regarded as having passed from the competence of the philosopher to that of the physicist. Nevertheless, there is still useful work for a philosopher to do in this area, in particular in considering the mereological properties of such possible prime matter, where we might after all recover the lost simplicity of extensional mereology.

The idea of ultimate matter is somewhat discredited philosophically because of the particular version of the doctrine found in Aristotle, who ran together the two notions of being a substratum of change on the one hand and being the bearer of properties on the other. Aristotle speaks—perhaps metaphorically, but the damage is done all the same—of 'stripping' attributes (form) from things to arrive at pure, i.e. formless matter, which can exist only potentially, not actually, since whatever exists is something with a form.[40] Now this bad metaphor of 'stripping' can perhaps be avoided, but there is in any case no need to follow Aristotle or the followers of Aristotle in linking substrata of change with bearers of properties (which then somehow manage *not* to have the properties). Ultimate matter, if there is such stuff, may well have its own characteristics, and it is an important question whether it could exist independently or only in something it constitutes.

There is nevertheless a good scientific reason for looking more closely at the stripping metaphor, namely that it turns out that the further we progress down the chain of constitution, the more limited become the ranges of characteristics exhibited by the objects encountered, merely by virtue of their greater simplicity. Micro-objects take characteristics from a very limited range of families. The tendency in the past has been to try to explain the variety of objects there are in terms of the multitude of different possible configurations of simpler objects, uncovering more profound complexity and extending the chain downwards. This process would reach bedrock if we could find a single kind of ultimate constituent; since qualitative differences also invite explanation, it would be ideal if all qualities could be explained in terms of the constituents' interrelations. This is the ideal stated dogmatically as fact in Wittgenstein's *Tractatus*, where he takes all material properties to arise from the configurations of ultimate constituents which are themselves 'colourless'.[41] But while the way in which ultimate constituents behave with respect to one another may be relational, such constituents do not thereby lack all characteristics. Even Tractarian simples are not 'bare particulars'—they have essential or internal properties, all of which are modal, and accidental or material properties, which are actual.[42]

By the definition of constitution, a constituted object is endangered by a wider range of circumstances than that which constitutes it. So

[40] *Metaphysics* Z3, 1 29a. I do not intend to go into the question whether Aristotle himself believed in prime matter: it suffices that such a theory can be conceived.

[41] WITTGENSTEIN 1961: 2.0231-2.

[42] Ibid., 2.0233.

the objects further down a chain of constitution are in this sense more hardy than those further up. It would therefore seem that ultimate matter should be indestructible. Again, this is traditionally associated with it, and is embodied in principles of conservation. However since the existence of higher-order constituted objects may be relatively independent of *which particular batch* of ultimate matter makes them up—if they are objects which are in flux—then such objects could survive even if ultimate matter were not sempiternal. The questionable attribute of sempiternality and indestructibility therefore has no place in our concept of a mass of matter.

Whatever happens further up the chain, ultimate matter cannot be composed of anything else, and therefore in particular cannot be composed of now this, now that. On the assumption that something can be in flux—can change its parts—only by virtue of a change in the proximate or remote matter of which it is made, this means that an ultimate constituent or mass of ultimate matter always has the same parts whenever it exists: it is *mereologically constant*. We recall the definition of this important predicate (which may apply to classes or masses as well as individuals):

CTD6 $\mathrm{MC}a \equiv \forall tt' \ulcorner \mathrm{Ex}_t a \wedge \mathrm{Ex}_{t'} a \supset \forall x \ulcorner x <_t a \equiv x <_{t'} a \urcorner\urcorner$

One theory of matter which fits in closely with the assumption that masses of matter are mereologically constant is the corpuscularism of Locke:[43]

> if two or more Atoms be joined together in the same Mass, every one of those Atoms will be the same. . . . And whilst they exist united together, the Mass, consisting of the same Atoms, must be the same Mass, or the same Body, let the parts be never so differently jumbled: But if one of the Atoms be taken away, or one new one added, it is no longer the same Mass, or the same Body.

Locke's masses of matter are not purely mereologically defined: they must be approximately connected, since they do not survive scattering—Locke's notion of being united resembles Chisholm's of being joined. Since this feature raised issues we would sooner skirt around, we henceforth ignore it. A mass is composed of atoms, or, as we shall call them, *particles*. The identity of a mass of matter is parasitic upon that of the particles of which it is composed; it is some kind of sum of them. The particles are assumed themselves to be mereologically constant, and they cannot be superposed or even

[43] *Essay* II, ch. 27, §3.

overlap, so in this case we can assume for particles that superposition entails not merely coincidence but identity. There is a tendency, helped by the use of the word 'atom', to regard particles as mereologically atomic, that is, without proper parts, but mereological constancy, though trivially a property of atoms, is compatible with non-atomicity of particles. Locke's sort of theory is easily represented in CT. We let the predicate 'Pc' mean 'is a particle' and add the principles

CTF10 $\text{Pc}a \supset \text{E!}a \wedge \forall t \ulcorner \text{Ex}_t a \equiv \text{Pc}_t a \urcorner$

(once a particle, always a particle—so long as it shall exist), and

CTF11 $\text{Pc}a \supset \text{MC}a$

CTF12 $\text{Pc}a \wedge \text{Pc}b \wedge a \circ b \supset a = b$

These principles entail neither sempiternity nor continuity (i.e. non-intermittent existence) of particles. These, like atomicity, are frequent concomitants, but are not strictly required. We must be careful now how to define 'mass of matter': if we essay the following definition:

CTD25 $^*\text{M}a \equiv \text{E!}a \wedge \forall x \forall t \ulcorner x <_t a \supset \exists y \ulcorner \text{Pc}y \wedge y <_t a \wedge y \circ_t x \urcorner \urcorner$

(compare the definition MD5 of 'fusion' in Leśniewski's Mereology), then we find we have defined a predicate meaning merely 'is always composed of particles', and something may be always composed of particles without always being composed of the *same* particles. For example, if *a*, *b* are particles such that *a* starts to exist before *b* does and *b* ceases to exist after *a*, and their lives overlap, then any *x* such that *x* SU *ab* is always composed of particles, but these vary from time to time. The definition we want is

$$\begin{aligned} \text{CTD26} \quad \text{M}a \equiv\ & \text{E!}a \wedge \forall x \forall t \ulcorner x <_t a \supset \exists y \ulcorner \text{Pc}y \wedge y \circ_t x \\ & \wedge \forall t' \ulcorner \text{Ex}_{t'} a \supset y <_{t'} a \urcorner \urcorner \urcorner \\ & \wedge \forall t \ulcorner \forall x \ulcorner \text{Pc}x \wedge x < a \supset \text{Ex}_t x \urcorner \supset \text{Ex}_t a \urcorner \end{aligned}$$

that is, *a* is a mass of matter iff *a* exists, every part of *a* overlaps some particle which is part of *a* for as long as *a* exists, and *a* exists as long as every particle which is ever part of it exists. This last clause ensures that masses of matter, unlike Locke's Bodies, may be scattered. To see further into the import of this definition, we can derive, assuming CTF12,

CTF13 $\text{M}a \wedge \text{Pc}b \wedge a \circ b \supset \forall t \ulcorner \text{Ex}_t a \supset b <_t a \urcorner$

This tells us that any mass of matter always contains any particles it ever overlaps (this is trivial if particles are atomic). The proof uses only the second conjunct of the definiens of CTD26, but putting the third conjunct together with CTF13 we get

CTF14 $\mathrm{M}a \supset \forall t \ulcorner \mathrm{Ex}_t a \equiv \forall x \ulcorner \mathrm{Pc}\, x \wedge x < a \supset \mathrm{Ex}_t x \urcorner\urcorner$

So a mass of matter exists at just those times when all of its constituent particles exist simultaneously. It is therefore inherently more frail than any one of them, and may be considered a sum of them, in a sense generalizing SM; we define

CTD27 $a\, \mathrm{SM}_t^x \ulcorner F_t x \urcorner \equiv \forall x \forall t \ulcorner x \circ_t a \equiv \forall y \ulcorner Fy \supset \mathrm{Ex}_t y$
$\wedge F_t y \urcorner \wedge \exists y \ulcorner Fy \wedge y \circ_t x \urcorner\urcorner$

when it is then tedious but straightforward to prove

CTT26 $\mathrm{M}a \supset \mathrm{E!}a \wedge a\, \mathrm{SM}_t^x \ulcorner \mathrm{Pc}x \wedge x <_t a \urcorner$

the converse implication being provable either under a general existence assumption for SMs, or at least under the last clause of the definiens of CTD26 as assumption.

It is easy to show that any mass of matter, including a single particle, is always composed of particles, and we have in addition

CTT27 $^*\mathrm{M}a \wedge \forall x \ulcorner \mathrm{Pc}x \wedge x < a \supset x <_t b \urcorner \supset a <_t b$

though this depends on the strong Supplementation Principle CTA10, and would have to be added axiomatically if we choose not to adopt this.

That masses of matter are mereologically well behaved is shown by

CTF15 $\mathrm{M}a \wedge \mathrm{M}b \wedge a < b \supset \forall t \ulcorner \mathrm{Ex}_t b \supset a <_t b \urcorner$

which can be proved from CTF11–12 and has the consequence

CTF16 $\mathrm{M}a \wedge \mathrm{M}b \wedge a <> b \supset \forall t \ulcorner \mathrm{Ex}_t a \vee \mathrm{Ex}_t b \supset a <>_t b \urcorner$

Masses of matter are still unconditionally mereologically constant with respect to other masses of matter: we have the following:

CTF17 $\mathrm{M}a \supset\!. \ \forall t t' \ulcorner \mathrm{Ex}_t a \wedge \mathrm{Ex}_{t'} a \supset \forall x \ulcorner \mathrm{M}x \supset\!. \ x <_t a \equiv x <_{t'} a \urcorner\urcorner$

which follows from CTF16. However, this is about as far as the good behaviour extends. Despite the assumptions of constancy (CTF 11) and extensionality (CTF12) for particles, these properties are not unconditionally carried over to masses of matter generally. It is here that we return again to the problem of Caesar's heart. Is this *part* of the mass of matter making him up at a particular time? If so, then this mass has the heart as part at one time and not at another, and is therefore not mereologically constant. We clearly cannot enjoy the benefit of *all* the following at once:

(1) The Coincidence Principle
(2) CTD26 as a definition of 'mass of matter'
(3) Mereological constancy of masses of matter.

Further, something which coincides throughout its life with a given mass of matter, and lasts just as long as it, is itself by CTD26 a mass of matter. But this coincidence might be accidental; the object concerned may have been of a kind such that it *could* have changed parts, but happened not to. But such a property ought not to belong to a mass of matter, which is an indication that CTD26 is too weak, in that it lacks a modal element. So, while CTF17 satisfies some of the intuitions we have about the mereological status of matter, it is clear that it is not enough to reflect what is forced on us by the idea that an object can change its parts only through change in its proximate or remote matter, which is what forces us in the direction of mereological constancy of masses of matter. The Coincidence Principle, on the other hand, treats constituting and constituted objects even-handedly as regards their mereological properties. Without it we could go on and add mereological extensionality for masses of matter:

CTF18 $\mathrm{M}a \wedge \mathrm{M}b \wedge a <> b \supset a = b$

thereby simplifying things considerably. The point is even independent of how we treat masses of matter. We chose to demonstrate the problem by a particle theory, but we could equally well have taken a prime stuff theory, treating 'mass of matter' as a primitive expression governed by the appropriate axioms:

CTF19 $\mathrm{M}'a \equiv \mathrm{E}!a \wedge \forall x \ulcorner \mathrm{M}'x \wedge x < a \supset \forall t \ulcorner \mathrm{Ex}_t\, a \supset x <_t a \urcorner\urcorner$

CTF20 $\mathrm{M}'a \wedge \mathrm{M}'b \wedge a <> b \supset a = b$

Independently of deciding the truth of the Coincidence Principle, we can undermine a chief reason for wanting mereological constancy of masses of matter, by showing that mereological change is not a sufficient condition for material change.

Consider an organism *a* which has a certain organ *b* composed of mass of matter *m* in a certain shape and fulfilling a certain function. The shape and function determine the kind of *b*. Now suppose the organism enters a process of internal change, involving the rearrangement of matter *m* so that it no longer composes *b* but now composes two different and separate organs *c* and *d* fulfilling different functions (we suppose the need for something like *b* has passed in the organism's life-cycle). Something similar happens, on a much more radical scale, in the metamorphosis of insects, though we have chosen a less radical change because we do not wish to put in question the continued existence of the organism *a*. Now we have every reason to suppose that *b* ceases to exist and *c* and *d* come to be out of the same matter, so that *a* is not mereologically constant. On the other hand, we can well suppose that not the slightest mass of matter has entered or left *a* in the process: *a* has remained *materially constant*, a notion we may define thus:

CTD28 $\text{MatC}a \equiv \forall tt' \ulcorner \text{Ex}_t a \wedge \text{Ex}_{t'} a \supset \forall x \ulcorner \text{M}x \supset (x <_t a \equiv x <_{t'} a)^{\urcorner\urcorner}$

from which it follows immediately, of course (by CTF17), that masses of matter are materially constant.[44] So the example shows that material constancy does not entail mereological constancy.

Of course, a die-hard mereological extensionalist would deny that there is any mereological change in the counter-example: he would describe it as simply a non-mereological change (viz. the scattering and rearrangement) of *m* within *a*. And if we limit the notion of mereology so that it is indifferent to what happens to components (which *b*, *c*, *d* are, but *m* is not), then again we preserve constancy. But since a component is a part which . . . (in the weaker sense of 'part'), it seems we can only do this by ignoring all but materially constant parts anyway, which is simply question-begging. It seems that a good deal of the intuitions behind demanding mereological constancy for masses of matter would be satisfied with material constancy, and indeed this is the safest conclusion we can draw out of Locke's position.

Likewise, we can define the notion of *material coincidence*:

CTD29 $a \,\text{mco}_t\, b \equiv \text{Ex}_t a \wedge \text{Ex}_t b \wedge \forall x \ulcorner \text{M}x \supset (x <_t a \equiv x <_t b)^{\urcorner}$,

a similar definition using 'Pc' instead of 'M' being equivalent under the assumption of CTT26. Clearly, coincidence entails material coincidence, but not vice versa. As before with the simpler notion of coincidence, the neatest link-up with spatial concepts would be to have material coincidence as a necessary and sufficient condition for superposition. Someone favouring Doepke's approach to super-

[44] We could have defined material—as indeed mereological—constancy of an object over an interval, allowing the object to change at other times. This would have perhaps been more appropriate for the present example. What we have defined is *permanent* constancy (of either type), while a further important notion to be defined is *essential* constancy.

position and accepting the existence of masses of matter would probably happily accept this solution.

6.7 Coincidence Again: A Resolution

It now looks as though the issue of the Coincidence Principle can be decided only by bringing in modal considerations, involving the notions of essential and accidental parts and the essential characteristics of components. These are brought into play by Doepke in his arguments against it:[45] one difference between Caesar's heart and the mass of matter currently making it up is that there are low-level scientific laws governing the anatomical and physiological properties of hearts, whereas these do not apply as such to masses of matter, even masses making up hearts, since it is in every case accidental to the mass of matter that it forms a heart and has the physical characteristics which go with something of that shape. And even if we bring in the role of mereological concepts in determining positions, so that, for example, we establish that the receptacle of a certain mass of matter comprising Caesar's heart is part of the receptacle of Caesar, since the mass of matter is superposed with the heart, which is itself part of Caesar, it is accidental to the mass of matter that its position is established as that of a heart; likewise, when we establish that the heart is in a certain mass of matter, namely that making up Caesar, we need not in principle bring Caesar himself in at all in individuating the mass of matter.

If these considerations speak against CP, it is worth noting that the introduction of modal concepts serves only to sharpen the opposition between proponents and opponents of CP, since the opponent must claim that the heart is not even *accidentally* a part of the mass of matter making up Caesar; components are then *essentially unable* to be parts of the matter composing their wholes. The mereological asymmetry thus reinforces the compositional and constitutional asymmetry.

Without adverting to modal considerations, however, it is possible to discuss the purely mereological merits of CP. We have so far been discussing the principle for a tensed part-predicate '$<$' which is not antisymmetric. For our final discussion we shall once again ignore tense and return to the use of '$<$' for the anti-symmetric predicate of

[45] DOEPKE 1982: 59, and in correspondence.

extensional mereology. We then recall two definitions from earlier:

SD1 $x < y \equiv x \ll y \vee x = y$

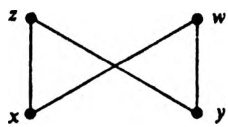
SD15 $x \leq y \equiv \exists z \ulcorner z \ll x \urcorner \supset \forall z \ulcorner z \ll x \supset z \ll y \urcorner \wedge$
$$\sim \exists z \ulcorner z \ll x \urcorner \supset x < y$$

The two-sided predicate obtainable from SD1 is of course just identity, but the two-sided predicate obtainable from SD15 satisfies

$$x \lesseqgtr y \equiv x = y \vee (\exists z \ulcorner z \ll x \vee z \ll y \urcorner \supset \forall z \ulcorner z \ll x \equiv z \ll y \urcorner)$$

We are thus faced with not one but *two* coincidence principles. The strongest says that superposed objects are identical. While this has often been held in the history of philosophy, under the motto 'No two things can be in the same place at the same time', we have seen ample reasons for rejecting it. The second coincidence principle concerns the symmetric predicate '$\lesseqgtr$', which we may call (in anticipation of things to come) *strong coincidence*. The strong coincidence principle says that superposed objects (better, superposed individual continuants) are strongly coincident.

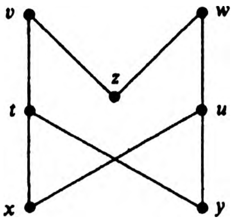
It is an advantage of rejecting this form of CP that our mereology is thereby made more flexible, and this is perhaps the best reason for rejecting it. Accepting strong CP but rejecting the extensionality of parts turns out to be an unstable option, as the following considerations show. It is the test of a non-extensional mereologist that he is prepared to accept something of the same general form as the four-element model (below) as in principle realizable under the intended



interpretation of 'part' ($\ll$ marked by a line or chain of lines going upward). The real-life examples which are likely to be accepted are more complicated, involving many more parts, but the principle is the same: eventually we arrive at some kind of bifurcation whereby the same smallest parts make up different objects. In the four-element model we might envisage z as being the sum of x and y, and w as being a complex formed of x and y in some constitutive relation. Now neither z nor w is part of the other in the sense of '$<$', but each is part of the other in the sense of '$\leq$'; they have the same proper parts, i.e.

strongly coincide. We have strong coincidence without extensionality, since the model violates the Proper Parts Principle.

However, once we accept such bifurcation, there is nothing to prevent it being embedded in a more complex model. Take for instance the seven-element model (below). Here both v and w are built



up from the same ultimate parts, namely x, y and z. Hence they will be superposed. But $t \ll v$, whereas $\sim t \ll w$, so v and w do not strongly coincide. If such models are acceptable, then the strong coincidence principle is unacceptable. How could we modify the seven-element model so that it satisfies the strong coincidence principle? Simply identifying v and w is unmotivated, for we have then *two* objects, t and u, which both yield the same object on the mere addition of z. This still leaves a bifurcation below, so the only way to motivate the identification of v and w is to also identify t and u, which leaves us with an extensional model. On the other hand, just identifying t and u leaves us with a bifurcation above, and this can once again be embedded in a more complex model giving rise to the same problem. It seems that there is no well-motivated half-way house between rejecting bifurcation altogether, i.e. accepting extensionality, and accepting models like the seven-element one, and therefore rejecting the strong coincidence principle.

This suggests that we look for a still weaker sense of coincidence in which v and w may be said to coincide. A natural suggestion arises out of one rather simple version of Doepke's idea of complete-composition. We take the case where an individual x is entirely made up from a class of basic building blocks a (which may be cells, molecules, bricks, or whatever). To eliminate the troublesome recursive clause in the definition of complete-composition, we give the purely mereological definition

$$x \operatorname{com} a \equiv \mathrm{E}a \wedge \forall y \ulcorner y \varepsilon a \supset y \leq x \urcorner \wedge \forall z \ulcorner z \leq x \supset \exists z \ulcorner z \varepsilon a \wedge z \circ y \urcorner \urcorner$$

(compare the Leśniewskian definition of sum, MD4 of §2.6.2, but note that because the context is no longer extensional we have taken the weaker predicate '$\leq$'). Doepke's leading idea is that individuals are superposed if and only if they are both completely-composed of the same whatever (individuals, mass of matter, the combinations we have mentioned.) In this simple case, say that individuals *weakly coincide* iff there are some building blocks of which both are composed in the sense just defined:

$$x \subset\supset y \equiv \exists a \ulcorner x \,\mathrm{com}\, a \wedge y \,\mathrm{com}\, a \urcorner$$

However, it can easily be shown that

$$\exists a \ulcorner x \,\mathrm{com}\, a \wedge y \,\mathrm{com}\, a \urcorner \equiv \forall z \ulcorner z \circ x \equiv z \circ y \urcorner$$

(for the implication right-to-left take the class of all z such that $z \leq x \wedge z \leq y$.) So weak coincidence as here defined is the same as overlapping the same individuals. It can be checked that in the seven-element model, v and w weakly coincide, as do t and u. This is the concept of weak coincidence we are looking for.

But now note that since weak coincidence is symmetric, it is natural to define a one-sided predicate of *weak inclusion*

$$x \subset y \equiv \forall z \ulcorner z \circ x \supset z \circ y \urcorner$$

and its counterpart sense of strict weak inclusion:

$$x \subset\subset y \equiv x \subset y \wedge \sim (y \subset x)$$

Note that it is only after rejecting extensionality that we could imagine we were here defining something new. Weak inclusion is a part-relation: it is reflexive (on existents), transitive, and satisfies the following strong supplementation principle

$$E!x \wedge E!y \wedge \sim (x \subset y) \supset \exists z \ulcorner z < y \wedge \sim z \circ x \urcorner$$

Interestingly, the definitions of overlapping obtained in the usual way from the concepts $<$, $\leq$, and $\subset$ are all coextensional, which strongly suggests that weak inclusion is a *stable* minimally weak part-predicate: any attempt to define a weaker predicate in terms of it in the way $\leq$ and weak inclusion were defined in terms of $<$ simply gives us $\subset$ back again. If we take together as theses the definition of $\subset$ in terms of $\circ$ and the obvious definition of $\circ$ in terms of $\subset$, we can derive all theses of extensional mereology (without infinitary operators) which do not include the identity sign.[46]

[46] I am grateful to the reader for the Press for pointing this out, and for suggesting basing CT on overlapping alone.

So we see what riches we miss by being extensional, since we now have not one but three concepts of part, which the Strong Supplementation Principle for < simply collapses together, and there may well be other concepts of part in between those we have considered. We also get another perspective on the materialism of extensional mereology, which effectively consists in identifying individuals made of the same matter. The issue of the coincidence principle is thus resolved: there are good reasons to reject it for the two stronger coincidence concepts (identity and strong coincidence), but no reason to reject it for weak coincidence, *provided* we consider only superposed *material individuals*. There is no reason to deny that Caesar's heart is weakly included in his matter—indeed, the point of the weak inclusion predicate is precisely that it gives us a concept of part which allows this. That stronger part-predicates have a different point is quite understandable. Note that because all three part-predicates yield the same concept of overlapping, Sharvy's mixtures provide a counter-example to even the weakest coincidence principle. This is a measure of the extreme nature of the example, and constitutes the only reason we can come up with for rejecting the weakest principle. What it seems to indicate, however, is not so much that the principle is false in the hedged-about form we have given, but that the components of such a mixture cannot be regarded as *material*; they are more like Leibniz's shadows, or the mixing of red and green light to give yellow. It is significant that Sharvy draws on the metaphor of projection when trying to make sense of perfect mixtures, and cites the wave conception of matter in this connection.[47]

In view of the relative stability of the overlapping predicate, it is worth seeing how we might reform the system CT, taking the predicate $\circ_t$ as primitive. A single axiom suffices:

CT*A $\quad a \circ_t b \equiv \mathrm{Ex}_t a \wedge \mathrm{Ex}_t b \wedge \exists z \ulcorner \mathrm{Ex}_t z \wedge \forall w \ulcorner w \circ_t z \supset w \circ_t a \wedge w \circ_t b \urcorner\urcorner$

Note, however, that the only part-predicate we can define back again in terms of $\circ_t$ is the weak inclusion predicate $\subset_t$: for stronger part-predicates more is required, and this additional content is probably not purely mereological. This suggests in retrospect that not all of the axioms of CT as given in Chapter 5 are equally plausible for all senses of 'part': in particular, CTA7 is wholly unsuspicious only for weak inclusion.

[47] SHARVY 1983*a*: 451ff.

Part III

Essence, Dependence, and Integrity

> There is, however, another artifice, by which we may induce the imagination to advance a step farther; and that is, by producing a reference of the parts to each other, and a combination to some *common end* or purpose. . . . The common end, in which the parts conspire, is the same under all their variations, and affords an easy transition of the imagination from one situation of the body to another.
>
> But this is still more remarkable, when we add a *sympathy* of parts to their *common end*, and suppose that they bear to each other, the reciprocal relation of cause and effect in all their actions and operations. This is the case with all animals and vegetables; where not only the several parts have a reference to some general purpose, but also a mutual dependance on, and connexion with each other.
>
> Hume, *A Treatise of Human Nature*, book I, part iv, section 6.

The discussion in Parts I and II makes it clear that a number of issues in mereology cannot be clarified without bring in modality. Modal mereology, if not completely new, is underdeveloped, and our first task is to develop sufficient for our main purpose, which is to clarify the concept of an *essential part*. Mereological essentialism is precisely the thesis that every part of every object is essential to it. The arguments for this thesis are found wanting, but as with mereological constancy and extensionality, there are certain objects for which the thesis holds. This then rounds off the discussion of mereological extensionality and its limits which began in Part I.

Our second topic in this part concerns mereological sums. Even if one is disposed for systematic reasons to accept the existence of such things, there is no doubting that they are in some way less paradigmatic as individuals than organisms, artefacts, and other bodies. The simple linguistic criterion according to which an individual is anything which can be designated by a singular term, coupled with the prejudice in favour of the singular, makes it appear a pseudo-issue to enquire into the extralinguistic *warrant* for using a singular term. Such a warrant may come in degrees, even for concrete objects.

We therefore need to give an account of what constitutes the unity and wholeness or integrity of the paradigm individuals, an integrity that sums lack. The schematic answer emerges from Hume's remarks:

it has to do with the interrelation of the *parts* of such objects, which makes it a mereologist's business. An object is more integrated the more its parts are in various respects interdependent and are not dependent on objects outside.

To clarify this idea, we have to make sense of the various notions of dependence which we need. There are in particular two general families of dependence concept: one concerns *ontological* dependence, the inability of an object to exist at all unless another object exists, the other concerns relations of dependence or determination among determinable characteristics of objects making up a whole. These relations belong to the general category of *functional* dependence. Notions of ontological dependence are modal notions, and it transpires that there is a whole cluster of such notions, some of them differing in respect of mereological concepts. The treatment of ontological dependence therefore builds on the modal mereology we develop, and leads naturally to a discussion of the traditional problem of substance. Substances, however these are understood, are philosophically the individuals *par excellence*. The discussion in this area owes much to unjustly neglected work by Husserl. In studying the properties of objects whose parts are in relations of functional dependence, we are able to build on largely forgotten work by Grelling and Oppenheim which does much towards clarifying the hitherto vague notions of Gestalt and integral whole.

7 Essential Parts

Since the rapid development of modern modal logic, there has been nothing to stop anyone interested from developing a modal extension of the calculus of individuals or Mereology. But in fact, those logicians interested in modal logic and those interested in part–whole theory overlap hardly at all, for largely ideological reasons: modal logic tends to be intensionalist and mereology extensionalist. This logical *apartheid* of modality and mereology is however a recent phenomenon and, it is to be hoped, a transient one; in the writings of Chisholm and Wiggins the two groups of concepts are once again brought together—once again, for, as we shall see, in the work of the earliest systematic mereologist of this century, Edmund Husserl, modal and mereological considerations are inexorably bound together. The whole of this third part may be taken as an attempt to carry further the project which Husserl began in the third of his *Logical Investigations.*

Our intention in the following sections is not to present a fully worked out theory, complete with formal system. Discussion of the topic is still too immature to permit this. Rather we intend to raise some of the most important issues concerning modal mereology. One is simply the question how best we go about formalizing typical intuitions on the subject. Another issue to confront is that of Chisholm's mereological essentialism. This chapter is therefore perforce somewhat tentative in its conclusions. The situation in general in Part III is the reverse of that in Part I. There we had plenty of formal systems to look at, and a problem as to how best to understand them in the light of our intuitions. Here, we need to consider first the facts and examples at our disposal before system construction can begin. It should be noted that we shall not attempt to follow up the manifold problems of modality and essence for their own sake, but simply clarify them sufficiently for our purpose.

7.1 Modality and Essence

Numerous kinds of modal part–whole sentences present themselves for consideration. We consider only those involving essential necessity: possibility is simply dual to this. For a start, there are generic sentences, such as

All boxers must have hands
A hand must be part of a living body
Every cycle of an Otto engine must include an ignition

which allow themselves to be mildly regimented into the forms

Every α must have a β as part
Every β must be part of an α.

Then there are sentences expressing essential part–whole relations between particulars as such, of such forms as

a must be part of *b*
b must have *a* as part

Starting with the generic sentences, there is an immediate problem as to the best way to capture them in a modal mereology. To see this, compare the arguments

(*a*) Every man must have a head
Tom is a man

∴ Tom must have a head

(*b*) Every boxer must have hands
Pat is a boxer

∴ Pat must have hands

The first argument is valid, and has true premisses and conclusion. The second is invalid. Although we can understand the premisses so that they come out true, the conclusion comes out false.[1] Pat could exist without hands. The obvious way to account for the difference between the arguments is to use a sentential necessity operator □ and distinguish the forms

$\forall x \ulcorner Fx \supset \Box Gx \urcorner$

and $\forall x \ulcorner \Box (Fx \supset Gx) \urcorner$

that is, account for the difference between (*a*) and (*b*) by finding an ambiguity in sentences of the form 'Every *F* must be *G*' which turns on a scope difference of the necessity operator. In argument (*a*), the first

[1] If we read the 'must' metalinguistically as going along with the 'therefore' and marking necessity of consequence, the second argument is valid too, but *then* its conclusion reads only 'Pat has hands', so we exclude such uses of 'must' from consideration.

premiss is of the first type, supporting the modal conclusion, while in (*b*) it is of the second type, supporting only a non-modal conclusion. However, the form '$\forall x \ulcorner Fx \supset \Box Gx \urcorner$' has instances of the form '$Fa \supset \Box Ga$', for example, 'If Tom is a man, then necessarily Tom has a head'. Since we have accepted it as a principle of continuant mereology that $a < b \supset \mathrm{E}xa \wedge \mathrm{E}xb$, so that we can also accept that this is necessarily true, then if 'Necessarily, Tom has a head' is true, so is 'Necessarily, Tom exists'. But that Tom exists is not a necessary proposition, so neither is that Tom has a head.

Another way of accounting for the difference between (*a*) and (*b*) would be to accept that both major premisses have the form '$\forall x \Box (Fx \supset Gx)$' but argue that the difference lies in the minor premisses. Tom could not but be a man, so it is necessary that he is a man, whereas Pat did not need to be a boxer. Thus the first argument would be filled out as follows:

(1) $\forall x \ulcorner \Box (Fx \supset Gx) \urcorner$	Major premiss
(2) $\forall x \ulcorner \Box Fx \supset \Box Gx \urcorner$	1, distribution
(3) $\forall x \ulcorner Fx \supset \Box Fx \urcorner$	('Whatever is a man must be a man')
(4) Fa	Minor premiss
(5) $\Box Fa$	3, 4
(6) $\Box Ga$	2, 5

This suggestion, while correctly highlighting the difference in status between 'Tom is a man' and 'Pat is a boxer' still wrongly imputes necessity to the true propositions that Tom is a man and Tom has a head. That Tom cannot exist other than as a man (that he is *essentially* a man) and that he cannot exist without his head (that it is an *essential part* of him) are not the same as its being necessarily true that Tom is a man, and that he has a head, respectively.

This means that we must distinguish the 'must' of necessity as applied to a proposition or state of affairs (*de dicto*) from the 'must' of essence, concerning the way in which an object has an attribute (*de re*). The question of the relation between these two kinds of necessity is the subject of much debate in the philosophy of modal logic.[2] The most usual tactic is to make use of the fact that a sentential operator, '$\Box$', has a scope within which there can be occurrences of variables which

[2] On attempts to reduce *de re* to *de dicto*, see PLANTINGA 1974: 29 ff., FINE 1978b. Reduction in the other direction has been suggested, but not carried through in detail; cf. CHISHOLM 1979: 369 ff., WIGGINS 1976: 301, 312.

are free within that scope. This can then be used to define a distinction between *de dicto* and *de re* without needing two operators.

How might we use '□' only and yet distinguish necessary propositions from propositions of essence? One possibility suggests itself from our gloss of '*a* must be *F*' above: *a* cannot exist unless *a* is *F*, that is, $\Box(E!a \supset Fa)$. This way of reading '*a* must be *F*' emerges informally even from the remarks of Wiggins, who has proposed an alternative treatment of the *de re* 'must' as a predicate modifier.[3] The present way of representing essential predications has the advantage of drawing the parallel between having hands being a necessary condition for Pat to be a boxer and having a head being a necessary condition for Tom to exist at all. In the case of argument (*a*), we can express the major premiss either as '$\Box\forall x\Box(Fx \supset \Box(E!x \supset Gx))$' or as '$\Box\forall x\Box(Fx \supset Gx)$'; under the assumption that whatever is a man must be a man: '$\Box\forall x\Box(Fx \supset \Box(E!x \supset Fx))$' the first follows from the second in the quantified S5 we shall use. But we prefer the first. We must distinguish between '*a* exists necessarily', expressed '$\Box E!a$', and '*a* exists essentially', expressed as '$\Box(E!a \supset E!a)$', and therefore always true. This means care in interpreting 'must': '*a* must exist' tends to suggest necessity rather than essence, whereas '*a* must be a man' or '*a* must have a head' tend to suggest essence rather than necessity. With such care we can circumnavigate one of the objections raised by David Wiggins to using '□' to express *de re* necessity, namely that such use does not distinguish between 'It is necessary that Cicero exists' (which is false) and 'It is essential to Cicero that he exist' (which is true).[4] Another objection Wiggins gives is this: if we accept that existential generalization on names like 'Cicero' gives us a necessary truth '$\Box$(Cicero is a man $\supset \exists x \ulcorner x$ is a man$\urcorner$)' and represent 'Cicero must be a man' by '$\Box$(Cicero is a man)', then we derive the result '$\Box(\exists x \ulcorner x$ is a man$\urcorner)$', which is false.[5] This objection is extendable to the rendering of 'Cicero must be a man' by '$\Box(\forall x \ulcorner x =$ Cicero $\supset x$ is a man$\urcorner)$', which is equivalent to '$\Box$ ($\exists x \ulcorner x =$ Cicero$\urcorner \supset$ Cicero is a man)', i.e. our formula for essential properties '$\Box$(E!Cicero $\supset$ Cicero is a man)'. If we again accept generalization on 'Cicero' we get the consequence '$\Box(\exists x \ulcorner E!x \supset x$ is a man$\urcorner$), which is again false. This shows that we must give up either the attempt to express *de re* modalities using '□' or necessary

[3] WIGGINS 1974; cf. 345, 348, 358, n. 41. In particular, the defence of the distribution of necessity over implication for the predicate-modifying operator NEC appears to make use of this reading.

[4] WIGGINS 1976: 301.

[5] Ibid., 301 f.

conditionals of the form '$\Box(Fa \supset \exists xFx)$'. In free logics such conditionals are precisely what is rejected, but Wiggins does not go along with free logic: he contends that 'Cicero' is here a 'good name';[6] such names are presumably precisely those that sustain existential generalization. To this, free logicians have the rejoinder that what distinguishes a good name from a bad one is nothing other than the truth of the existential proposition '$\exists x \ulcorner x = a \urcorner$', and it is this which allows us to generalize, on the weaker schema

$$\exists x \ulcorner x = a \urcorner, \quad Fa \vdash \exists xFx$$

which free logic accepts. If modal logic allows (as it ought) that not everything which exists exists necessarily, it would seem as though modal and free logics were made for one another.

Wiggins has employed these and other arguments to support the idea of representing the *de re* 'must' by means of a predicate modifier 'NEC' operating on λ-abstracts rather than employing the sentential operator '$\Box$' and distinguishing different types of occurrences of terms in modal sentences. Wiggins's approach suffers from a number of drawbacks by comparison with the more familiar approach, chief of which is that it is very difficult to decide what formal principles hold for it or to give intuitive justifications for them. As a result, there is no fully developed semantics for the operator. A further unsolved problem is the relation between the two operators. Some of Wiggins's own informal glosses suggest that NEC is indeed superfluous.[7]

By contrast, there is now a well-established semantic framework for dealing with quantified modal logic, that of possible-world semantics. While one may entertain doubts as to whether there really are such things as possible but non-actual worlds, they are a highly convenient way to formalize our discourse about what might have been or what must be the case. Their assumption as a convenient fiction is justified by the ease with which we can formulate modal intuitions in terms of them. We trade the unfamiliarity of the modal operators for the familiarity of quantifiers ranging over worlds, just as the 'at *t*'

[6] Ibid., 302. Cf. WIGGINS 1980: 132 n.

[7] Cf. those mentioned under note 3 above. QUINE 1977: 236 says: 'I can detect a confusion in the rendering "NEC$[(\lambda x)(\lambda y)(x = y)]$" for "that relation which any *r* and *s* have iff they are necessarily identical" (293). The way to write it is "$(\lambda x)(\lambda y)\Box(x = y)$"'. Cf. LOWE 1985, who questions whether a modal predicate operator indeed makes sense at all.

modification coupled with quantification over times can replace more insecure intuitions about tense.[8]

Since our aim is to explore the virtually uncharted territory of modal mereology, it is heuristically advantageous that our navigational aids be as familiar as possible, and for this reason alone it is worth following the conservative strategy of sticking with the more widely accepted modal framework based on '$\Box$'.[9] There is no consensus as to exactly which form a quantified modal logic should take, but since our interest is not modal logic for its own sake, we may again choose on grounds of expediency. Recent work by Fine and Forbes[10] demonstrates that a relatively simple modal predicate logic can be successfully used to formulate and test informal intuitions about various kinds of essentialist thesis. This logic is quantified S5 (QS5), with actualist quantifiers and contingent existence. The Fine–Forbes semantics for QS5 is summarized in the appendix to this chapter. Of course, modal *logic* as such should not decide questions of modal mereology, the justification of whose principles rests as usual on intuition and argument. If our modal framework should prove prejudicial to this aim, it must be revised. But this is something which can only be established after we have tried an approach out.[11]

With these points behind us, let us consider how to express the idea that an object is essentially thus and so. For a particular individual a, to say that a must be F, that a is essentially F, we can simply use

$$\Box(\mathrm{E!}a \supset Fa)$$

as mentioned before. That is, a is essentially F iff a cannot exist unless a is F. This is a *de re* formula concerning a because 'a' occurs within the scope of the modal operator. The other kind of *de re* formula which we shall consider is that where an occurrence of a bound variable is within

[8] For further comparisons between tense and modality see e.g. PRIOR and FINE 1977, FORBES 1985: 38 ff.

[9] In earlier versions I attempted to use Wiggins's NEC operator. The relative simplicity and intuitiveness of '$\Box$' by comparison with 'NEC' convinces me this was a mistake. I am grateful to the Press's reader for urging me to reconsider.

[10] FINE 1977, 1978*a*, 1978*b*, 1981*a*, 1981*b*, FORBES 1985.

[11] One problem on which more can be said is the interaction of mereology and vagueness. Despite good general discussions of vagueness (see GOGUEN 1969, FINE 1975, WRIGHT 1976, FORBES 1983, 1985: ch. 7), there is little on vagueness in mereology, although the discussion in FORBES 1985 just cited is applicable. Vagueness in mereology leads straight to identity problems: if '$a \circ b$' has no definite truth-value, then while '$a+b = a-b$' is definitely false, neither '$a = a-b$' nor '$a = a+b$' has a definite truth-value.

the scope of a modal operator whereas the quantifier binding the variable is outside the scope of this operator.[12]

To this analysis of essential properties it might be objected that according to it any necessarily non-existent object, such as the round square or the greatest prime number, will have *all* properties essentially, and not just the ones it actually has. To answer this objection fully would involve a comprehensive discussion of Meinongian theories of impossible objects, for which we do not have the space.[13] We have two excuses for not facing the objection. First, for the purposes of discussing mereology, impossible objects are of vanishingly little interest, so we are justified in ignoring them. Secondly, the implausibility of their assumption places the burden of showing that they must be taken into account onto their supporters. For present purposes we may therefore rest content with the traditional rationalist notion of object which admits *possibilia* but excludes *impossibilia.*[14]

A second consequence of the above rendering of 'a must be F' is that many things are trivially essential to objects, such as existing, being self-identical, being red if coloured, and being such that $2+2 = 4$. But the essential properties a mereologist is interested in are not of this trivial sort, and that is all that matters.[15]

The essential attributes of an object are not, so to speak, a brute fact about it as a particular; an object has the essential properties it has in virtue of being the *kind* of object it is.[16] It is not accidental to the objects of a given kind that they have the essential properties the kind lays down: any possible member of the kind must have these properties essentially. This motivates the following modal scheme due

[12] Since we do not have free variables (their place is taken by parameters), these are the only cases arising.

[13] Cf. PARSONS 1980, ROUTLEY 1980, LAMBERT 1983, and ZALTA 1983. Only Routley mentions mereology (704, 738, very briefly and in unimportant places.)

[14] A classic formulation is '*Ens* dicitur, quod existere potest, consequenter cui existentia non repugnat' (Christian Wolff, *Philosophia prima siva ontologia*, § 134). It is against *this* background that Meinong's ideas are revolutionary.

[15] In the semantics we employ, existence is essential but not necessary, since objects do not exist in every world, but self-identity is necessary: '$a \approx a$' is true even in worlds in which a does not exist.

[16] As stressed in WIGGINS 1980, and before him by Husserl, in his third *Logical Investigation*. The difference between them is that Wiggins denies that truths of essence are all a priori (1980: ch. 4.)

to Forbes:[17]

$$\Box\forall x\Box\forall x_1 \ldots \Box\forall x_n\Box (Kx \wedge Fxx_1 \ldots x_n \supset \Box(E!x \supset Fxx_1 \ldots x_n))$$

Here K indicates the general category of object concerned (such as being a continuant, occurrent, set, or property), and F specifies a further attribute which is stated to be essential. The complex prefix of alternating necessity operators and quantifiers indicates that the ensuing conditional necessarily applies to all *possible* objects. Since such prefixes occur frequently, it is convenient to have an abbreviation which avoids the need to write them out. If $A(x, y, \ldots)$ is a formula in which the individual variables x, y, ... occur freely, then we abbreviate its *necessary universal closure*:

$$\Box\forall x\Box\forall y \ldots \Box(A(x, y, \ldots))$$

by

$$(\Box)A(x, y, \ldots)$$

To form the necessary universal closure of a formula, one simply takes the free variables in it in alphabetical order, strings prefixes of the form $\Box\forall z$ in this order together, and adds an innermost '$\Box$'.

The scheme for essential properties given above is not always strictly adhered to in what follows, but the variants all bear an obvious family resemblance to it.

7.2 Some Theses of Modal Mereology

On the assumption that the most basic properties of the part–whole relation hold of logical necessity, it is admissible to assert modally strengthened versions of the principles expressing these properties. In the same way, the definitions employed may be given modally strengthened forms. Taking definitions first, if a non-modal mereology expresses the definition of a predicate F as follows (where initial quantifiers are left tacit):

DS $\quad Fxx_1 \ldots x_n \equiv A$

then its modally strengthened version is obtained by prefixing the above with '$(\Box)$'.

In the case of definitions employing definite descriptions, a different approach is necessary. Since definite descriptions are not modally

[17] FORBES 1985: 97.

rigid, it is preferable, in evaluating formulae in which operators occur which are introduced by descriptions, to eliminate the descriptions using the Russellian scheme

$$(\Box)(A(\iota xB) \equiv \exists x^{\ulcorner}\forall y^{\ulcorner}B[y/x] \equiv y = x^{\urcorner} \wedge A(x)^{\urcorner})$$

We can here skirt round the difficult problem of how to interpret descriptions in quantified modal logic[18] because the basic mereology we are considering does not involve any axioms concerning such operators. But the problem is one which must be faced by anyone attempting to give a modal mereology making stronger assumptions than those we are here making.

Because rather few axioms of non-modal mereology are secure from all reproach, we shall advance only very few modal strengthenings of non-modal principles. For non-temporally modified mereological predicates, we have

MA1 $(\Box)(x \ll y \supset \sim y \ll x)$

MA2 $(\Box)(x \ll y \wedge y \ll z \supset x \ll z)$

MA3 $(\Box)(x \ll y \supset \exists z^{\ulcorner}z \ll y \wedge z \lfloor x^{\urcorner})$

The corresponding set of axioms for a temporally modified part-concept are obtained in the fashion of

MA1*t* $(\Box)(\forall t^{\ulcorner}x \ll_t y \supset \sim y \ll_t x^{\urcorner})$

from MA1–3, i.e. by suffixing the part-predicate and slipping a universal quantifier '∀*t*' in between the last necessity operator and its matrix. The need to consider these two cases separately arises from our discussion in Part II.

It might be wondered whether the introduction of modal considerations would lead to a distinction, parallel to that between continuants and occurrents, between objects which do and objects which do not have *modal* parts. What would an object with modal parts be like? Following the analogy with spatial and temporal parts, we must take it to be an object which is *extended* in the modal dimension, so that the whole object is not present in any one possible world, but rather only a part of it is present in any world in which it figures at all. For example, a class of individuals drawn from more than one world may be such that in any world only a subclass of these individuals exists. Excluding those worlds where none of the class exists, we have a case where the class has modal parts. A natural use for this conception is to

[18] Cf. RUSZA 1981 for some of the issues involved.

distinguish what C. I. Lewis called the *comprehension* of a term, which is the class of all individuals, actual or not, to which the term may apply, from its *denotation*, which is the class of individuals to which it actually applies. The *extension* of a term at a world is the intersection of its comprehension with that world's domain.[19] The analogy is somewhat strained because the same part (non-empty subclass of the comprehension) may be in more than one place (world). There is also perhaps an analogy between continuants, which can be wholly present at different times, and objects for which genuine trans-world identities hold, on the one hand, and occurrents, at any one time only one part of which is wholly present, and objects for which counterpart semantics holds. In Lewis's (D., not C. I.!) counterpart semantics, no individual exists in more than one possible world, but it is connected by cross-world relations of similarity to other objects, and its most similar object in a world is its counterpart there. If we take an object *a* and form the class of all its counterparts (assuming this can be done), then the resulting class might be seen as a whole having different modal parts.

In neither of these cases is it clear that there is more than a somewhat strained philosophical analogy, and in any case talk of modal parts is unlikely to clarify the more immediate modal–mereological issues with which we are dealing. Accordingly, we shall restrict the part-predicate to an 'intra-world' use. This may be formulated as

MA4 $(\Box)(x \ll y \supset \mathrm{E}!x \wedge \mathrm{E}!y)$

with its counterpart for continuants

MA4*t* $(\Box)(\forall t\ulcorner x \ll_t y \supset \mathrm{Ex}_t x \wedge \mathrm{Ex}_t y\urcorner)$

(Compare both these formulae with FA6 of Chapter 2.) The connection between them may be made by noting that it is a necessary truth for continuants that to exist is to exist at some time: if '*C*' stands for 'is a continuant' this may be expressed as

$$(\Box)(Cx \supset . \mathrm{E}!x \equiv \exists t\ulcorner \mathrm{Ex}_t x\urcorner)$$

Following the terminology of Fine,[20] we may call the principles MA4(*t*) *Falsehood Principles* for the two part-relations. The name derives from the fact that in a world in which an object does not exist, any sentence to the effect that it is or has a part is there false, and not

[19] LEWIS 1944: 238 (1970: 305).
[20] FINE 1981a, 293 f. Cf. FORBES 1985: 30 ff.

true or truth-valueless. It follows from MA4(t) that a falsehood principle applies also to certain other mereological predicates, in particular to overlapping, but not to disjointness, if this is defined in the usual way as the contradictory of overlapping. Objects will be disjoint at a world if either or both of them do not exist there. This is not very disturbing, but a stronger disjointness predicate may be defined by stipulating that objects exist together in a world in which they are disjoint. Such a predicate trivially satisfies a falsehood principle, but it is now only a contrary, not the contradictory, to overlapping as usually defined. The situation here is rather like that which we faced when forging definitions for continuants; we may define a strong sense of disjointness which requires the simultaneous existence of both objects.

If we now define '$<$' as usual (cf. SD1 of Chapter 1), then we do not have a falsehood principle for it, since it is a feature of the Fine–Forbes semantics we are using that self-identity applies to all objects in all worlds, irrespective of whether they exist or not. Similarly, the predicate '$\leq$' (SD15 of Chapter 3) is necessarily reflexive. For a reflexive part-predicate which can be used in non-extensional mereology and for which a falsehood principle holds, we must modify SD15 to

$$(\Box)\ (x \leq y \equiv \mathrm{E!}x \wedge \mathrm{E!}y \wedge (\exists z \ulcorner z \ll x \urcorner \supset \forall z \ulcorner z \ll x \supset z \ll y \urcorner) \wedge (\sim \exists z \ulcorner z \ll x \urcorner \supset x \ll y \vee x = y))$$

The continuant version is obtained as usual by slipping '$\forall t$' after '($\Box$)' and modifying all the modifiable predicates (note that 'Ex' replaces 'E!', but '$=$' is not modifiable).

The mereological operators of sum, product, difference, upper bound, and universe were introduced in Chapter 1 by means of definite descriptions. So expressions for them are not modally rigid. For example, 'U' designates in any world *whatever it is* in that world which has everything as part there. This by no means entitles us to assert that the referent of this term in one world is identical with its referent in some other world. Such identifications are plausible only in very special circumstances. Suppose, for instance, we have a classical extensional mereology which is atomistic, and suppose we have two worlds in which the very same atoms exist. Then since (by the sum principle) every subclass of the class of all atoms forms an individual, all the sums of atoms, including, *a fortiori*, the maximal sum, exist in both worlds. On the assumption that *there are no other objects apart* *from atoms and sums of atoms*, we could then identify the referents of 'U' in both worlds. This additional assumption rules out the possibility that some objects exist not just in virtue of the matter (atoms) they contain but also in virtue of their configuration (the relations the atoms have to one another, roughly). Prima facie this additional assumption appears false, since what distinguishes a man from the mass of matter making him up is precisely the configuration of this matter, which is accidental to the matter but essential to the man. It is because the additional assumption has the effect of reducing all objects to the same level, namely heaps or masses of matter, that we may term it a kind of materialism. (Cf. the criticisms in Chapter 3.)

The non-rigidity of mereological operators has corrective consequences for our discussion of coincidence in Chapter 3. Suppose we were to take '$a+b$' as being modally rigid whenever 'a' and 'b' are rigid, and suppose further that it is contingently true that $a \leq b$, in that in some alternative world a and b both exist but a is not part of b. On extensional principles we should expect it to be contingently true that $a+b = b$; but if '$a+b$' is rigid the identity is necessarily true, so there is no such alternative world. Since such a case appears unproblematic, we conclude that '$a+b$' is not modally rigid. So even if '$a \leq b$' is contingently true, a sentence containing '$a+b$' is to be analysed so that this term disappears: this applies in particular to '$a+b = b$', which is to be treated like 'Reagan = the President'. It therefore seems appropriate to regard sums and the like as lacking in autonomous identity conditions. This has repercussions for our discussion of Wiggins's example of the cat Tibbles and the sum Tib + Tail. If Tibbles does not lose her tail, what is wrong with the contingent but analysable identity 'Tibbles = Tib + Tail'? This, however, overlooks a chief point of the example, which is that if Tibbles *does* lose her tail, the divergent histories of Tibbles and Tib + Tail show that they cannot ever have been identical, even at times where they coincided. All our discussion of the modal behaviour of complex mereological terms shows in this case is that there is a lack of parity between the cat, which is a natural object with autonomous identity conditions, and the sum. This lack of parity is an obvious feature of the example anyway, and does not undermine the case for coincidence, which rests firmly on cases where coincident objects are equally natural, like the statue and the bronze, or the person and his body.

We have already seen that non-modal mereology must approach continuants and occurrents differently because of the different forms of mereological predication appropriate to each. The divergence extends, as we have seen, to the modal case. When we investigate the modal mereological behaviour of continuants and occurrents, we find that these diverge even more markedly. This justifies discussing questions of essential parts for each case separately.

7.3 Essential Parts for Continuants

Everyday propositions about what parts an object must have come in different sorts, of different logical strengths. Take the proposition that every bicycle must have a wheel. One way to understand this is that it says that any possible object in any possible world has a wheel as part if it is a bicycle. Overlooking the temporal dimension for the moment, we may express this as follows (where we are assuming that whatever is a bicycle exists):

$$\Box \forall x \Box (Bx \supset \exists y \ulcorner Wy \wedge y \ll x \urcorner)$$

This is a rather weak claim. It tells us nothing about whether a given bicycle could have had different wheels from the ones it actually has. So it is not an essentialist claim. If we now take time into account, we may specify the claim in at least three different ways:

$$(\Box)(Bx \supset \forall t \ulcorner B_t x \supset \exists y \ulcorner W_t y \wedge y \ll_t x \urcorner \urcorner)$$

$$(\Box)(Bx \supset \exists y \forall t \ulcorner B_t x \supset W_t y \wedge y \ll_t x \urcorner)$$

$$(\Box)(Bx \supset \exists y \exists t \ulcorner B_t x \wedge W_t y \wedge y \ll_t x \urcorner)$$

These three specifications make quite different claims. The first says that whenever something is a bicycle, it has a wheel then, but this need not be the same wheel at all times (since there are two, one can be changed and the bicycle still be a bicycle during the change). The second says that there is something which is a wheel and part of the bicycle at all times at which it is a bicycle. This rules out a complete change of wheels. And the third makes the much weaker claim that every bicycle must have some wheel at some time, and seems the most acceptable of the three.

The example concerns artefacts, concerning which there is some doubt as to their having essential parts. In addition the necessity in question appears to be no more than analytic—something which never had a wheel as part would by definition not be a bicycle. So let us turn to an example of a natural kind in our search for essential parts.

The proposition that every helium atom must contain two protons as parts may be understood in just the same way as the bicycle case. But in this case we may state something stronger. Any given helium atom could not have had protons other than the ones it actually has. Its component protons are essential parts of it. If we confine our attention to the more common isotope ^{4}He, we can say that the neutrons of any atom of this isotope are likewise essential parts of it. On the other hand, it is not essential to any of the four nucleons of a helium 4 atom that it be part of this atom. The essential dependence is asymmetric. So, taking account of the time factor, and picking a particular helium atom *h* and proton *p* of it, we may assert that

$$\Box(\text{E!}h \supset \forall t\ulcorner \text{Ex}_t h \supset p \ll_t h\urcorner)$$

The proton is thus not just an essential sometime part of the atom; it is an essential permanent part of it. Note that the formula above is compatible with the atom's existing intermittently. If the proton detached itself from the other nucleons and then reattached itself again, it is open to us to say that the same atom comes back into existence. Whether such a process is physically possible is not something we need decide.

That every helium atom necessarily has a proton as an essential part may then be expressed by

$$(\Box)(Hx \supset \exists y\ulcorner Py \wedge \Box(\text{E!}x \supset \forall t\ulcorner \text{Ex}_t x \supset y \ll_t x\urcorner)\urcorner)$$

This example may be generalized to all atoms: their protons are essential parts. Although for a heavy atom the loss or gain or exchange of a proton makes relatively less difference in mere numerical terms, it changes the element. Strictly speaking, even the loss or gain of a neutron must be regarded as spelling the end of a given atom. For electrons, the matter is different. As physicists and chemists use the term 'atom', an atom's electrons are not central to its identity in the way that its nucleons are.

It seems, then, that even in this simple case we may distinguish between those parts of an individual which are essential to it and those which are not. It is our chief contention in this section that such a distinction between essential and accidental parts applies everywhere, or almost everywhere, among continuants. In particular, it continues to apply as we move from simple natural kinds to organisms and then to artefacts, but the proportion and importance of essential parts diminishes as we move through these cases. Further, the boundary between essential and accidental parts becomes progressively vaguer.

Organisms, like chemical elements, fall into natural kinds, but the boundaries of the extensions of these kinds are inherently vague for evolutionary reasons. Species diverge and develop out of one another without sharp discontinuities. The existence of marginal, defective, and mutilated members of biological species provides a rich fund of counter-examples to putative essentialist theses in biology. Nevertheless, it is still possible to find essentialist mereological truths for organisms. The weakest sort is like 'Every man must contain at least one cell (DNA molecule, carbon atom)'. These have the same form as the bicycle-wheel example and are thus rather uninteresting. Can we find parts which are essential to organisms in the way in which a proton is essential to an atom?

One essentialist thesis about organisms which has aroused much discussion is Kripke's claim that organisms could not have originated from any other initial germ cells than those from which they did in fact originate. This is called the 'necessity of origin' thesis.[21] So an oak must have come from the acorn it came from, a human being must have come from a given zygote, which in turn must have come from just the sperm and egg from which it came. To cover cases of sexual and asexual reproduction under one heading, Forbes calls any organic antecedent from which an organism develops in this way a *propagule* of the organism, and gives good arguments for the necessity of origin thesis.[22] Some organisms may have more than one propagule (like the zygote), others may have just one, and sometimes two or more organisms may share the same propagule, as do two amoebae which come from a parent by splitting.

Suppose, then, that we accept the necessity of origin thesis. Does this give us cases of essential *parts*? Certainly they are not essential permanent parts in the way the proton is an essential permanent part of the atom, since propagules typically cease to exist quite early in the development of an organism. A zygote ceases to exist when it divides, as does an amoeba. In the case of the amoeba, the parent cannot be a part of the offspring, both because it ceases to exist when they come into being, and because it is too big. We have partial continuity, but of matter. The zygote case is more promising. If we can assert that the zygote *is* initially part of the organism (indeed an improper part), then we can assert (letting '*o*' and '*z*' stand for 'Organism' and 'Zygote')

$$\Box(\mathrm{E!}o \supset \exists t \forall t' \ulcorner t' \text{ is not later than } t \supset .\ \mathrm{Ex}_{t'} o \supset z <>_{t'} o \urcorner)$$

[21] KRIPKE 1972: 312 ff., 1980: 110 ff.; cf. also McGINN 1976, FORBES 1985: ch. 6.
[22] FORBES 1985: 133.

which says that *o* and *z* must coincide in the earliest phases of the latter's existence. From this we may derive the weaker but more perspicuous

$$\Box(E!o \supset z \leq o)$$

to the effect that *o* must have *z* as a sometime part.

Can we justifiably say that the zygote is (initially) part of the organism? McGinn has argued that the two are in fact identical, and that 'zygote', like 'child' or 'adult', is a phase-sortal, i.e. a term applied to an individual during a certain part of its life. But biologists do not use 'zygote' in this way. A zygote is a certain kind of single cell arising by fertilization and ceasing to exist (normally, not always) by division.[23] The most unstrained description is that the organism and zygote come into being together at the time of fertilization, since they then coincide, while the zygote alone (normally) ceases to exist when it divides, though the organism keeps on. It would be implausible to say the zygote predates the organism, since the decisive genetic discontinuity takes place at fertilization. We may thus conclude that it is essential to an organism arising by sexual reproduction that it has its zygote as initial improper part. Because further development of the organism is dependent on circumstances and is subject to termination, it is plausible that the *only* sometime essential parts of an organism are *original* parts. And not all original parts are essential. A carbon atom more or less in the cell wall is neither here nor there.

If we restrict our attention not just to organisms *per se*, but to adult or developed organisms of a species, then we may find essential parts which must accompany the organism throughout its adult life. Most of the parts of an organism are accidental to it because of the flux of minute parts. Further, the advances of transplantation surgery have shown that many organs are not essential, although their replacement is a delicate business. But is there at least one part of an adult organism which it *cannot* do without? It is natural to think of the brain of a man as an example. If medical techniques advanced far enough for brains to be transplanted out of human bodies into other bodies or artificial support systems, would this be a brain transplant or rather a body transplant? We seem to face three alternatives in such a case. (1) We may deny that any man survives the change. (2) We regard the brain, not the rest, as the basis for reidentifying the man. (3) We regard the

[23] MCGINN 1976: 133. Cf. FORBES 1985: 134 ff. against McGinn's view.

body, not the brain, as the basis. This last case is by far the least plausible, and it is the only one which would provide a counter-example to the claim that his brain is essential to a man.

That we are less firm ground here is due to the open texture of the concept *man*. We have very little idea what future medical and technical advance may make possible, so the concept has not been faced with the kind of test cases which would fix its boundary in this respect. Perhaps we might say simply that their brains are essential to men *as we know them*, and leave speculation to the science fiction writers.

When we turn to artefacts, it is again original parts which are the best candidates for being essential. It is hard to conceive how an object could have as essential a part which was attached at some time after the object had come into being, since the part could quite easily have ceased to exist in the meantime, without prejudice to the already existing artefact. In the case of an object assembled from pre-existing components, one may claim that, at least for some of the more important parts, *that* object would not have existed had *this* not been part of it. The term 'important' is deliberately vague, but we seem to find clear cases of essential parts well away from the indeterminate zone. Its blade is essential to a sword, its frame to a bicycle, it engine to a car. These essential parts may in turn have parts which are not essential to them or their wholes—we just take some atom near the surface. But an essential part of an essential part is an essential part of the whole. If the engine block is an essential part of the engine, and this of the car, then the block is essential to the car. It can be checked that transitivity is a property of the strong essentialist form

$$\Box(\mathrm{E!}a \supset \forall t \ulcorner \mathrm{Ex}_t a \supset b \leq_t a \urcorner)$$

and likewise of the essential original part-form given above for the case of the organism and the zygote. Essential parts of both these kinds appear to be met with. Permanent essential parts are like the bicycle's frame. Its wheels might be original but not permanent essential parts: it could survive wheel replacement, but had another wheel been used in the initial construction, this particular bicycle would not have been made. If we are here less then certain what to say, it is probably because the elements of vagueness and convention are more markedly present for artefacts than for natural objects.[24]

[24] As stressed by WIGGINS 1980: 90 ff.

7.4 Mereological Essentialism for Continuants

Chisholm's doctrine of mereological essentialism was discussed above in connection with mereological constancy, but as its name implies, it is primarily a modal doctrine. It was noted in § 5.3 that Chisholm in fact formulates two distinct doctrines, a weak one and a strong one. The weak principle may be formulated as follows

WME $(\Box)(x \ll y \supset \Box(\mathrm{E!}y \supset \exists t\ulcorner x \ll_t y\urcorner))$

Here, quantification is implicitly restricted to what we might call 'proper continuants'. The adjective is required because Chisholm is prepared to countenance continuants for which mereological essentialism does not hold (*entia successiva*), but accords these the status of logical constructions. Note the optical similarity between WME and RCA3 of § 5.3. The stronger thesis is

SME $(\Box)(x \ll y \supset \Box(\mathrm{E!}y \supset \forall t\ulcorner \mathrm{E}x_t y \supset x \ll_t y\urcorner))$

(cf. RCA3′ of § 5.3.) Again, quantification is restricted to proper continuants.

Both versions are propounded by Chisholm, who draws on his theory of *entia successiva* to account for common or garden cases of part-replacement and also for cases of apparent contingency of parts. We have already rejected his view on continuants which change their parts; in so far as Chisholm regards the mereological constancy of proper objects as following from mereological essentialism, we have, as he would see things, already rejected the latter also. However, it is worth looking separately at the modal thesis.

Chisholm begins his case with a disarmingly simple example:[25]

> Let us picture to ourselves a very simple table, improvised from a stump and a board. Now one might have constructed a very similar table by using the same stump and a different board, or by using the same board and a different stump. But the only way of constructing precisely *that* table is to use that particular stump and that particular board. It would seem, therefore, that that particular table is *necessarily* made up of that particular stump and that particular board.

It is worth noting that, in *Person and Object*, this example constitutes the *sole* direct evidence that Chisholm brings for the principle. In addition, he cites a number of authorities from the history of philosophy and shows the implausibility of three rival views, to which

[25] CHISHOLM 1976: 146.

we come presently. The table example is, as it stands, indeed convincing. If we call the stump '*a*', the board '*b*', and the resulting table '*c*', the claim is that

$$\Box(\mathrm{E}!c \supset \forall t \ulcorner \mathrm{E}x_t c \supset a \ll_t c \wedge b \ll_t c \urcorner)$$

Now even if a sum of *a* and *b* were to have existed despite their not being joined (which Chisholm would deny) or by virtue of them being joined in a way inappropriate for a table (which he probably would not), such a sum would not be *c*, which is a *table* made by joining stump and board in the appropriate way. Admitting a sum irrespective of joining entails recognizing the superposition but non-identity of it and *c*. As it is, since *a* and *b* can be disjoined and rejoined, Chisholm faces the dilemma of intermittent existence, which he himself sees as a problem.[26]

Let us agree with Chisholm's analysis of this example. This by no means commits us to the general doctrine, since it would seem that he has chosen a particularly favourable case and illegitimately generalized from it. In itself, all our acceptance of the example commits us to is the existence of essential permanent parts, which we are happy to countenance. If we complicate the example only slightly, its plausibility vanishes. How is the stump fixed to the board? In Chisholm's case, it would have to be either simply laid on or else somehow dovetailed to it. Suppose instead that the joiner uses four screws. Then according to mereological essentialism, these four screws are also essential parts of the table. Had other screws been used, or had nails and glue been used instead, we should not have had this table but a distinct one, varying slightly in its parts from this one. Nor could we remove one screw and replace it by another without destroying the table. Now I think this is implausible. The board and stump may be essential, but then they belong to the essential kernel of the table. The minor parts could belong to the periphery. Chisholm's view is that any proper object is all essential kernel. But if we consider some vastly complex machine like an aircraft, containing literally millions of components, he is committed to saying that the merest screw or washer is as essential to the identity of the machine as its wings and fuselage. Of course this does not correspond to our actual individuative practice for aircraft, but Chisholm can reply that this is not strict.

Suppose, however, that there are no further components to the table apart from its board and stump, which, we agreed, are essential

[26] Ibid.: 90.

parts of it. They are far from being the only parts, however, since the board and stump themselves have further proper parts, and other parts of the table are brought into existence by joining the board and stump, parts straddling the join. But suppose the joiner, in fashioning the board, had shaved an extra fraction of a millimetre off one side. Would this lost shaving prejudice the identity of the table? Chisholm must answer that it does. Not only the table, but also the board (if it is a proper object) is subject to mereological essentialism, down to the smallest part, which means that a chip or scratch shaving even a few molecules off the table spells its destruction. Finally, there are the parts which arise by joining. Had the stump been joined to the board in a slightly different position, or at a slight angle of rotation from the angle of actual joining, different parts would arise by joining, so we should not have had the same table. In all of these cases, as Chisholm has to admit, untutored common sense goes against mereological essentialism. There is the further problem, unforeseen by common sense, that it may not be possible at the micro-level to determine exactly which microparticles are parts of the table and which are not—in which case we must admit, according to Chisholm's principle, that there simply is no such proper object as the table.

None of these objections shows up a contradiction in Chisholm's view; they serve rather to show that proper objects are further removed from our experience than the table example might suggest, raising thereby the question whether they are worth the expense in securing mereological essentialism.

Chisholm contrasts his view with its polar opposite, extreme mereological inessentialism. In fact, he gives two accounts of what this phrase means, which are not equivalent. The first is the view that for any whole *w*, *w* might have been made up of any two things whatever.[27] We may plainly agree that this is absurd. The other version is:[28]

> There is no *x* and no *y* such that *y* is necessarily such that it has *x* as a [proper] part

This may be read as denying WME or (more weakly) as denying SME. In either case, our acceptance of Chisholm's table example and our discussion of examples in the previous section show that we have no quarrel with his dismissal of the positions. However, the views are

[27] Ibid.: 147. [28] Ibid.: 149.

only contrary, not contradictory to the two versions of mereological essentialism, so there is (at present) room between them.

Chisholm further rejects two compromise positions suggested by Plantinga.[29] The first is:[30]

> For every x and y, if x is ever a part of y, then y is necessarily such that x is part of it at some time or other

which is in fact nothing other than WME, so we happily agree with the rejection. The second is:[31]

> For every x, y, and t, if x has y as a part at t, then x necessarily has y as a part at t

This is new: we may call it *temporally rigid mereological essentialism.* The best way to formulate it is probably

TRME $\quad (\Box)(\forall t \ulcorner y \ll_t x \supset \Box(E!y \supset y \ll_t x)\urcorner)$

This position allows mereological variation, but places it under the upwards restriction that an object could not have failed to have at any time the parts it actually then had, although it might have had more parts then. This restriction seems one-sided, and the position is subject to the further objection that it makes an object's time of existence essential to it, whereas it seems that many objects might come into existence and (more particularly) cease to exist at times other than those at which they actually do. If a person is a proper object and has parts, then that person must live at least as long as she actually does. We can therefore agree with Chisholm in rejecting this alternative also.

There is a further *via media* rejected by Chisholm, namely the view that in general some parts are essential to their wholes and others are not. This is our view, so we must examine Chisholm's arguments against it. His rejection is based on two points. First, it is shown that one kind of argument for the view is fallacious. Secondly, it is claimed in general that the denial of mereological essentialism leads into philosophical difficulties. Taking these in turn, Chisholm remarks that arguments like[32]

Necessarily all cyclists have feet
ergo All cyclists are necessarily such that they have feet

are fallacious. We already recognized this move as illegitimate. But his

[29] In PLANTINGA 1975. [30] CHISHOLM 1976: 150.
[31] Ibid. [32] Ibid.: 149.

second example is

Necessarily all cars have engines
ergo All cars are necessarily such that they have engines

In this case, not only are both premiss and conclusion true, but it seems that we can get a formally valid argument by adding the plausible additional premiss that whatever is a car is essentially a car.

(1)	$(\Box)(Cx \supset \Box(E!x \supset Cx))$	A car must be a car
(2)	$(\Box)(Cx \supset \exists y \ulcorner Ey \wedge y \ll x \urcorner)$	Necessarily, all cars . . .
(3)	$(\Box)(Cx \supset \Box(E!x \supset \exists y \ulcorner Ey \wedge y \ll x \urcorner))$	from 1, 2 by logic

That a car must have an engine is true; that it must have the particular engine it does have is not *so* obvious. The engine is an important part of the car, and certainly a change of both engine and body at once would be car replacement rather than car repair. As we pointed out in Chapter 5, there is both vagueness and room for conventions concerning the essential kernel of artefacts. However, by changing the example from cars and their engines to helium atoms and their protons we get a clear case of essential parts. So Chisholm's point is not ultimately telling.

The second, dialectical point is relevant to our uncertainty whether the particular engine a car has is essential to it. Without mereological essentialism, Chisholm argues, one object *a* could have been composed of all the parts of another object *b*, and vice versa. The point relates closely to the Ship of Theseus problem and the Paradox of the Heap. The latter, ancient paradox tells us that even a single grain of wheat is a heap of wheat, since whenever we have a heap, we still have a heap upon removal of a single grain. Applied to the carriages of Plato and Socrates, if we consider what happens when we successively exchange small and insignificant parts of the carriages so that we still have qualitatively similar carriages, then eventually we can exchange all the parts. So even if we do not carry the procedure out, one carriage *might* have had all the parts of the other, and vice versa. Chisholm blocks this by rejecting the first move: as soon as the smallest part is exchanged we no longer have the same carriage in the strict and philosophical sense, which is simply a consequence of mereological essentialism.[33]

[33] It is unimportant here whether we formulate the argument counter-factually or in terms of actual part-exchange.

It is certainly legitimate to point out this consequence of denying mereological essentialism, since it is one which many will find unpalatable. One may quibble slightly with the example, since carriages might have some important parts belonging to an essential kernel. However, if there are any objects which do not have an essential kernel, Chisholm's point would apply to them, which is enough. In my view, the problem here mentioned is a special case of a far wider one, namely the existence of vague predicates and, correlatively, degrees of truth for sentences involving such predicates. There is nothing especially mereological about this issue, since it appears with predicates, for example colour terms like 'red', which have nothing to do with part and whole. The difficulties arise not because of any illegitimacy of vague predicates as such, but rather because the bivalent logic of exact predicates is illegitimately applied to sentences containing them.[34]

If we take the heap example, the important sentence

> For any heap x and any grain y of x, the result of removing y from x is also a heap

is *slightly* less than perfectly true, so that from a clearly true sentence 'a is a heap' we obtain, by repeated applications of Modus Ponens, a chain of sentences of gradually diminishing truth-value, until we reach the false 'b is a heap', where b is obtained from a by removing all grains but one. The characteristic truth-value drops occur only in penumbral zones: in the case of huge heaps we can remove one grain and still have a heap. Similarly, for carriages we might remove a scratch or a screw and still have the same carriage. But because of the truth-value drop, we block the move even from '*Any* part can be exchanged and we still have the same carriage' to '*Every* part can still be exchanged and we still have the same carriage'. Once we get beyond a certain grey area, we no longer have the same carriages, but two new hybrids. And when all parts have been exchanged, we can happily say we have *re-created* from two hybrids the original two carriages (understanding 'carriage' here in the matter-constant sense, of course).

So Chisholm's dialectical point can be countered by admitting freely that ordinary objects do not have exact mereological identity and survival conditions, but accepting this as part of our conceptual apparatus which it would be impractical to throw away. Now this is

[34] Cf. FORBES 1983: 241 ff., 1985: 169 ff., and GOGUEN 1969, whom he cites.

not metaphysically binding, since it is open to a metaphysician to insist, as Chisholm does, that our actual conceptual scheme is inappropriate for metaphysics. He would sooner relegate most or all of the objects of experience to the status of logical constructions and preserve bivalence for proper objects than admit the vague, moderate, partial mereological essentialism we advocate, which allows us to treat objects of experience more directly, although it has the consequence, pointed out above, that the identity conditions for objects may sometimes be partly indeterminate. What inclines us to pursue our approach rather than Chisholm's, even though they might ultimately be complementary, is the fact that most of the paradigmatically individual objects studied by the natural sciences, such as stars, planets, organisms, and volcanoes, are *both* natural units or wholes *and* mereologically variable, which seems to leave room for something intermediate between a strict Chisholmian continuant and an object whose parts are arbitrarily or conventionally determined: a naturally unified object. In the last chapter we shall attempt to give a schematic account of what it is for an object to be naturally unified.

Might there nevertheless be kinds of continuant for which mereological essentialism holds without reservations? Richard Sharvy thinks so.[35] He distinguishes first between a count sense of 'part', in the predicate 'is *a* part', and a mass/plural sense of 'part', in the mass predicate 'is part of' and the plural predicate 'are part of'. He then claims that, while mereological essentialism fails to apply to the former, it does apply to the latter. A car may indeed change its parts, but whatever is part of or are part of something is/are part of it at all times and in all possible worlds in which the latter exists:[36]

For example, the matter of my car will not change any part when I replace my carburetor. Rather, that quantity of stuff will become scattered, and will cease to satisfy the description 'matter of my car'. Where is the snow of yesteryear? All around us and in us.

With the distinction between a count sense of 'part' and a mass/plural sense we have, of course, no quarrel. However, Sharvy supposes that the latter sense is that of 'some of', the sub-mass or subclass relationship. This is an assumption which we also made in Chapter 4, but we revised it in the light of the linguistic facts in §6.4, where it appeared as quite natural to say, for example, that the metal in the

[35] SHARVY 1983b. [36] Ibid.: 237.

carburettor is part not only of the stuff making up the car but also of the car itself. So it is not the mass/plural sense (or senses) of 'part' to which mereological essentialism applies, but rather to the *narrower* sub-mass and subclass senses. Mereological essentialism applies, in other words, to masses *of matter* (i.e, where constituent change is ruled out), and to classes or sets in the extensional sense, where membership change is ruled out.

Let us spell this out for classes in greater detail. We use Leśniewskian predicates, but quantifiers ranging only over existents, and a distinctive run of singular variables for perspicuity. Since the predicate 'ε' ('is one of') is purely formal, it is tenseless, and this goes for the other predicates defined in terms of it. The essentialist principle we are looking to establish is *subclass rigidity*. We write it out in full:

MEC $\quad \Box \forall a \Box \forall b \Box (a \sqsubset b \supset \Box (\mathrm{E}b \supset a \sqsubset b))$

What plausible modal principles for classes allow us to derive MEC? Once again we can draw on work of Fine and Forbes, which, although done with a modal form of Zermelo–Fraenkel set theory rather than low-brow class theory in mind, nevertheless dissects the extensional nature of sets in a way which can be carried over, since it is independent of the details of set formation.[37] Assuming we accept the necessitations of the usual Ontological definitions of existence (E) and singular existence (E!) in terms of 'ε', we get a falsehood principle for 'ε' quite trivially:

$$(\Box)(x \varepsilon a \supset \mathrm{E!}x \wedge \mathrm{E}a)$$

and similarly for '$\sqsubset$' (again using the usual Ontological definition):

$$(\Box)(a \sqsubset b \supset \mathrm{E}a \wedge \mathrm{E}b)$$

The first substantial modal principle we need is *membership rigidity*:[38] a class could not have members other than those it has in any world in which it exists:

MR $\quad (\Box)(x \varepsilon a \supset \Box (\mathrm{E}a \supset x \varepsilon a))$

This is exactly analogous to the mereological essentialist claim for subclasses in general. The problem is how to get from the particular case of membership rigidity to the general one of part (i.e. subclass) rigidity.

It turns out that we need a further principle which is partly

[37] FINE 1981*b*, FORBES 1985, ch. 5. Cf. also PARSONS 1983: ch. 11.
[38] FINE 1981*b*: 179.

constitutive of the idea that a class can be nothing more than its members. We call it the principle of *class existence.*[39] Expressed in terms of worlds, it says that if in some world there is a (non-empty) class, and all the members of this class exist in some *other* world, then the class itself exists in this other world. To express this in modal language without mention of worlds requires that we have a subscripted 'actually' operator which picks up the anaphoric reference to the 'other world'. Writing this operator 'A', the principle then runs:[40]

CE $\Box \forall a \Box_1 (\Diamond(\mathrm{E}a \wedge \forall x \ulcorner x \varepsilon a \supset \mathrm{A}_1(\mathrm{E!}x)\urcorner) \supset \mathrm{E}a)$

So if *a* is part of *b* in some world, then in any world in which *b* exists, so do all its members (by membership rigidity), and since these members include all those of *a*, it follows by class existence that *b* exists in this world with *b* and is there part of it. As is often the case, writing the derivation out in terms of worlds is by far the easiest way to check its validity. So we may derive MEC: mereological essentialism holds for classes, taking '$\sqsubset$' as the part-relation. The principles MR and CE have an immediate plausibility independent of our wish to derive MEC. Indeed, CE is less problematic for classes in the low-brow sense than its analogue for sets, since if we have all the members, we *thereby* have the class (as many), whereas for the case of sets there still remains something to do, namely to form (comprehend) the set (class as one) from its members.[41]

The plausibility of mereological essentialism for classes carries over to atomistic Mereology, since to each class of atoms there is exactly one individual which is its sum, and there is a one–one correspondence between individuals and classes of atoms. In the case of atomless Mereology, the essentialist principles appear to have no independent justification; they are, rather, constitutive of what extensionality for part–whole comes to.

[39] Ibid.: 180 calls the corresponding principle 'set existence'.

[40] On 'actually' cf. FORBES 1985: 90 ff. and the literature he cites. Subscripted 'actually' operators are due to PEACOCKE 1978: 485 ff. The way the subscripts pick up the previous reference makes this look like quantification over possible worlds under a different notation—but perhaps an interpretation can be given which does not commit one to the existence of worlds, e.g. a substitutional one.

[41] Cf. § 4.10. PARSONS 1977 expresses doubts about this step. Cf. FORBES 1985: 113 f. on 'automatic set formation'.

7.5 Essential Parts for Occurrents

In an examination of essentialist theses about events, Forbes has argued that, since the case for a reductionist analysis of events is open in a way that one for continuants is not, there is room for scepticism about whether there are any irreducible *de re* truths about events, for instance, that their participants or their kind and time of occurrence are essential to them.[42] To the extent that events and other occurrents are genuine objects, there appears to be no reason to deny that there are essential truths about them, for example that the particular event that was Princip's killing of Archduke Franz Ferdinand essentially involved Princip and the archduke. The problem appears to be that because occurrents are usually specified by means of descriptions, like 'Princip's killing of the Archduke', it is unclear to what extent occurrents have autonomous identity conditions across worlds—in plain English, it is unclear what a given event might otherwise have been. An alternative world in which Princip kills the archduke with a bomb rather than a gun is one in which the above description is fulfilled, but I do not think we should want to identify the one killing with the other, because the two differ too much in kind. But suppose we take an alternative world which is just like ours except that the sequence of bullets in the assassination weapon is different, so that it is a different bullet which is fatal. Would we then identify the events or not? We may well be uncertain. The only guide seems to be that the nearer a counter-factual situation is to the actual one in terms of the nature of the event and the participants, the more inclined we are to accept an identification. But this suggests that identification is a matter of weighted similarity and convention rather than a hard and fast fact. But identity is not conventional and does not come in degrees. This suggests, then, that the proper semantics for modal discourse about temporal objects forswears trans-world identity and works instead with counterparts *à la* David Lewis.[43] If this is so, then strictly speaking no occurrent could exist in more than one possible world, so mereological essentialism for temporal objects would be true, but trivial.

However, it appears to be too strict to deny that one and the same event or process or state could exist in more than one world. The identity of worlds depends on *everything* that exists and takes place in

[42] FORBES 1985: 205 ff. [43] LEWIS 1968.

them, and we can imagine two worlds which are exactly alike in respect of a certain event and its causal antecedents, differing only in respect of something causally unconnected with this event, such as an event on the other side of the universe. Another very important case for the study of time and counter-factuals concerns worlds whose initial histories are exactly alike, but which later diverge. We can imagine a world which is indistinguishable from the actual one up until a certain time. There is no reason here to deny the identity of the events in the initial, exactly like stage, which can then be regarded as literally shared.[44]

This may be granted without denying mereological essentialism for occurrents, since our inclination to identify part of one world with part of another will be crucially disturbed if we find the two supposedly alike stages differ in respect of some property of some event, other than relational properties connecting the event to things that happen after the point of divergence. To find a counter-example to mereological essentialism for occurrents, we need to find an occurrent and a part of it which is inessential to it, as the flourish on a signature is inessential, or a mole on someone's cheek. Take the event of someone's signing their name with a flourish. Is the part which consists in the execution of the flourish an inessential part of *this* event? But if it had not taken place, it would surely have been a different signing. Or again, is it essential to a particular performance of Bartók's Fifth String Quartet that the violist scratch his nose during his initial three bars' rest in the *scherzo*? Certainly not—but then the nose-scratching is not *part* of the performance, any more than are the digestive processes simultaneously going on in the violist. The performance has its limits, and these events lie beyond them. Where we might be uncertain, this is not because of inessentialism but because of vagueness, which is a different matter. So it still looks as though mereological essentialism holds for occurrents.

Note that the converse essentialism for containers does not hold. According to this, it is essential to an occurrent that it be part of just those occurrents of which it actually is part. But while the performance of the *scherzo* of Bartók's Fifth Quartet is an essential part of the complete performance, the performance of the *scherzo* need not be part of a complete performance, since we may have worlds which

[44] Cf. this approach to worlds in general in BRODY 1980.

diverge after the *scherzo* is played, as when the violist's peptic ulcer catches up with him during the *prestissimo*. There are nevertheless certain occurrents which are essentially such as to be part of some other occurrent. This applies particularly to temporal boundaries. The particular temporal boundary which is the *Titanic*'s sinking beneath the waves could not have taken place except as part of the complex process of the *Titanic*'s going down. Boundaries provide an example of existential dependence, which we discuss in greater detail in the ensuing chapter.

At this point it may be wondered whether we have not undermined the argument of §3.2.4, which said that the move to four-dimensional objects does not save extensionality of parts, because a cat process and a cat-part process may actually have the same parts but need not. If they are occurrents, either mereological essentialism is false for these after all, or at least one of the expressions for the processes is a description and fails to yield autonomous identity conditions.

Now if a cat has autonomous identity conditions, then it is reasonable to suppose its four-dimensional surrogate has them. And it will only be a surrogate, a *cat*–process, if its modal behaviour parallels that of the cat. A cat could have lost its tail, or have died earlier than it did, in which case the cat–process would have had parts other than those it has. So either mereological essentialism does not hold for occurrents after all, or the cat–process is not an occurrent. Faced with this dilemma, I would take the first horn rather than give up the argument in §3.2.4. This could then be used as an argument against mereological essentialism for occurrents.

But the second horn seems to me to be right. Mereological essentialism has been defended for occurrents as we know them, as part of an ontology including also continuants, where most of the expressions designating occurrents are descriptions which mention some continuant or other. It is by no means clear that spreading a cat out over the fourth dimension gives us an occurrent in this sense. The defender of four-dimensional objects cannot allow their identity conditions to turn on those of continuants, since then continuants have not been successfully dispensed with. The fact that cat-processes could not obey mereological essentialism is, then, a good indication that they cannot be processes in the sense with which we are familiar, which deepens the mystery of the alternative ontology still more.

We must conclude that modal mereology distinguishes continuants and occurrents even more than non-modal mereology.

7.6 Normal Parts

It is very easy (especially for philosophers) to forget that between the mere fact of something's being part of another, and something's being an essential part of another, there is an important third case. This is when an object which is *normal of its kind* always has such and such parts. There is as yet no formal theory of normal mereology, and we content ourselves here with informal remarks. It was, as usual, Aristotle who picked up the connection between mereology and the idea of a normal or well-formed member of a kind. For Aristotle, an object is called *mutilated* if it is physically connected, if it normally has parts, which are both salient and unregenerable, which belong in certain positions of the whole, and if one or more of these parts is missing.[45] This notion clearly makes most sense for organisms, although it applies elsewhere as well, and it is in many respects perhaps more important than that of the out-and-out essential part.

The idea of a normal or well-formed member of a kind has been used by Nicholas Wolterstorff to explicate the notion of a musical work of art. A norm kind is a kind for which it is possible that there be well-formed and improperly formed exemplars.[46] An exemplar of a kind can only be improperly formed when one or more of its normal parts is missing or misplaced, or if there are more parts of a kind than there normally are. The notion is therefore somewhat wider than Aristotle's concept of mutilation. The application of Wolterstorff's concept to music works allows us to say that a performance of a certain work with a false note is still a performance of *that* work, albeit a slightly imperfect one, and not, as Goodman has argued, a performance of a different work. Goodman, of course, has his reasons, which may strike, so to speak, a familar note:[47]

> The practising musician or composer usually bristles at the idea that a performance with one wrong note is not a performance of the given work at all; and ordinary usage surely sanctions overlooking a few wrong notes. But this is one of those cases where ordinary usage gets us quickly into trouble. The innocent-seeming principle that performances differing by just one note are instances of the same work risks the consequence—in view of the transitivity of identity—that all performances whatever are of the same work. If we allow the least deviation, all assurance of work-preservation and score-preservation is lost; for by a series of one-note errors of omission, addition,

[45] *Metaphysics* Δ27, 1024a.

[46] WOLTERSTORFF 1980: 56.

[47] GOODMAN 1969: 186 f.

and modification, we can go all the way from Beethoven's *Fifth Symphony* to *Three Blind Mice.*

Goodman here emerges as the Chisholm of musical works, and for the very same sort of reason. Like Chisholm, he recommends a metaphysically tough line, keeping bivalence and strict identity conditions at the expense of deviation from normal usage. The answer is also the same as that to Chisholm: musical works do not have strict identity conditions, so it is bound to lead to difficulties if we apply inferential procedures which are appropriate only for a strictly bivalent logic. A performance of Beethoven's Fifth is *more or less* perfect. One with two notes wrong is less perfect than one with only one of these wrong; one with a salient note (the first note of an exposed entry, for example) wrong is less perfect than one with a note tucked cosily away in the middle harmony wrong. 'Perfect' here has little to do with artistic qualities. Scores leave enough indeterminate for performances of the same notes to differ widely in artistic value, and it may be that—anathema though this may be to purists—some works may be artistically improved by deliberate deviation from the score. The perfect performance of a work is therefore one which complies in every minute particular with the instructions contained in the score. Perfect performances may, as we said, nevertheless be phenomenally different. A performance may approach or approximate such perfection to a greater extent and still remain a performance of that work, provided we take the work as a norm kind in Wolterstorff's sense, with the perfect performance as norm. This norm provides a *focus* for admissible deviations, to which there are no exact boundaries. 'Performance of Beethoven's Fifth', like 'red', has a clear focus and a vague periphery of application, and while the periphery may shift somewhat from generation to generation, the focus stays put.[48]

Similar remarks apply to other 'repeatable' works of art, such as literary and dramatic works; a complete performance of Shakespeare's *Hamlet* may be the exception rather than the rule, and most literary works appear in editions with misprints, so the normedness has nothing to do with statistical frequency.

As we had occasion to remark briefly in Part II, works of art have a slightly exceptional position among artefacts, in that deviation from

[48] For numerous valuable remarks on the relationship between a musical work and its performances, and the indeterminacy involved, cf. INGARDEN 1962: ch. 1.

the mereological norm is more restricted than for objects of use. Concerning general artefact species, there is rather little use for the notion of a normal part. A razor as such must have a cutting edge, but this is an analytically essential part. Yet, even here, something may be more or less of a razor, depending on how good its cutting edge is at cutting. However, when we turn to artefacts produced to a design or plan, such as mass-produced articles, the idea of a normal part is very apt. Grossly malformed or mutilated exemplars of a brand of automobile or an edition of a book are precisely what good quality control is intended to weed out. Despite this, many more or less flawed exemplars get onto the market, and the perfect exemplar may again be the exception.

Among organisms there is no such thing as *the* perfect member of a species, if by that we mean that any other perfect member must be physically indistinguishable from it. As with performances of a musical work whose score leaves certain elements of performance undetermined, two members of a biological species may be equally good exemplars, and have no betters, but still differ from one another in numerous respects. The focus of a biological species is itself not a point, but an imprecisely marked volume in a multi-dimensional space of possible variations, with less well-formed exemplars falling off from this norm along one or more dimensions of variation, until the penumbral zone of uncertain membership is reached. By virtue of the vagueness of even the focus, it is possible for what is normal among members of a biological species to shift over a number of generations, for here what is normal is not kept fixed by an eternal blueprint like the musical score, but rather shifts with the population itself, and is not independent of statistical frequency. If most men are now between five and six feet tall, whereas a million years ago they were mostly between four and five feet tall, then what is a normal height for a man has changed in that time. It is this which allows species to evolve and diverge, and shows us that it is inappropriate to apply a strict bivalent logic to species considered over many generations. Even if at any one time the members of the species are nearly all clearly marked off from one another, over the generations what were mere variations either side of a single norm develop into divergent foci, so that in the temporal dimension species emerge as plural normed continuants without exact membership conditions. Of course, the biologist is interested in factors other than the purely anatomical ones, but he must take account of normal parts.

Even among non-biological natural kinds there may be room for norms. Deuterium atoms deviate from the norm for hydrogen by virtue of having two nuclear neutrons where the normal hydrogen atom has only one. As with biological species, and divine blueprints aside, what is normal is here a purely statistical matter.

7.7 Appendix: Quantified S5

The following brief account of the modal logic presupposed in our discussions in this chapter and the next is based on works by Fine and Forbes,[49] though the principal ideas, of course, go back to Kripke. The reader is referred to these works for details.

7.7.1 Models

If we have a set Γ of sentences of quantified modal logic, an S5 model $\mathfrak{M}$ for these sentences is given by

(1) A non-empty set W of possible worlds, from which one world w is selected as the actual world.
(2) A non-empty set D of possible objects.
(3) A function d from W to the power set of D which assigns a subset d(u) of D to each world u in W. This is the set of objects which exist in u. Every object in D is in some d(u).
(4) For every *n*-placed atomic predicate occurring in Γ, it is specified which *n*-tuples in D^n are in the extension of it at u, for each u in W.
(5) For each individual constant occurring in some sentence of Γ, an assignment of a member of D to it as its referent.

A sentence is true in $\mathfrak{M}$ iff it is true at w in $\mathfrak{M}$. A sentence (argument) is S5-valid iff it is true in all S5 models (the conclusion is true in all models in which the premisses are all true). For a recursive definition of truth at a world for a particular language, see the works by Fine and Forbes.

7.7.2 Axioms

We shall not give the syntax of the object language in detail; we may suppose it is of the customary sort. In addition to the metalinguistic symbols used in Chapter 2, we have $\Box$ and $\Diamond$ for necessity and

[49] FINE 1978*a*, FORBES 1985.

possibility respectively. We modify the runs of metavariables as given in § 2.2.1 as follows:

singular individual terms: s, s_1, s_2, . . . (leaves t, etc. for times if needed)
singular individual variables: x, y, z, x_1, . . .
(singular) world variables: u, v, w, u_1, . . .
world constant: ω (for the actual world)
arbitrary (n-place) predicate parameters: P, Q, R, P_1, . . . (it is assumed the number of arguments following such a metavariable tells us how many places it has)
formula parameters: A, B, C, A_1, . . .

We have distinguished (somewhat laboriously) between singular individual variables and singular world variables. We stress 'singular' because it would be of advantage to extend the treatment to neutral variables in the fashion of Leśniewski. We shall not carry out such an extension here. Another extension would introduce a third sort of variable for times.

Definition

QS5D1 $\Diamond A \equiv \sim \Box \sim A$

Axioms

QS5A0 Any set sufficient for classical (non-modal) propositional logic

1 $\Box(A \supset B) \supset . \Box A \supset \Box B$

2 $\Box A \supset A$

3 $\Diamond A \supset \Box \Diamond A$

4 $\forall x A \supset . \mathrm{E!}s \supset A[s/x]$ (universally quantified if s is a variable)

5 $\forall x \mathrm{E!}x$

6 $\forall x \ulcorner A \supset B \urcorner \supset . \forall x A \supset \forall x B$

7 $A \supset \forall x A$ (x not free in A)

8 $\Diamond \mathrm{E!}s$ (s not a variable)

9 $s \approx s$ (as for A4)

10 $s_1 \approx s_2 \supset . A \supset A[s_1//s_2]$ (as for A4 if either term is a variable)

Rules

QS5R1 $A, A \supset B \vdash B$

2 $A[s/x] \vdash \forall x A$

3 $A \vdash \Box A$

This system is due to Kit Fine,[50] who proves its completeness with respect to S5 models as given.

7.7.3 Translation Between Modal Logic and Predicate Logic With Worlds

We give here just the easy direction, from modal logic to predicate logic quantifying over worlds. The reverse translation is not so easy. For details, see once again Fine and Forbes.

The basis of the translation scheme is the idea of the *relativization* of a formula to a world. We add to the language for non-modal predicate logic a run of world variables and a constant denoting the actual world. For each n-placed predicate $P\xi_1 \ldots \xi_n$ (except identity) we obtain an $n+1$-placed world-indexed predicate $P^w\xi_1 \ldots \xi_n$, and we add the world-indexed existence predicate $E!^w\xi$. The relativization $[A]^w$ of a formula A to a world w is then defined recursively as follows:

(1) $[Ps_1 \ldots s_n]^w = P^w s_1 \ldots s_n$

(2) $[E!s]^w = E!^w s$

(3) $[s_1 \approx s_2]^w = s_1 \approx s_2$

(4) $[\sim A]^w = \sim([A]^w)$

(5) $[A \wedge B]^w = [A]^w \wedge [B]^w$

and likewise for the other truth-functional connectives

(6) $[\forall x A]^w = \forall x \ulcorner E!^w x \supset [A]^w \urcorner$

(7) $[\Box A]^w = \forall u \ulcorner [A]^u \urcorner$

The translation of A is then simply $[A]^\omega$. By this procedure, modal formulae are replaced by non-modal formulae, with atomic predicates relativized to worlds and quantification over worlds. A modal formula or argument is S5 valid iff its translation is valid in first-order predicate logic with identity.

[50] FINE 1978*a*: 131 ff. The minor differences are due to our having separate runs of bound variables and parameters.

8 Ontological Dependence

8.1 Introduction

Implicit in the criticism of mereological theories which permit the existence of arbitrary sums is the view that something cannot count as an individual, as *one* object, unless it is possessed of a certain degree of integrity or internal connectedness. There are problems with this criticism, however. The first, which we have already mentioned, is that arbitrary sums may be bizarre or ontologically wasteful, but they are algebraically neatening and appear not to lead to contradictions. More seriously, the properties said to keep individuals on the right side of respectability are hardly very clear. Integrity or connectedness, whatever they are, presumably come in degrees. Yet it seems to be counter-intuitive to suppose that there are degrees of being an individual. Also, the lesson of history seems to be that philosophers who have talked about integral or organic wholes, Gestalten or *Ganzheiten*, have never managed, even where they tried, to get very clear about what such a whole was, which may suggest that the whole area is better left alone.

This would be well enough were it not that both ordinary and scientific thinking is marked through and through by an interest in picking out or individuating units or single objects, whether there are clear limits to what counts as an individual or not. This applies whether we think about the most obvious kinds of individual like organisms and well-demarcated material bodies or the more subtle individuals such as molecules, cells, and genes uncovered by the sciences. Whatever these have that makes them natural objects of interest and study, is not something arbitrarily wished on them by our activity. There are natural units which we discover as such, and what naturally makes them unitary is also something for us to find out. It does not help that modern philosophy is still under the spell of Frege's view that an individual is anything that has a proper name.[1] For there are proper names and other definite terms that are not singular. Certainly the use of a singular term signifies an *intention* to refer to a

[1] This myth is beginning to crumble even within analytic philosophy. Not only are empty names now recognized, but some philosophers such as Black and Sharvy are aware of the existence of plural reference (cf. also MANSER 1983: 74 f.)

single object. But such reference need not be successful. The usual mereological calculi throw up singular terms in abundance: the question is which of them designate. A suggestion with a long pedigree is that integrity belongs to objects which are spatio-temporally continuous. This idea obviously arises by reflecting on the difference between integral material bodies and scattered aggregates. But, as has been pointed out often enough, the very bodies which seem most continuous are in fact microscopically scattered,[2] while we are also willing to allow the one-ness of many macroscopically scattered objects, such as countries like Indonesia, or galaxies. Very few philosophers would be willing to adopt the radical stance of Leibniz, who regarded nothing as an individual which was not absolutely atomic.[3] It will turn out that spatial or spatio-temporal continuity or near-continuity is only one form of integrity, and probably not the most important.

All of these arguments may be accepted without one's thereby being disposed to welcome arbitrary sums. Particularly where the summands come from different categories, the sums appear nothing more than a philosopher's concoction. How a number, a sigh, a poem, a person, a galaxy, and a thunderstorm could comprise and exhaust a single individual seems beyond understanding. Cross-categorial sums as such are, however, not unknown in philosophy. A body and the events which befall it are intimately connected, and one—albeit hopeless—way to try and understand the mysterious four-dimensional blocks of Chapter 3 might be as a mereological sum of the body and its life. Some realist theories of universals may construe propertied particulars as sums of concrete bearers and abstract properties. Whether such sums are found illuminating or not, they are at least not arbitrary.

The problem facing the proponent of a distinction between arbitrary sums and non-arbitrary unified wholes is to give an account of the latter which is neither too vague and unsystematic to be of use (as is almost all of the *Ganzheitsphilosophie* written in German between the wars) nor too specific to cover all cases, as are

[2] Cf. QUINE 1960: 98: 'Even the tightest object, short of an elementary particle, has a scattered substructure when the physical facts are in.'

[3] Cf. the *Monadology*, opening remarks, and §8.7 below. The later account of complex substances and the *vinculum substantiale* fits ill with this position, though it may be said that complexes have phenomenal rather than real status in the correspondence with Des Bosses, and Leibniz was never very happy with them.

contemporary discussions of personal identity. Finding the *via media* is by no means easy, and I am conscious of the provisional nature of my own efforts, which take as their starting point certain remarks of Aristotle and Husserl.

What is it about an integrated whole in virtue of which it hangs together in the way that an arbitrary sum does not? In schematic outline, it seems as though the explanation must refer to some kind of relation between the *parts* of an integrated whole which they have to one another, while such relations do not obtain between the parts of a mere sum. This notion of 'hanging together' is obviously not a specific one, but should be given a formal characterization. That is attempted in the next chapter. What kinds of relation are involved in integrated wholes? Here we are on much more slippery terrain. Aristotle gives certain samples, notably continuity, rigidity, uniformity, qualitative similarity, being of like kind, being of like matter.[4] This covers more than just the most obvious case: (as he notes) it includes the unity not only of individuals but also of species and genera. Nevertheless the list is only impressionistic, and it does not mention one of the most important connecting relations, namely (efficient) causation. Husserl, whose *Logical Investigations* treat of many Aristotelian problems without mentioning the Philosopher by name,[5] essays there a definition of a *pregnant whole* as one, all of whose parts are connected with one another by relations of *foundation*.[6] This term 'foundation' is not current in modern philosophical discourse, and requires explanation, but we may roughly characterize it as meaning a kind of existential or ontological *dependence*. With this it becomes clear that the sort of wholes Husserl is considering have much to do with the traditional philosophical notions of *substance*, as being objects which are in some sense ontologically *independent*. So, while not all natural wholes are pregnant in Husserl's sense, discussion of this rather special case is of relevance to traditional philosophical concerns. The concepts of foundation and ontological dependence which we require to explicate it turn out to have interest of their own, and since having an essential part is a special case of being ontologically dependent, it links up with the topic of the previous chapter. Since other connecting relations of integrated wholes may also be styled kinds of dependence,

[4] *Metaphysics* Δ, 6, 1015b f.

[5] On the Aristotelian background to Husserl's theory, cf. SMITH and MULLIGAN 1982.

[6] HUSSERL 1984: *investigation* III, § 22. Cf. SIMONS 1982*a*: 141.

we outline first the general idea of dependence, before turning to ontological dependence and its ramifications.

8.2 Dependence in General

There is no one way in which one thing is dependent on another or others. The terms 'dependent' and 'independent' are syncategorematic. Sentences of the form '*a* is dependent on *b*' or 'αs are dependent on βs' are incomplete. We must add something about the type of dependence, or the respect in which one thing is dependent on another. 'Dependence' connotes merely a rough schema, a *form* of connection between objects or kinds of objects, which may be variously filled out.

To try to find a common thread running through various notions of dependence, we may consider the following approximate definitions:

(1) Person *a* is financially dependent on person *b* iff *a* cannot be solvent unless *b* is solvent (financial dependence).
(2) Skill *A* is dependent on skill *B* iff *A* cannot be mastered unless *B* is mastered (practical dependence).
(3) Judgement *a* (e.g. that roses are red) is dependent on idea *b* (e.g. the idea of roses) iff *a* cannot occur to a person unless *b* occurs to that person (Brentanian psychological dependence).
(4) Person *a* is dependent on drug *B* iff *a* cannot survive unless doses of *B* are regularly administered (physiological dependence).
(5) Clause *a* of this contract is dependent on clause *b* iff *a* cannot apply unless *b* applies (legal dependence).
(6) Detonation *a* of this mine is dependent on its priming *b* iff *a* (the detonation) cannot take place unless *b* (the priming) previously takes place (causal dependence).
(7) Accident *a* (e.g. this whiteness) is dependent on substance *b* (e.g. this piece of paper) iff *a* cannot exist unless *b* exists (ontological dependence).
(8) Proposition *p* is dependent on proposition *q* iff *p* cannot be true unless *q* is true (one form of logical dependence).
(9) The pressure *P* of a fixed mass of ideal gas is dependent on its temperature *T* and volume *V* iff *P* cannot vary in value unless at least one of *T* and *V* varies in value (functional dependence).
(10) The life-expectancy *E* of a male born in 1960 is dependent on the country of his birth iff males born in 1960 cannot differ in their life-expectancy unless they were born in different countries, *other things being equal* (one form of statistical dependence).

In each case the labels we have attached are suggestive only, and it is not implied that all things nameable by such labels have the form given. In particular, there are several somewhat different forms of logical dependence,[7] and we shall soon see that there are different notions of ontological dependence. Certain of the examples involve reference to kinds ((2), (4), signified by upper-case letters), but the schema we wish to abstract involves only particulars as terms of the dependence relation, although kinds also play an essential role. The examples (9) and (10) also involve what we might call variable quantities or magnitudes, where we are dealing with relationships between the values such magnitudes have. The theory of the various kinds of functional dependence is very underdeveloped,[8] and since it has no great role to play in our considerations of integrity we also leave it on one side. The scheme we may extract from the remaining examples is then

DS1 *a* is dependent as *F* to *G* on *b* iff *a* cannot be *F* unless *b* is *G*

where '*a*' and '*b*' hold places for definite terms and '*F* ', '*G*' hold places for monadic predicates.[9]

We have left the pattern in semi-English because the exact force of 'cannot . . . unless—' varies. It can mean, for example, psychological, practical, physical, legal, deontic, metaphysical, logical impossibility, according to the case. If we replace the 'cannot . . . unless—' scheme by the handier

Necessarily: if *a* is *F* then *b* is *G*

we leave just as much open because 'necessarily' may vary in force.

8.3 Ontological Dependence

Whereas logical dependence concerns the relationships between propositions or other truth-bearers, ontological or existential dependence, our chief concern here, is about relationships between objects in general. The ontological dependence of one object on another or

[7] Cf. SIMONS 1981*a*.

[8] Cf. the papers by Grelling and Oppenheim, in particular GRELLING 1987, and my commentary on them, SIMONS 1987*a*. It must be admitted that there is still no adequate theory of functional dependence.

[9] In SIMONS 1982*a* I use variables for common nouns, which is closer to Husserl. The use of monadic predicates is here a concession to simplicity; it would take us too far afield to explain why common nouns should be treated differently from other monadic predicates (for an argument, cf. GUPTA 1980).

others is one of *de re* necessity: the object itself could not exist if others did not exist. The causal dependence of events and states on other events and states seems in many cases to be less compelling, itself dependent on contingencies and laws of nature. However, it may well be that there are cases of causal dependence which are also cases of ontological dependence in the strict sense. Causal dependence, where it concerns the existence (or occurrence) of events, differs from other forms of ontological dependence perhaps only in the strength of its associated 'must'. Other forms of causal dependence seem better subsumed under the heading of functional dependence, concerning the variation of quantities.

Taking our clue from the second schema, we may attempt a first definition of the ontological dependence of an individual a on another individual b as follows; anticipating § 8.6, we call it *weak foundation*:

$$\Box\,(E!a \supset E!b)$$

This is a rather weak notion for at least two reasons. For one thing, every individual is ontologically dependent on itself. For another, if some individual exists necessarily (a proposition on which we shall remain agnostic, but which we therefore do not rule out of court), then necessarily every individual is ontologically dependent on it. Now in traditional theology God exists necessarily *and* everything on earth is ontologically dependent on God. But it is not just because God exists necessarily that everything depends on him, but rather because he created everything on earth, and presumably nothing could have come into being without his creating it. Varying the example, if we hold that certain abstract objects like numbers, universals, or propositions exist necessarily, we do not want to say that therefore everything depends ontologically on them. Pythagoreanism aside, I am not ontologically dependent on the number 23. Since we are confining attention only to concrete entities, we can eliminate these unwanted cases by excluding the case that b exists necessarily. Excluding also the trivial case of self-dependence, we obtain the more useful concept of *weak rigid dependence*. For 'x is weakly rigidly dependent on y' we write '$x \mathrel{\not{7}} y$':[10]

DD1 $(\Box)(x \mathrel{\not{7}} y \equiv x \neq y \wedge \sim \Box E!y \wedge \Box(E!x \supset E!y))$

Cases such as those from the previous chapter where an object has an essential proper part are thus cases of ontological dependence,

[10] The notation is chosen to be like that of SIMONS 1982*a*, although the definitions have been changed in the meantime. $\not{7}$ is meant to suggest 'f' for 'foundation'.

assuming (as is plausible in all the cases dealt with) that the essential part does not exist necessarily.

It is my view that this concept of ontological dependence and those related to it have wide application in the discussion of philosophical issues. We may take just two examples. On one form of subjective idealism, the physical world is ontologically dependent on me. (In weaker forms, it is dependent on subjects in general, but this case can be accommodated also among our definitions.) Similarly, in one form of realism about universals, universals are generically dependent ontologically on instances. Another case of ontological dependence, that of accident upon substance, is discussed at greater length below. Philosophical discussion of issues involving one form or other of ontological dependence would therefore benefit from more exact analysis of the concepts involved. For example, the most detailed modern discussion of the realism/idealism problem, that of Ingarden,[11] gains much by his careful discrimination of different senses of dependence and independence.

The form of ontological dependence defined above is confined to individuals. However, it can be broadened to include the cases of masses and classes by using the broadest existence predicate, covering all three cases. It is obvious how this goes. For example, the existence of a class depends on that of each of its members, and accordingly on that of each of its non-empty subclasses (cf. the membership and subclass rigidity conditions from § 7.4.)

In giving instances of ontological dependence for particular objects we have to be careful how we formulate the point. Take the proposition that the largest satellite of Jupiter cannot exist unless Jupiter exists. In one sense this is true: if Jupiter did not exist, nothing could be its satellite. But Ganymede is Jupiter's largest satellite, and it is false that Ganymede could not exist unless Jupiter exists. We can distinguish the two senses by taking account of scope. For a sentence of the form $\iota x \ulcorner Sxa \urcorner \not\rightharpoondown a$ to be true in one sense, we require only the necessary truth of '$\exists x \forall y \ulcorner Sya \equiv y = x \urcorner \supset \mathrm{E}!a$', which holds trivially if the predicate S satisfies a falsehood principle. On the other hand, if we give the description wide scope we get

$$\exists x \ulcorner \forall y \ulcorner Sya \equiv y = x \urcorner \wedge \Box (\mathrm{E}!x \supset \mathrm{E}!a) \urcorner,$$

[11] Cf. INGARDEN 1964–5: §§ 10 ff. Of Ingarden's concepts of dependence (he gives four), two can be handled by the conceptual apparatus given here. The idea of being produced or created is not here dealt with, and Ingarden's concept of *heteronomy*, of an object's having the foundation of its being in something else, remains opaque to me.

which is false. The dependence of Ganymede on Jupiter is merely *notional*; it could not be *described* as Jupiter's largest satellite unless Jupiter existed. This is important, because the forms of expression for ontological and notional dependence are easily confused with one another: both can take the form 'an *F* cannot exist unless a *G* exists'. It is thus important to regiment these two notions differently.[12]

First, note that a given individual might be only generically rather than rigidly dependent on objects of a certain kind. For example, a man cannot exist for an instant unless the pressure at his epidermis is above a certain minimum. He is therefore ontologically dependent on matter which exerts such pressure. But which particular parcel of matter (which consignment of gas molecules) actually presses down on his epidermis is not important. Similarly, a man cannot exist without carbon atoms, but *which* carbon atoms exist (and are part of him) is immaterial.[13]

We may express the generic dependence of the individual *a* on objects of sort *G* by

$$\Box(\mathrm{E!}a \supset \exists x \ulcorner Gx \wedge x \not\Vdash a \urcorner) \wedge \sim \Box\, \exists x Gx$$

(We assume that *Gs* do not exist of necessity to rule out unwanted trivial cases.) Note that if $a \not> b$ and *b* is essentially a *G*, then *a* depends on *Gs* in this latter sense too. So now we define what it means for *Fs* to depend generically on *Gs*: they do so if necessarily every *F* is generically dependent on *Gs*:[14]

$$\text{DD2} \quad \Box(F \not> G \equiv \Box \forall x \Box (Fx \supset \Box(\mathrm{E!}x \supset \exists y \ulcorner Gy \wedge y \not\Vdash x \urcorner)) \wedge \Diamond \exists x Fx \wedge \sim \Box \exists x Gx)$$

In this sense human beings are dependent on carbon atoms. Now *notional* dependence is typified by such statements as 'There cannot be a husband without a wife'. Such a statement may be rendered by the

[12] Cf. CAMPBELL 1976: 30, where a distinction is made between dependence in existence and dependence in description. Here, too, concepts of dependence and mereology come briefly into contact. HINCKFUSS 1976 tries to show that ontological dependence, as distinct from notional, does not occur, but the attempt has such a small basis of examples that it does not succeed.

[13] It is important that the dependence here is not just causal. Something that could withstand zero pressure without special equipment could not be a man, nor could something that did not contain carbon. That causal and essential necessity overlap is powerfully argued in WIGGINS 1980: chs. 3 f.

[14] The clause saying *Fs* may exist again rules out trivial cases. In the case of '$\not>$' we did not need a clause '$\Diamond\mathrm{E!}a$', since this is automatically guaranteed by our modal semantics.

simpler

$$\Box \forall x \Box (Hx \supset \exists y^{\ulcorner} Wy \wedge x \neq y^{\urcorner})$$

(Compare the initial weak formulation of the claim that every bicycle must have a wheel at the beginning of § 7.3.) Assuming that *F*s may exist and *G*s need not, and that *F*s are notionally dependent on *G*s in this last sense, this still does not suffice to show that *F*s are generically ontologically dependent on *G*s, as the husband/wife case shows. There can be no husbands without wives, but for no husband is it true that *he* (that very man) could not exist unless there were wives—the institution of marriage need not have existed among human beings. Notional dependence of *F*s on *G*s implies ontological dependence only under the special further condition that *F*s are essentially *F*s: $(\Box)(Fx \supset \Box(E!x \supset Fx))$.

While every case of ontological dependence (of the form defined in DD2) is a case of notional dependence, the converse is not true. However, there is one way of *generating* a case of ontological dependence out of every case of notional dependence, which maybe taken if we adopt a suggestion of Kit Fine and introduce the idea of a *qua* object.[15] Fine's idea is rather controversial, but it is well worth considering because it affords a unified framework within which to discuss such philosophically important issues as the relationship between a material object and its matter, and the relation between an act and its associated bodily movement. The idea is this: suppose we have an object *x* which has the property *F*. Then, in addition to plain *x* we also have *x qua F*, or *x under the description F*. The underlying object *x* is called the *basis* and the property *F* is called the *gloss*. The object *x qua F* is, according to Fine's account, always distinct from its basis *x*.[16] This seems to me to be too strong a position; it would for instance distinguish not only *x* and *x qua F* but also (*x qua F*) *qua F*, and so on, for further additions. Further, it would distinguish also *x* from *x qua self-identical*, *x qua object identical with x*, and also *x qua F* for essential properties of *x*. Be that as it may, the adjustment required is not so important as the fact that *qua* objects, in most cases unlike their bases, have their glosses built into them (which is not the same as saying they have their glosses as properties). The existence condition

[15] FINE 1982.
[16] 'A qua object is distinct from its basis' (ibid.: 100).

for *qua* objects looks like this:[17]

$$\Box \forall x \Box \forall t \Box \, (\mathrm{Ex}_t(x \; qua \; F) \equiv \mathrm{Ex}_t x \wedge F_t x)$$

There are many things that can be said about this startling formula,[18] but for present purposes we note only that *qua* objects are what yield an ontological dependence for a notional one. For if we define the derived property of being a *qua-F*-object thus:

$$(\Box)(^{*}Fx \equiv \exists y \ulcorner Fy \wedge x = y \; qua \; F \urcorner)$$

then every $^{*}F$ is essentially an $^{*}F$, and if Fs are notionally dependent on Gs then $^{*}F$s are ontologically dependent on Gs in the sense of DD2—provided Fs may exist and Gs not (if Fs can exist, so can $^{*}F$s.)

It is natural to view the teeming universe of *qua* objects with some suspicion, but some candidate *qua* objects are old friends: what is a fist if not a hand *qua* clenched, or a statue if not some bronze *qua* shaped in such and such a way? Another case is that matter-constant ship from Part II—this matter *qua* (having the form of a) ship. The same goes for any Aristotelian compound of matter and form. As the examples make clear, *qua* objects lead a precarious existence, and exist intermittently in cases where their basis alternately gains and loses the gloss of the corresponding *qua* object. *Qua* objects and their bases are obviously superposed, and it is natural to say the basis is an improper part of the corresponding *qua* object. Whether the converse is also true depends on our attitude to the coincidence principle. On the basis of §6.7 we would suggest that the *qua* object is at least weakly included (improperly) in its basis. As Fine notes, *qua* objects inherit many properties from their bases,[19] which explains their conceptual usefulness as well as their perceptual indistinguishability from their bases at any time at which they exist.

[17] 'The qua object x qua φ exists at a given time (world-time) if and only if x exists and has φ at the given time (world-time)' (ibid.).

[18] Fine notes the similarity of his views with those of Aristotle (104), but another historical comparison is worth making. WHITEHEAD and RUSSELL (1927: 43) have a theory of the truth of atomic sentences which includes Fine's *qua*-objects as a special case (the monadic one). Consider the two-placed case. The two objects a and b exist, and a stands in the relation R to b iff there exists a *complex*, a-in-the-relation-R-to-b. Call this complex '$[aRb]$'. Then we have

$$\mathrm{E!}[aRb] \equiv \mathrm{E!}a \wedge \mathrm{E!}b \wedge aRb$$

which is the analogue of Fine's existence condition. Russell and Whitehead's complexes went on to haunt Wittgenstein's *Tractatus*: cf. SIMONS 1985a.

[19] For the inheritance conditions, see FINE 1982: 100 f. Inheritance is also important in the theories of DOEPKE 1982 and CHISHOLM 1976 (*entia successiva*).

Despite having some interesting applications, the theory is at present insufficiently tested for us to be clear which of the putative *qua* objects are to be taken seriously. The use of the particle '*qua*' is also something of a hindrance, since in many cases it clearly does not serve to form new singular terms from old. The sentence 'Socrates *qua* philosopher was successful but Socrates *qua* husband was unsuccessful', far from telling us about two *qua* objects based on Socrates, simply tells us two things about Socrates—that he was a successful philosopher and an unsuccessful husband. So we shall make no further use of the idea in what follows.[20]

The only form of dependence we have so far described at the level of kinds is generic dependence, but it is possible to define the rigid dependence of members of one species on members of another:

$$\text{DD3} \quad \Box(F \not\rightharpoonup ! G \equiv \Box \forall x \Box (Fx \supset \exists y \ulcorner \Box (E!x \supset E!y \wedge Gy \wedge x \neq y)\urcorner) \wedge \Diamond \exists x Fx \wedge \sim \Box \exists x Gx)$$

In this sense every smile is dependent on a face, for not only can no smile exist unless some face exists, but every smile is rigidly dependent on *its* face; it could not have existed except as a configuration of just the face it is in. Helium atoms are likewise rigidly dependent on protons.

It is an interesting question whether there are any species which are self-dependent in this strong sense, i.e., are such that one cannot have just one member of the species in any possible world. Notional self-dependence is common-place: the species sibling, spouse, associate, river-bank are all such that if one thing falls under them, so does another. But as in the husband/wife case, it is clear that this falls short of ontological dependence, since the objects which are siblings, spouses, etc. could have existed without being siblings, spouses, and so on. There are cases of self-dependence where the necessity falls short of the logical; for instance, magnetic poles seem to always go around in pairs, and quarks in threes. Likewise, it falls slightly short of being a logical truth that every human must have human parents, for evolutionary reasons. If we accept abstract objects, too, we have cases where no one of a kind could exist alone—numbers, for example. But

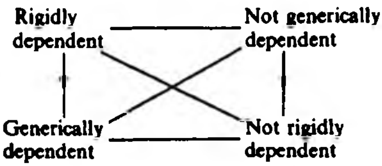
[20] FINE 1982: 100 makes it clear that many phrases of the form '*x qua* φ' are not referential. There is a large medieval literature on '*qua*'; the operation was known as 'reduplication'. Cf. BÄCK 1987.

for concrete objects it is hard to find a clear example. Perhaps there is a deep metaphysical reason for this.

8.4 Independence, and More Dependence

With each of the concepts of dependence we have mentioned we may correlate concepts of independence. So *a* is ontologically independent of *b* if *a* can exist without *b*, if there is a possible world in which *a* exists and *b* does not. Realists will say that the physical world could exist without me, indeed without any subjects. Platonists will say that universals may exist whether or not they are exemplified. And so on. The stronger the concept of dependence involved, the weaker the correlated concept of independence, and vice versa.

It is also of interest to consider absolute (de-relativized) concepts of dependence and independence. So *a* is rigidly dependent (*tout court*, *schlechthin*) iff it is rigidly dependent on something, and independent iff it is dependent on nothing, and likewise for generic dependence. We thus get a square of opposition for objects (below). The strongest form



of absolute independence thus belongs to an object which is not generically dependent. The weakest version of generic dependence we can concoct for an individual is expressed by

$$\Box(E!a \supset \exists x \ulcorner x \neq a \urcorner)$$

Negating this, we arrive at the startling

$$\Diamond(E!a \wedge \forall x \ulcorner x = a \urcorner)$$

An object is independent in this sense if it could be all there is. This is such a strong condition that it is not clear that even God would fulfil it (what of his thoughts?). At all events, any object fulfilling this lonely condition would, in the world in which it is alone, be absolutely atomic, for the proper parts of an object are distinct from it. This might explain some of the appeal of the idea that God is a monad.

The very strength of this concept of independence limits its usefulness. But there is something attractive about the idea that an object could comprise the whole universe, and this idea can be given a less rarefied interpretation by requiring, not that an object be the only thing there is, but merely that there be nothing disjoint from it. We may express this using weak inclusion as

$$\Diamond(\mathrm{E!}a \wedge \forall x \ulcorner x \subset a \urcorner)$$

What, then, does it mean for an object to be *not* independent in this weaker sense? Negating back again, we find it means that the object is essentially such as to have something disjoint from it:

$$\Box(\mathrm{E!}a \supset \exists x \ulcorner x \wr a \urcorner)$$

This condition should not be misunderstood as entailing that whatever is disjoint from *a* is spatially outside it. *a* might be a continuant which cannot exist without a process going on in it: the process is disjoint from *a* by necessity—a continuant and a temporal object are categorically incapable of sharing a common part— but the process is not spatially outside the continuant in which it occurs.

Brentano called an object which could be the whole universe a 'something-for-itself'—*ein Etwas-für-sich*—and defined it in the following way:[21]

$$\Diamond(\mathrm{E!}a \wedge \sim \exists x \ulcorner a \ll x \urcorner)$$

This formulation is equivalent to the previous one only under certain assumptions. What is important here is not so much how to render all the possible versions equivalent as to notice that, because of the possibility of substituting either '<' or '≤' for '⊂' in the above definition of absolute independence, we have several slightly differing conditions for dependence and independence. It is not easy to decide in each case which concept is most suitable in application to a particular case. Since not a great deal seems to turn on spelling out all the possible variants for our present purposes, we shall give representative definitions using the medium-strength part-concept '≤' only, and note alternatives only when they are important for the case under discussion.

One obvious form of dependence which is stronger than all those covered by the previous remarks is the dependence of an object which is essentially part of another:

$$\Box(\mathrm{E!}a \supset a \ll b)$$

This entails but is not entailed by the object's not being an *Etwas-für-*

[21] Cf. BRENTANO 1933: 170, 1981: 128. Cf. CHISHOLM 1978: 208, 1982: 14.

sich:

$$\Box(\mathrm{E!}a \supset \exists x \ulcorner a \ll x \urcorner)$$

Corresponding to our previous definitions of rigid and generic dependence for individuals and members of species, we can give the following analogous definitions (DD4 defines *strong rigid dependence*):

DD4 $(\Box)(x \mathbin{7} y \equiv \sim \Box\,\mathrm{E!}y \wedge \Box(\mathrm{E!}x \supset \mathrm{E!}y \wedge \sim y \leq x))$

DD5 defines '$F \mathbin{7} G$'; replace '$y \neq x$' in DD2 by '$\sim y \leq x$'

DD6 defines '$F \mathbin{7!} G$'; replace '$x \neq y$' in DD3 by '$\sim y \leq x$'

These definitions exclude cases where objects have essential parts as cases of dependence; the idea is that an object is dependent if it requires the existence of something which is not a part of itself.

With these concepts of dependence we are very much in the same territory as Husserl was exploring with his notion of *foundation.* Husserl's definition of 'foundation' runs:[22]

> If an α as such by essential law can only exist in a more comprehensive unity which associates it with a μ, we say that *an α as such requires foundation by a μ*, or also, *an α as such is in need of supplementation by a μ.*

Despite the inexactness of the natural-language formulation, this is clearly very close in meaning to the concept defined in DD5. The only slight amendment which might be necessary to capture every nuance of Husserl's version is to note that talk of association (*Verknüpfung*) with a μ suggests that the more comprehensive unity is such as to have both the α and the μ as proper parts, so that neither the α nor the μ is part of the other. There are occasions when this amendment might be important, but for most purposes it is not. These and other subtleties make Husserl's actual text difficult to interpret, and in particular he does not trouble to distinguish between rigid and generic dependence. He also rather slurs over the distinction between relations between individuals like that given by DD4 and relations between kinds (species) like that given by DD5 or DD6, since he holds, plausibly enough, that individuals are dependent by virtue of being the kind of individuals they are. Since, however, he does not distinguish rigid from generic dependence, he does not distinguish the rigid dependence of one individual on another (DD4) from the weaker relation of *support* which a G has to an F when Fs are generically dependent on Gs

[22] HUSSERL 1984: 267 (A254, $B_1$261), 1970: 463.

(DD5) but it is not important *which* Gs do the job for a given *F*. This relation of support is an obscure one, and it is doubtful whether it can be given a *formal* characterization. In particular cases we may be able to say what the support amounts to—in the case of the matter exerting pressure at a man's epidermis, for instance, it amounts to little more than being in the right place at the right time and in sufficient quantity. These and other difficulties of Husserl's text have been discussed in greater detail elsewhere,[23] but it is still imperative to call attention to Husserl's unique pioneering efforts at bringing clarity into an obscure topic.

The relationship of rigid foundation between individuals defined by DD4 is perhaps the most important one. If *a* ⁊ *b*, we follow Husserl in saying that *b founds a, a is founded on b*, or again that *a is a moment of b*. In such cases we also say that *a* is a *moment* and *b* its *fundament*. A moment is thus precisely something which is founded on something else.[24] Examples of moments are legion: smiles, headaches, gestures, skids, collisions, fights, thoughts, and many other events are moments because they are founded on their participants (the continuants involved in them). A more philosophically controversial case is provided by boundaries, which cannot exist except in so far as there are objects which they bound. And more controversial still are individual quality instances, like the particular snubbedness inhering in Socrates' nose, which is numerically distinct from all other particular snubbednesses.[25] Frege's functions, too, are described as 'in need of supplementation' (*ergänzungsbedürftig*), since they require their arguments to complete them and give an object. There could be no functions if there were no objects. But it is not quite clear as to whether the dependence of functions is rigid or generic.[26] It is my view that moments have a bright future in ontology, but whether this view is correct will take some time to establish.[27]

[23] SIMONS 1982*a*, SMITH and MULLIGAN 1982.

[24] This is the definition given in MULLIGAN *et al.* 1984: 294, except that there moments are confined to existents.

[25] For a list of subscribers to individual qualities, cf. MULLIGAN *et al.* 1984: 293. The list continues to grow. Cf. REID 1895: 395, for instance (thanks to Kevin Mulligan for unearthing this one). I am now of the opinion that belief in individual qualities is the rule rather than the exception.

[26] If a function is one-placed, one object will saturate it. How can it then remain unsaturated for every other object as well? Cf. SIMONS 1981*c*, 1983*a* for more on the connection between Fregean functions and dependence.

[27] MULLIGAN *et al.* 1984 argue that moments have a role in truth theory, and that they are perceived.

8.5 Some Related Concepts: Substance, Accident, Disturbance

So far we have left time out of the picture when discussing ontological dependence. The possibilities of combination which the introduction of temporal considerations open up are so extensive that it is easy to lose one's way. For this reason we introduce only very few notions which promise to have ready and interesting application.

For occurrents, a formula like '$a \leq b$' is complete as it stands, whereas for continuants it abbreviates '$\exists t \ulcorner a \leq_t b \urcorner$' (CTDS). The attempt to suffix mereological predicates applying to occurrents by temporal indices yields nonsense. Nevertheless, it is a helpful unifying device to use the same sign now with, now without a temporal suffix; and where it is without and applies to continuants, we may understand it according to the above convention. Similarly, while in English it is barbarous to say of occurrents that they *exist*, rather than that they take place, happen, obtain, hold, or go on (according to the case),[28] nevertheless we want some way to express that a given occurrent a is taking place, going on, or obtaining (as the case may be) at time t. For this we shall simply purloin the expression '$\mathrm{Ex}_t a$' from our previous discussions of continuants. The squeamish may read this as they wish. What is important is that, for occurrents, to exist, in the philosophers' (barbarous) sense, is to take place, go on or obtain *at some time*, so we may carry over the previous equivalence from Chapter 5:

$$(\Box)(\mathrm{E!}x \equiv \exists t \ulcorner \mathrm{Ex}_t x \urcorner)$$

This also allows us to give a more unified account of the varieties of dependence, which may cross the category line between continuants and occurrents. So it is characteristic of occurrents but not of continuants that they satisfy

$$(\Box)(\mathrm{Ex}_t x \wedge x \leq y \supset \mathrm{Ex}_t y)$$

If an occurrent is going on at t, then any occurrent of which it is part is going on at t. By contrast, a continuant x may exist at t and be a part of some other continuant at some time, but this other continuant need not exist at times at which its sometime part exists. Examples are obvious.

The most obvious kind of moment is one which is *permanently* dependent on its fundament: *whenever* it exists, so does its fundament. We call such a moment an *accident* of its fundament, and the

[28] This point of usage is (over-)stressed in HACKER 1982.

fundament may be called a *substratum* of its accident:

DD7 $(\Box)(x \operatorname{acc} y \equiv \Box\forall t\Box(\mathrm{Ex}_t x \supset \mathrm{Ex}_t y \wedge \sim y \leq x) \wedge \sim \Box \mathrm{E!}\, y)$

The part-predicate is here unsuffixed: among occurrents we just mean 'is not part'; among continuants we mean 'is never part'. Since no occurrent is part of a continuant and no continuant is ever part of an occurrent, we cover the mixed cases as well.[29]

A smile is an accident of its face, a headache or a thought of its bearer, a quality-instance of the object it qualifies. Many of the events involving particular continuants are accidents of them. This shows that, contrary to a venerable philosophical tradition, we may meaningfully talk of relational accidents, that is, accidents with a leg in more than one substratum. A collision between two bodies is just such a relational accident. *It* could not have occurred between other bodies (modal rigidity) and both bodies exist at the time of the collision (temporal rigidity), even if one or both of them are destroyed in the collision. Similarly, weddings, divorces, and football matches are more or less complex relational accidents. There is nothing especially mysterious about such things.

Modal rigidity may be present without temporal rigidity. It is, we may suppose, essential to a given human being that he or she have the parents he or she actually has, but most offspring outlive their parents. Cases of temporal without modal rigidity are harder to find. The following might be one: we might suppose that the adult form of a parasite could not survive without the simultaneous existence of its host, but it may have been a matter of chance that the egg of the parasite found its way into this host rather than another.

The definition of 'accident' leaves it deliberately open whether an accident is a continuant or an occurrent. In some cases it is not easy to tell which we are dealing with. Take, for example, a particular smile of Mary. We may say that this smile that it gets steadily broader—which is a way of speaking appropriate to continuants—but also that it goes on for a long time—which is more appropriate for a process or state. In this case I think we do need to distinguish two things, one of which is an action or period of activity of smiling, which is an occurrent with a beginning and an end, and the visible product of this activity, the contortion in the face we call a smile, which is a short-lived continuant which comes into being when the activity or action begins, alters or

[29] The decision to have one set of variables for both continuants and occurrents is heuristic, but there are other reasons favouring a two-sorted approach.

remains the same while this continues, and then ceases to exist when the activity comes to an end. This smile is a certain kind of disturbance in the medium of the face, and smiles are accidents of the owners of their faces. So, however, are the actions of smiling. It seems that we use the expression 'smile' both for the action and the facial disturbance, which explains the initial difficulty above. Let us agree to call the disturbance a 'smile', the action a 'smiling'. What is the relation between them? Normally a smile exists because a smiling occurs, though a qualitatively similar disturbance could have other causes, such as a nervous disorder. But a 'genuine smile' is the product of a smiling, and an action of smiling, though it does not include the smile as a part, is only a smiling because it successfully causes the face to become so disturbed.[30] So there cannot be a smiling without a smile (any more than there can be a hitting without something hit), and, provided we confine ourselves to *true* smiles (and not similar expressions, which are sometimes called 'smiles', with other causes), there cannot be a smile without a smiling. But the two are not to be identified. A smiling is an event whose effects include the appearance of a smile, and which is a smiling precisely because it has this effect (had the effect been blocked, the same event would have been at best an attempt at smiling). A smiling, being an event, has causes and effects, whereas the smile thereby produced is a continuant, and has itself neither causes nor effects.

Both are further to be distinguished from John's state of having a smile, which obtains as long as the smile exists, that is, as long as the tensed sentence 'John has a smile' remains true. The difference here seems to be that the state is a mere reflection of the truth of the sentence, whereas the smile is what makes the sentence true.[31]

I am sceptical of there being any general recipe connecting continuants, processes, and states among accidents. In some cases, as in the case of the smile, the continuant accident is produced by some event and maintained by a state or process, without which it would collapse. For instance, when a person keeps hold of a heavy suitcase, the shape of the hand is maintained by a sustained effort, which requires a lot of behind-the-scenes biochemical activity (processes). In this case, the person's state of having the hand shaped so is supervenient on these temporal objects and the continuant they

[30] On this account of smiling as a transitive action, cf. HORNSBY 1980: ch. 1. FINE 1982 takes actions to be *qua* objects, which is a related approach.

[31] On truth-makers in general, cf. MULLIGAN *et al.* 1984.

maintain. On the other hand, the shape of a metal bar which is in static mechanical equilibrium with its surroundings does not rest on underlying occurrents. It is characteristic of living things that the maintenance of themselves and their continuant accidents rest on processes which drag in energy from the environment. Even many non-living things remain in a certain state only thanks to processes they are involved in: for instance, normal stars are in a dynamic equilibrium whereby the tendency to implode under their own gravity is balanced by the energy produced internally by thermo-nuclear fusion finding its way out. Aristotle's classification of stars as living proves itself to be a good analogy, and it would appear that there are other structures, such as thunderstorms and river systems, whose energy exchange with their environment, ensuring their continuance, is analogous to that of organisms.[32]

A smile, like a knot in a piece of string, is a disturbance in a medium and static to the extent that it does not move around in the medium from one part of it to another. Other disturbances, however, are mobile: the most obvious kind is propagated waves, but thunderstorms are another example. Such disturbances are often thought to present ontologist with problems.[33] They are not substances (the medium is the substance) nor are they accidents (which cannot wander from one substance to another). Nor are they obviously events or processes. It is tempting to say that the wave is a process which starts from the point of origin of the wave and lasts until the wave breaks on an obstacle or dies down. That overlooks the fact that we attribute to waves positions and other characteristics at a time, and that we regard the wave as a whole as so characterized, and not a temporal part of it. But if a wave is a continuant, it is not a wandering accident, first since accidents cannot wander, and secondly because it is contingent *which* consignment of medium the wave actually traverses. The wave is maintained by a process transferring motion from particle to particle of the medium, but it is not identical with this process: we cannot say that the process has a certain shape or velocity of propagation. But there is no ontological problem of the status of moving disturbances; they are simply a special and interesting kind of continuant: moments which continuously change their fundaments. The difficulty only arises if the palette of ontological categories (for example substance,

[32] Cf. PANTIN 1968: 37 f.

[33] One who has recognized the importance of disturbances as an ontological category is Toomas Karmo; cf. KARMO 1977.

accident, relation) at one's disposal is too meagre to find a place for such things.

Nothing in the definition of 'accident' precludes an accident from being itself a substratum. So we may have accidents of accidents. The timbre of a note played on a violin, the boundary of a stain in some cloth, are examples.

A fist—like any other *qua* object—is one-sidedly, rigidly, and permanently dependent on its hand, and fails to be an accident of it only because the hand is an improper part of the fist. This corresponds to the Brentanian rather than the Aristotelian concept of accident. For Aristotle the accident would be something like the individual clenchedness inhering in the hand. This is an accident in our sense too. The Brentanian accident is a combination of the form, clenchedness, with the matter, the hand. But such entities occur in Aristotle as well, being called 'accidental unities', for example, 'musical Coriscus'.[34]

A substratum which is not an accident we may call a *substance:*

DD8 $(\Box)(\text{sub}\ x \equiv \exists y \ulcorner y \text{ acc } x \urcorner \wedge \sim \exists y \ulcorner x \text{ acc } y \urcorner)$

In other words, a substance is an ultimate substratum. It is the intention behind this definition to capture something like the Aristotelian conception of primary substance, according to which common or garden objects like people, chairs, and stars are substances. Whether the definition succeeds in this endeavour is open to scrutiny. Take the case closest to home, that of human beings. It is clear that these have accidents; the question is whether they *are* accidents. The definition of 'moment' rules out dependence on necessary beings and on essential parts, while that of 'accident' rules out dependence on one's ancestors. There remain two obvious test cases. If a human being is rigidly dependent on some continuant of which he is essentially part, then he is an accident. 'Tis certain, as Hume would say, that no man can be the whole world—a man is not an *Etwas-für-sich*—which suggests that he may be dependent on the totality of all that is, the Universe. But now, either the Universe is a necessary existent, which we have already excluded, or the term 'the Universe' is a description rather than a proper name, which means that even though there must be *something* beyond the man (gas molecules, energy sources, and the like), *which* ones there are is immaterial. In other words, the suggestion is that the dependence of a

[34] *Metaphysics* Δ, 6, 1015b. On Brentanian accidents, which have their substances as parts, cf. BRENTANO 1933 (1981), CHISHOLM 1978.

man on his surroundings is generic rather than rigid. The other test case involves the life processes which go on within the man—his respiration, heartbeats, and so on. These are clearly not part of him (although they take place within his boundaries). Is he then *rigidly* dependent on them? Again, I would venture to say not. *Some such* processes are necessary to maintain him in being, but of no such process or part-process is it necessary that *it* take place within him for him to exist. If we can fend off such putative counter-examples, it looks as though our reconstruction of the Aristotelian concept is not too wide of the mark.

Note that substances in this sense need not be in any sense absolutely independent: organisms clearly are not. We return at the end of this chapter to the idea of an absolutely independent object, a concept which has also gone under the title of 'substance'. To clarify the connection of our efforts to define 'substance' with the main topic of these last two chapters, the drawing of lines between integral wholes and arbitrary sums, we may note that a substance is such that everything on which it is modally and temporally rigidly dependent (in the sense obtained from DD7 by replacing '$\sim y \leq x$' by '$x \nmid y$') is a part of it. Its most stringent ontological requirements do not reach out beyond it; in this (somewhat weak) sense, it is a self-contained unit.

8.6 Non-modal Theories of Foundation and Dependence

Although some notion of ontological dependence is almost as old as philosophy, our considerations take their orientation from the pioneering work of Husserl, whose third *Logical Investigation*, in which he considers dependence and mereology, has been described as 'the single most important contribution to realist (Aristotelian) ontology in the modern period'.[35] It was indeed the discrepancy between Husserl's approach to mereology in this investigation and that of extension part–whole theory which first interested me in the topic of this book. Despite the importance which Husserl attached to this study, it remained sadly neglected by phenomenologists and ontologists. Exceptions to this are essays on Husserl and dependence by the Polish philosopher Eugenia Blaustein Ginsberg,[36] and on Husserl's part–whole theory by Robert Sokolowski.[37] Although

[35] SMITH and MULLIGAN 1982: 37.
[36] GINSBERG 1929, 1931 (1982).
[37] SOKOLOWSKI 1967–8, 1974: ch. 1.

Husserl explicitly says that the theory of dependent and independent parts is formal, and himself takes the first faltering steps in the direction of formalization, the first concerted attempt to envisage a formalization was that of my somewhat over-ambitiously titled 'The Formalisation of Husserl's Theory of Wholes and Parts'.[38] The principal aim of this exploratory essay was to show that Husserl's difficult text could be given a consistent and reasonable interpretation, where Ginsberg had claimed that his views were confused. The aim of the essay constrained me to stay (perhaps too) close to Husserl's text. The essay differed from the present treatment most in not distinguishing notional and ontological dependence, which was a definite shortcoming. I also there employed Wiggins's predicate-modifying necessity operator, which was at least a heuristic shortcoming, and perhaps also a philosophical one. The concepts of foundation and dependence are, however, interesting enough in their own right to be worth investigating in advance of attaining final clarity about necessity and essence. This interest has motivated the investigation of non-modal, non-sortal, first-order theories of individual foundation. One theory is due to Gilbert Null,[39] the other comes from as yet unpublished work by Kit Fine.[40] Both of them are axiomatic theories (Fine presents several equivalent axiomatizations), and are of independent interest. We present them briefly for completeness, and to facilitate comparisons with the ideas above.

Symbols used in the systems do not necessarily mean the same as they do in previous sections, except that we take over the usual mereological symbols. Neither Fine nor Null considers the interrelations of his concepts with temporal matters, which keeps things simple.

8.6.1 Systems of Fine

Fine employs as basis an extensional part–whole theory where '$<$' is a partial ordering with sums of non-empty sets of particulars guaranteed in the form of least upper bounds: if X is a non-empty set, we write the least upper bound of X as '$\sigma' X$'. The full strength of CEM is not used: in particular, Strong Supplementation is not assumed. Fine

[38] SIMONS 1982*a*. [39] NULL 1983.

[40] Fine's ideas are set out in detail in FINE and SMITH 1983, hence the names of the axioms begin with 'FS'.

offers a number of interesting equivalent formulations of his theory, and justifies his choice of principles by reference to Husserl's text, but we present only the bones of the theory.

Husserl used—without distinguishing them—two different concepts of foundation among particulars: a strong concept whereby an object cannot be founded on a part of itself, and a weak one where it always is founded on itself. We have already noted this distinction above. Based on strong foundation, the Fine theory has six axioms:

FS1A1	$x \text{ fd } y \supset y \nless x$	(Apartness)
FS1A2	$x \text{ fd } y \wedge y \text{ fd } z \wedge z \nless x \supset x \text{ fd } z$	(Qualified transitivity)
FS1A3	$x \text{ fd } y \wedge x < z \wedge y \nless z \supset z \text{ fd } y$	(Addition on the left)
FS1A4	$x \text{ fd } y \wedge z < y \wedge z \nless x \supset x \text{ fd } z$	(Subtraction on the right)
FS1A5	$X \neq \emptyset \wedge \forall y \in X \ulcorner x \text{ fd } y \urcorner \supset x \text{ fd } \sigma' X$	(Union on the right)
FS1A6	$x \text{ fd } y \supset x \text{ fd } (x +' y)$	(Left-to-right addition)

Axioms 1–4 define the algebraic structure of a complete semi-lattice, and the terminology comes from lattice theory.

In terms of foundation, the following concepts are readily defined:

FS1D1	$\text{dep } x \equiv \exists y \ulcorner x \text{ fd } y \urcorner$	(Dependence)
FS1D2	$\text{ind } x \equiv \sim (\text{dep } x)$	(Independence)
FS1D3	$x \text{ dep } y \equiv \exists z \ulcorner z < y \wedge x \text{ fd } z \urcorner$	(Relative dependence)
FS1D4	$x \text{ ind } y \equiv \sim (x \text{ dep } y)$	(Relative independence)
FS1D5	$x \text{ deppt } y \equiv x < y \wedge x \text{ dep } y$	(Dependent part)
FS1D6	$x \text{ indpt } y \equiv x < y \wedge x \text{ ind } y$	(Independent part)

These concepts can be used to prove the six theorems that Husserl ventures in his Investigation, which turn out to be relatively uninteresting in themselves. With this, the Fine theory fulfils the same goal as my earlier article, and with a simpler logical basis, though the depth of analysis is not as great as attempted here or in my previous essay, since the essential–modal aspect is not brought out. The repercussions of this will be discussed below.

Fine bases a different system on weak foundation as primitive. This has the same mereological basis as before, and the following set of axioms:

FS2A1	$x \text{ wf } x$	(Reflexivity)
FS2A2	$x \text{ wf } y \wedge y \text{ wf } z \supset x \text{ wf } z$	(Transitivity)

FS2A3 $x \,\mathrm{wf}\, y \wedge x < z \supset z \,\mathrm{wf}\, y$ (Addition on the left)

FS2A4 $x \,\mathrm{wf}\, y \wedge z < y \supset x \,\mathrm{wf}\, z$ (Subtraction on the right)

FS2A5 $X \neq \emptyset \wedge \forall y \in X \ulcorner x \,\mathrm{wf}\, y \urcorner \supset x \,\mathrm{wf}\, \sigma' X$ (Union on the right)

Strong foundation can be defined in terms of weak

FS2D1 $x \,\mathrm{fd}\, y \equiv x \,\mathrm{wf}\, y \wedge \sim y < x$

and vice versa

FS1D7 $x \,\mathrm{wf}\, y \equiv x \,\mathrm{fd}\, y \vee y < x$

and the two systems FS1 and FS2 with these definitions are equivalent. Fine offers two further equivalent formulations. The third system is based on the notion of an independent object (ind) with two axioms

FS3A1 $\mathrm{ind}(\sigma' \mathrm{V})$, where V is the set of all objects (independence of the universe).

FS3A2 $\forall x \in X \ulcorner \mathrm{ind}\, x \urcorner \wedge \mathrm{E}!\pi' X \supset \mathrm{ind}(\pi' X)$

(non-empty intersections (nuclei) of independent objects are independent.)

Under FS1D2 and

FS3D1 $x \,\mathrm{fd}\, y \equiv \forall z \ulcorner \mathrm{ind}\, z \wedge x < z \supset y < z \urcorner$

FS1 and FS3 are equivalent.

The final formal equivalent is in some ways the most interesting. It is based on the notion of the *foundational closure* $\mathrm{f}(x)$ of an object x. It is the idea of closure which is behind both the definition of substance given in the previous section and various concepts of an absolutely independent object which we consider in the next. Both implicitly use the idea of a closure system, which we define in the final chapter. The special axioms for foundational closure are very simple:

FS4A1 $x < \mathrm{f}(x)$

FS4A2 $\mathrm{f}(\mathrm{f}(x)) < \mathrm{f}(x)$

FS4A3 $x < y \supset \mathrm{f}(x) < \mathrm{f}(y)$

These axioms define a structure called a *pre-closure algebra* which arises naturally in connection with the topological concept of the closure of a set; Fine and Smith conjecture that Husserl might have been distantly aware of the analogies, and perhaps even prepared to accept a fully topological structure. At any rate, their conclusion is that

It is therefore perhaps no exaggeration to say that, once all the extraneous material is removed, the mathematical structure underlying all of Husserl's thought on dependence is that of a pre-closure algebra.

Foundational closure is readily interdefinable with *weak* foundation:

FS2D2 $f(x) = \sigma'\{y: x \,\mathrm{wf}\, y\}$

FS4D1 $x \,\mathrm{wf}\, y \equiv y < f(x)$

so systems FS2 and FS4 are equivalent, and therewith so are all four systems. Foundational closure is neatly connected with independence by the formula

FS4D2 $\mathrm{ind}\, x \equiv x = f(x)$

and with strong foundation by

FS4D3 $x \,\mathrm{fd}\, y \equiv y < f(x) \wedge \sim y < x$

which are derivable in the systems FS3 and FS4 as well.

8.6.2 System of Null

Null bases his first-order theory of foundation and dependence on a stronger extensional mereology than Fine, namely full CI, with σ as the sum and Strong Supplementation. However, the important results are made independent of strong sum axioms, about which Null too has reservations. The weak reflexive concept of foundation is employed, with the axioms

NA1 $x \,\mathrm{wf}\, x$ (Reflexivity)

NA2 $x \,\mathrm{wf}\, y \wedge y \,\mathrm{wf}\, z \supset x \,\mathrm{wf}\, z$ (Transitivity)

NA3 $x < y \supset y \,\mathrm{wf}\, x$ (The whole is founded on its parts)

NA4 $\exists xy \ulcorner x \,\mathrm{wf}\, y \wedge \sim (y \,\mathrm{wf}\, x) \urcorner$ (Existence of one-sided foundation)

NA5 $\exists xy \ulcorner x \,\mathrm{wf}\, y \wedge y \,\mathrm{wf}\, x \wedge x \neq y \urcorner$ (Existence of mutual foundation)

The first three axioms are equivalent to either FS2A1–3 or FS2A1–2, 4, showing that either FS2A3 or FS2A4 is not independent of the other together with FS2A1–2. Null has nothing corresponding to the sum principle FS2A5. The axioms NA4–5, making existential claims, are not ontologically neutral.

Null offers a set of definitions, some of which are similar to those of Fine, and a long series of theorems, among which are interpretations of Husserl's Six. Null's definition of relative dependence is different from that of Fine

ND1 $x \operatorname{dep} y \equiv \exists z \ulcorner z < y \wedge z \rfloor x \wedge x \operatorname{wf} z \urcorner$

but, given that his system incorporates Strong Supplementation, his definition is provably equivalent to Fine's in his system, although in a system without Strong Supplementation it is a stronger concept. His definitions of ind, deppt, and indpt are then the same as that of Fine, a noteworthy convergence all round.

Null defines a number of further interesting concepts. An *individual* is any whole, all of whose parts are foundationally connected

ND2 $\operatorname{indiv} x \equiv \forall yz \ulcorner y < x \wedge z < x \supset y \operatorname{wf} z \vee z \operatorname{wf} y \urcorner$

This sense of 'individual' is, as he points out, narrower than that of Goodman, and corresponds rather to Husserl's already mentioned concept of a *pregnant* whole.[41] Not all sums are individuals in this sense: this applies especially to plural particulars. Independent individuals are what Null calls individual *substances*; dependent individuals he calls *qualities*. *Higher-order wholes* are objects which are not substances but have a substance as part. Higher-order qualities are *accidents*; *x* is *accident of y* if *x* is an accident of which *y* is a part and *x* has a maximal independent part which is also part of *y*. The concepts of substance and accident here are those of Brentano's *Kategorienlehre* as interpreted by Chisholm,[42] and are not to be found in the work of Husserl himself. Here Null supports a conjecture I would also support, namely that the neo-Aristotelian ideas of Brentano and the different neo-Aristotelian ideas of his pupil Husserl could be combined in a single theory.[43]

8.6.3 Comparisons

In contrast with the complexity of our ontological deliberations, the axiomatic systems of Null and especially of Fine are delightfully simple. We concentrate on the latter, which is the more mathematically elegant of the two approaches. One source of this simplicity has already been mentioned: the systems prescind from temporal considerations. This is excusable: one need not try to get everything right at once. But the theories also contain no modal aspect, and this is not

[41] Cf. SIMONS 1982*a*: 141, and note 6 above.
[42] CHISHOLM 1978.
[43] Steps in this direction are taken in SMITH and MULLIGAN 1982.

so easy to accept, since everything we have seen about dependence makes it clear that the concept is modal. In an axiomatic theory in which modal operators do not occur, even if the intended interpretation of the primitive is a modal concept, other, non-modal interpretations cannot be ruled out. The case of Fine makes this clear: since he is at pains to point out the analogies between his concepts and those of topology, it is natural to look for a non-modal interpretation in topology. The fourth axiomatization gives the clue as to how this should go: interpret the variables as ranging over the non-empty subsets of a set on which a topology is defined, take '<' to mean set-inclusion, and 'f' to be the topological closure operator. Of course, formal theories cannot be protected from unintended interpretation, but the question is whether such an interpretation is so wide of the mark that the axiomatization be seen to fall short of adequacy as an analysis of the intended interpretation. On the topological interpretation, the difference between independent and dependent objects emerges as the difference between closed and non-closed sets, whereas the intended meaning of dependence/independence marks not a qualitative but a modal-existential difference: dependent and independent objects exist in different ways. On the topological interpretation, all sets, closed and non-closed, exist in the same way. A similar problem would seem to arise for *any* non-modal axiomatization of a modal concept.[44]

Suppose we attempt to circumvent this criticism by embedding the theory of Fine in a modal theory, defining his modal concepts appropriately. This would preserve what is of value in his approach and at the same time screen out a number of unwanted interpretations. For example, take the second axiomatization, based on 'wf', and embed it in a modal mereology by means of the definition

$$(\Box)(x \text{ wf } y \equiv \Box(\mathrm{E}!x \supset \mathrm{E}!y))$$

which is clearly what is understood by weak foundation. Then axioms FS2A1–2 (reflexivity, transitivity of 'wf') are satisfied all right, but we have difficulty with the remaining ones. To see this most clearly, note

[44] Talk of ways of existing suggests an anti-realist attitude to possible worlds, which I would endorse. Cf. the discussions in FINE 1977 and FORBES 1985. But even the realist about worlds and *possibilia* must find the distinction between closed and non-closed sets modally misplaced.

the following theorem (cf. NA3):

FS2T1 $x < y \supset y\,\mathrm{wf}\,x$

which follows from FS2A1 and either FS2A3 or FS2A4. On the interpretation of 'wf' just given, this ought not to be a theorem. Just because x is part of y it does not follow that y could not exist without x. At least, it does not follow if one does not subscribe to mereological essentialism. Assuming the authors do not wish to subscribe to it, we see that there is a discrepancy between the modal concept of weak foundation and the non-modal concept of part which their non-modal axioms cannot put right.

This can be rectified if we give '$<$' likewise a modal interpretation: let us take it to mean, not 'part', but 'essential part', i.e. the same as '$<!$', defined by

$$(\Box)(x <! y \equiv \Box(\mathrm{E}!y \supset x < y))$$

With this interpretation, and assuming the falsehood principle for 'part', FS2A3–4 drop out as theorems. And under the reasonable assumption that $\sigma'X$ satisfies

$$(\Box)(\mathrm{E}!\sigma'X \equiv X \neq \emptyset \wedge \forall x \in X \ulcorner \mathrm{E}!x \urcorner)$$

then the final axiom FS2A5 holds as well, provided we interpret the quantifiers possibilistically (as ranging over all possible objects, not just those in a world), and understand the set-membership predicate as likewise not world-relative. That we can understand the Fine theory modally such that its principles all hold is a satisfying result.

What is the price of buying modality into what on the face of it is not a modal theory? It is that the theory is incapable of dealing with matters of fact and actual existence, but remains throughout at the level of essential attributes and necessary truths. A realist about worlds and their inhabitants would say that only world-independent attributes of *possibilia* are mentioned. In fact, this high level of abstraction fits well with the attitude evinced by Husserl in the text upon which Fine and Smith comment (the third Logical Investigation). Husserl is *completely* disinterested in mere facts, like the fact that one object happens to be part of another. He is concerned with *essential* connections like foundation and being an essential part. This is what makes the text so alien to those who approach it from extensional mereology. If this interpretation is right, then the Fine approach is more apt for understanding Husserl than either that of Null, who introduces extraneous existential assumptions, or that of my previous essay, which got bogged down in the connections between factual and essential properties.[45]

Nevertheless, while an ontologist, in particular a mereologist, needs the mathematics of essential structures (as Husserl repeatedly emphasized), he also needs to get his feet muddy in that area where essence meets fact. Connecting up mathematical structures with their realizations in the world therefore requires, among other things, being *explicit* about the relations between modal and non-modal truths. This much in defence of messiness.[46]

8.7 Unconditional Existence

Where independence is valued positively, as it is in many philosophies, dependent objects are second-class, and many philosophers, especially those with an interest in theology, have spent much effort in trying to analyse what it is to be an absolutely independent object. We have already mentioned Aristotle's concept of substance, which is in our terms a rather weak form of independence. Aristotle accorded first substances priority in all respects, including priority of nature. When we ask what priority of nature is, we get a familiar-sounding answer:[47]

> Some things are called prior and posterior . . . in respect of nature and substance, i.e. those which can be without other things, while the others cannot be without *them*.

This is insufficiently exact for us to be able to pin down which concept of ontological dependence best fits it; but it shows that dependence is a perennial philosophical theme. Its prominence in modern philosophy may be due in part to Descartes, who defines 'substance' as follows:[48]

> By *substance* we can understand nothing other than a thing which exists in such a way as to depend on no other thing for its existence.

Descartes actually defines 'substance' twice; one case allows, the other overlooks, dependence on God (cf. our considerations leading up to DD1). In the strictest sense, only God is a substance; in the weaker sense, minds and bodies are substances, and everything else is a quality

[45] Many of the difficulties which arise in SIMONS 1982*a* stem from just this source. These include the notion of support, and a definition of dependence according to which x depends on y iff x is founded on a part of y. If, whenever Husserl says 'part', we understand 'essential part', these worries do not arise.

[46] But in moderation. Earlier versions of this chapter were much messier.

[47] *Metaphysics* Δ, 11, 1019a.

[48] *Principles of Philosophy*, book I, § 51.

or attribute. Now Descartes's *definition* of 'substance' may be accepted, but until we know what dependence is, we cannot be clear as to what objects are substances. It is remarkable that Descartes's great rationalist successors, Spinoza and Leibniz, held views on what are substances which are as far apart as can be imagined. For Spinoza, there is but one substance, which comprises all there is, while for Leibniz there are infinitely many, and each is perfectly atomic. The reconstruction I should like to give of this opposition is that Leibniz and Spinoza effectively understood 'substance' as Descartes did, but differed on what they meant by 'dependence'; to a reasonable approximation, Leibniz understood weak rigid dependence (⁊̸) while Spinoza understood strong rigid dependence (⁊). Thus the apparently minor mereological addition which distinguishes the definitions of these two concepts has hefty consequences.

Consider Leibniz's one-line proof for the existence of monads:[49]

> There must be simple substances, because there are compounds; for a compound is nothing but a collection or *aggregatum* of simples.

There is much going on beneath the surface here. One problem concerns whether a compound is an *individual* with the monads as parts or a *class* with the monads as members. It is not quite clear which reading is preferred. If the class reading is preferred, we have a reason for thinking the compound has its members essentially. But fortunately, Leibniz makes it unnecessary for us to decide for our purposes, since he accepts mereological essentialism for individuals anyway:[50]

> we cannot say—with complete fidelity to the truth of things—that the same whole continues to exist if a part of it is lost.

Let us now make two further suppositions. The first is that Leibniz would understand dependence in the sense of '⁊̸'. This has the consequence that everything is dependent on its essential parts, which means, since Leibniz accepts mereological essentialism, that everything is dependent on all its (proper) parts (Leibniz always understands 'part' to mean 'proper part'). The second assumption is that

[49] *Monadology*, §2. That Leibniz really did hold the view we attribute to him on dependence can be seen from the following passage in a letter to Des Bosses: 'That it depends upon a substance does not suffice to define the nature of an accident, for a composite substance also depends on simple substances or monads' (letter of 20 Sept. 1712, *Philosophische Schriften*, ed. C. I. Gerhardt, vol. II, 457, LEIBNIZ 1969: 605.)

[50] *New Essays on Human Understanding*, book II, ch. 27, §11.

anything which is dependent on anything else is dependent on something which is independent. In other words, any chain of dependencies *x* 7̵ *y* 7̵ *z* . . . has a terminus. Call this the assumption of *groundedness*. Then we can reconstruct the argument as follows:

(1) There exist compounds (i.e. objects with proper parts)
(2) Every part of a compound is essential to it (mereological essentialism)
(3) Therefore every compound is dependent on its parts (2, Df. '7̵')
(4) If every object has proper parts, then every object stands at the beginning of an ungrounded chain of proper parts (from 3)
(5) But every chain of dependencies is grounded (Groundedness)
(6) Hence if something is a compound, it has simple parts (from 4, 5)
(7) Hence there are simples (monads, atoms) (1, 6)

Now there are two places where this argument can be attacked. The one is mereological essentialism, about which we have said enough. The other is the Groundedness Principle. In the form in which it is used here, have we any reason to believe it? The belief is not peculiar to Leibniz, but informs all philosophies which contend that there is some metaphysical bedrock of primitive, unconditioned objects or stuff out of which everything else is built. We can find both the atomistic form, as in Leibniz and in Wittgenstein's *Tractatus*, and a continualistic form, as in Aristotle's theory of *materia prima*. Commenting on Wittgenstein's version of atomism, Black calls the view that not all existence is conditional a 'metaphysical prejudice'.[51] If one takes dependence in the sense given, there is reason in what he says. Similar appeals to forms of groundedness have informed unsuccessful proofs of the existence of God, a first cause, a prime mover, a first moment of time. But if we understand 'dependent' in the sense of '7', i.e., if we exclude dependence on parts, then we may defend unconditional existence, because even if we have an ungrounded sequence *x* 7̵ *y* 7̵ *z* . . . arising because *x* ≫ *y* ≫ *z* . . . , its first member *x* may be independent in the sense of '7'. It may be also that the absolutely independent object we are looking for is not an individual at all, but a plurality. If we understand 'the Universe' as comprising roughly speaking *everything* there is, then the Universe could not be dependent in the sense of '7', for if it were, there would be something outside it. This does not tell us much about what 'the Universe' means, because we have been deliberately vague in saying it *comprises* everything; it

[51] BLACK 1964: 60. Wittgenstein's version is at WITTGENSTEIN 1961: 2.021 ff.

could be the class of everything there is, or its sum, or perhaps some hybrid of these.

There is a rather nice cosmological proof which appeals implicitly to something close to this: it is by Bolzano, and deserves to be better known. It is too theologically unspecific to count as a proof of *God's* existence, but it can be regarded as an argument for the existence of an ontologically independent object, or, as Bolzano says, an object whose existence is unconditioned. Bolzano's argument neatly sidesteps the problem of grounding which beset earlier arguments by appealing, as we did, to classes. The argument runs as follows:[52]

(*a*) There is something real, e.g. my thought that this is so.
(*b*) Take any real thing, say A. If it is unconditioned in its existence, we are home.
(*c*) So suppose A is conditioned. Then form the class of all conditioned things A, B, C, . . . This is possible even if the class (*Inbegriff*) is infinite.
(*d*) The class of all conditioned real things is itself real. So is it conditioned or unconditioned? If the latter, we are home again.
(*e*) Suppose it is conditioned. Now everything conditioned presupposes the existence of something else, whose existence conditions it. So even the class of all conditioned reals, if conditioned, presupposes the existence of something conditioning it.
(*f*) This other thing must be unconditioned, since if it were conditioned, it would belong to the class of all conditioned real things.
(*g*) Hence there is something unconditioned, i.e. a God.

Notice that the argument does not make use of grounding; step (*c*) leaves the possibility of an infinite chain open. But two steps may be objected to. The first is the assumption, made in (*c*), that we may form the class of all conditioned objects. One might have qualms about this in a post-Russellian age. But the objects Bolzano is concerned with are real (*wirklich*) and we can for our purposes choose this to mean that they are spatio-temporally located. Let us allow step (*c*), since, for the simple classes we are interested in, the kind of reflexive formation which leads to paradoxes is not used.

The second objection must be to the argumentation in step (*f*).

[52] BOLZANO 1834: i, 178 f. (§ 67). The following rendering omits only the flourishes in the proof's rhetoric. Thanks to Heinrich Ganthaler for bringing the gem to my attention.

Why can the class of all conditioned things not itself be conditioned by something within the class? This would in turn be conditioned, and so on, but any attempt to put a stop to such a regress would appeal once again to a form of grounding, and Bolzano has tried to get round that by appeal to the class of all conditioned things. I suggest we can help Bolzano out at this point by making an additional assumption, which may be called the *Conditioning Principle*:

If a class X is such that every dependent element of it has all the objects on which it depends also in X, then X is not dependent.

This allows infinite chains of dependencies. Indeed, one kind of infinite circle arises when we have mutual dependence of two obejcts on each other. I claim that the Conditioning Principle is plausible. For if every element of X has all its needs catered for within X, how can X be dependent on something *outside* itself (i.e. on something which is not a member of X)? We may now complete Bolzano's argumentation. Suppose we have argued as far as (*e*). We now go on:

(*h*) If the class of all conditioned things is conditioned, then there is an element of it which is dependent on something which is not an element of this class (contraposing Conditioning Principle)
(*i*) Then such an object is not a member of the class of all conditioned things, and is therewith unconditioned.
(*j*) Hence in all cases, there exists something unconditioned.

This is to my mind a far better argument for the existence of independent objects that anything an atomist could produce, and the crucial Conditioning Principle is a reflection of what is arguably the best way to extend the concept of strong dependence (7) to classes, i.e.

$$(\Box)(a \, 7 \, x \equiv \exists y \ulcorner y \varepsilon a \wedge y \, 7 \, x \urcorner \wedge \sim x \varepsilon a)$$

So it seems to me that by employing 7 instead of $\not 7$ as our dependence concept, we stand a much better chance of turning Black's charge that belief in unconditioned existence is nothing more than a prejudice.[53] The argument is also free from worries about the dubious status of the Universe in the knock-down Spinozistic proof sketched before.[54]

[53] It is surprising that Black does not consider this possibility, since in BLACK 1971 he defends the low-brow conception of classes, which makes it easier to accept.

[54] The argument is also mentioned in MULLIGAN *et al.* 1984: 292, but it is more simple-minded than anything in Spinoza himself. The best passage reflecting Spinoza's view on dependence is: 'But all things which are, are in God, and so depend on God, that without him they can neither be, nor be conceived' (*Ethics*, book I, proposition XXVIII, Scholium.)

However, the argument does not prove the existence of God. For one thing, it is only an existence proof; uniqueness is not proved (we may note that Bolzano says carefully '*a* God').[55] For another, there is no reason given in the argument to suppose that independence entails divinity.

[55] In the earlier *Athanasia* (BOLZANO 1827: 11 f., 2nd edn.: 22), Bolzano uses the same form of argument to show that there is at least one substance: the sole difference is that the conditioning relation is replaced by the slightly different relation of 'adherence' which an attribute (*Beschaffenheit*) has to its bearer. Here the uniqueness assumption is of course inappropriate.

9 Integral Wholes

9.1 Sums and Complexes

In his well-known criticisms of the uses of the terms *Gestalt* and *Ganzheit* in the 1930s,[1] Moritz Schlick argues that there is no *ontological* difference between sums on the one hand and *Ganzheiten* on the other, but simply a difference between two modes of representation of the same objects, so it is not possible for *Ganzheit* theorists to use the opposition to distinguish two *kinds* of object, mere sums or aggregates on the one hand, and the essentially different (and usually more interesting) unities, wholes, or totalities on the other. Schlick defends a form of micro-reductionism which is rather extreme for modern tastes, though his judgements on the value of the *Ganzheit* approach of Driesch and others have by and large stood the test of time.[2] However, on the point that the opposition sum/*Ganzheit* has no ontological repercussions, but is solely of a methodological nature, we must disagree with him. The notions of 'sum' which we have employed are not vague *ad hoc* carry-overs from arithmetic, but concepts clearly defined within mereological theories. The original concept, that of Leśniewski, Tarski, and Leonard and Goodman, has impeccable formal credentials, and while the equivalence of the various formulations breaks down when the principles of the theory are weakened, each formulation has a clear sense, as do the various related concepts of sum definable in a mereology for continuants.

That the distinction between sums and non-sums—which we may call *complexes*—is an ontological one may be seen by comparing their existence conditions. For sums these are minimal: the sum exists just when all the constituent parts exist (what this 'just when' in detail amounts to varies according to the concept of sum involved—SUM or SM, etc.) By contrast, a complex constituted of the same parts as the sum only exists if a further constitutive condition is fulfilled. In the case of pluralities, the difference is that between a mere class and a group. The difference is most obvious for continuants, where the

[1] SCHLICK 1935 (1979).

[2] It should be noted, however, that Schlick never disputes the *utility* of holistic descriptions, whether in physics, biology, astronomy, meteorology, history, or psychology; in the latter, indeed, he suggests the Gestalt approach is 'the only promising one' (1935: 265 (1979: 398)).

constitutive condition of the complex may cease to hold whereas the parts do not cease to exist, so the complex ceases to exist while the sum continues, showing the sum and the complex to have been superposed rather than identical.

We do not need to confine this result to objects which change, so long as we introduce a modal element. Suppose we consider the objects of the world all to be made up of elementary constituents, none of which could have failed to exist, but any of whose configurations or interrelations might have been different. For each actual complex, the holding of its constitutive relation is a contingent fact, so its existence is contingent. On the other hand, the existence of its ultimate constituents and therefore of their sum is necessary. No mention is here made of change, but only of modal status.[3] However, this static example makes micro-reductionism more tempting than it is in the case of continuants, where, for example, the interest in tracing the careers of successive consignments of molecules is inexplicable until it is made clear that they successively constitute a single organism.

Schlick argues that if it is a part of the concept 'sum' that the parts in no way interact, as Driesch seems to suggest, then nothing in nature is a sum.[4] Now of course there are things in nature which do not interact, such as point-events e_1 and e_2 with a space-like space–time separation, or, thinking of continuants, bodies b_1 and b_2 such that every event in the history of b_1 has a space-like separation from every event in the history of b_2. Essentially the same point—that nothing is a sum, so the opposition sum/complex is scientifically worthless—is made by Popper in a brusque dismissal of the claims of any sort of Gestalt theory.[5] However, a mereological theory postulating the existence of *individual* events like $e_1 + e_2$ or continuants like $b_1 + b_2$ does provide an (albeit weak) alternative to any kind of natural complex. Of course, that something is a sum implies neither that its parts are totally connected nor that it is a sum *through and through*. Köhler's paradigm of a sum, three pebbles chosen at random, lying in three different continents, has components which *are* natural wholes, and which presumably exert a tiny gravitational attraction on one another. They are all parts of *one* natural whole, the earth. It is the idea of an

[3] Cf. the account in WITTGENSTEIN 1961.
[4] SCHLICK 1935 (1979): 259 (393 f.)
[5] Cf. Popper's remarks in MEDAWAR and SHELLEY 1980: 75.

individual comprising *just them and nothing besides* which appears so superfluous.

That Schlick and Popper are prepared to deny that *any* natural objects are sums is a prima-facie argument (from authority, of course) that mereological concepts of sum, while perhaps metaphysically unobjectionable, are of negligible scientific importance. The metaphysical problem, therefore, shifts to that of saying something general but sensible about what *makes* something a natural (or other) complex, which entails giving a schematic account of the constitutive interrelations of parts characteristic of such complexes.

9.2 Integrity in General

Aristotle described some objects as more truly one than others: for example, a straight line is more truly one than a bent one, a rigid body more than a jointed one.[6] What is meant cannot be that individuality comes in degrees. It is not that some things are more individual than others. What comes in degrees is the *integrity* or *wholeness* of something. We can see this from examples. A spatially extended object like a country is more integrated if it is in one continuous piece than if it is in several scattered pieces. Similarly, temporal objects which are scattered in time—i.e. have temporal gaps in them, like interrupted discussions or chess games—are less unified than those without gaps. We need not confine ourselves to spatial or temporal continuity. A confederated state like Switzerland has a looser internal political structure than a more highly centralized one like France; a community divided into antagonistic cliques or warring factions is less unified than one where social harmony prevails; and Aristotle's rigid body is *kinematically* more integrated than a flexible or jointed one.

It is apparent from this brief selection of examples that integrity comes not only in degrees but also in many varieties. What is highly integrated in one sense may be loosely structured in another—for example, Switzerland is politically 'loose', but, unlike Indonesia, geographically 'tight', linguistically less integrated than many lands, but racially fairly homogeneous, and so on. The integrity of something in some respect is clearly a matter of certain specific relations among the parts of the object, these relations being characteristic of the

[6] *Metaphysics* Δ, ch. 6.

respect in which the object is integrated. Since this tells us rather little, we consider a representative example.

What property do spatial objects which are all of a piece have which is lacking from those which are in two or more disconnected pieces? Overlooking the microscopic discontinuity of matter, we may get an answer by looking at the *topological* properties of such objects, these properties being independent of their particular shape. The topological concept of *connectedness* is less intuitive in this respect than the slightly stronger concept of *path-connectedness*. A space is path-connected iff any two points in it are the end-points of some path or arc lying wholly within it, a path being the homeomorphic image of an interval, i.e. a set for which the removal of any single point except one of the two end-points separates it into two. Path-connectedness is an equivalence relation on the points of a topological space, partitioning it into maximally (path-) connected sub-spaces or *components*. Every point in such a component is connected to every other point in it, and to no point outside it. A path-connected space is one with a single component.

Similarly, in graph theory a graph between any two vertices of which there is a *chain* of vertices and edges is called *connected*.[7]

This pattern may serve to give a schematic characterization of integrated wholes. First, if *w* is any object, a *division* of it is any class of parts completely exhausting it:

ID1 $a \operatorname{div} w \equiv \forall x \varepsilon a \ulcorner x < w \urcorner \wedge \forall x \ulcorner x < w \supset \exists y \varepsilon a \ulcorner x \circ y \urcorner\urcorner$

while a *partition* is a disjoint division:

ID2 $a \operatorname{ptn} w \equiv a \operatorname{div} w \wedge \forall xy \varepsilon a \ulcorner x \circ y \supset x = y \urcorner$

and a *basic division* (definition not given) is a division whose members form a base for *w* in the sense of § 1.6. Characteristic for integral wholes is the fulfilment of or approximation to the following condition:

> Every member of some division of the object stands in a certain relation to every other member, and no member bears this relation to anything other than members of the division.

[7] WILSON 1972: 19, 27 f. A graph is connected iff it is not the union of several graphs. A topological space is connected iff it is not the union of several closed, disjoint, non-empty sets. Path-connectedness implies connectedness but not vice versa; there are connected spaces which are not path-connected.

To clarify this, we introduce some concepts governing such relations.[8]

Let R be a binary relation and a a class of objects. We say a is *closed on the left under* R if no R-relationship leads from outside a to inside it, i.e. nothing outside a bears R to anything in a:

$$\text{ID3}\quad \text{cll}\langle R \rangle a \equiv \forall xy \ulcorner y \varepsilon a \supset . xRy \supset x \varepsilon a \urcorner$$

For instance, if no one else (no third party) owes either Jack or Jill any money, the pair Jack and Jill is closed on the left under indebtedness. Note that either may still owe the other or some third party.

Conversely, a is *closed on the right under* R if no R-relationship leads from inside a to outside it:

$$\text{ID4}\quad \text{clr}\langle R \rangle a \equiv \forall xy \ulcorner x \varepsilon a \supset . xRy \supset y \varepsilon a \urcorner$$

Jack and Jill are closed on the right under indebtedness if neither owes any money to a third party, irrespective of whether they are creditors to third parties or each other. If R is a symmetric relation, closure on the left and the right coincide.

Putting both conditions together, we say a is (simply) *closed under R* (*R-closed*) if no R-relationship crosses a's boundary in either direction:

$$\text{ID5}\quad \text{cl}\langle R \rangle a \equiv \forall xy \ulcorner x \varepsilon a \supset . xRy \vee yRx \supset y \varepsilon a \urcorner$$

Since the disjunction of R with its converse, $R \dot{\cup} \breve{R}$, is symmetric, this amounts to saying that a is closed on the left (or right) under this disjunction. So if neither Jack nor Jill owes or is owed money with respect to third parties, the pair Jack and Jill is closed under indebtedness, though either may owe the other. If R is already symmetric, all three conditions coincide.

If every member of a bears either R or its converse to every member of a, we say a is *connected under R* (*R-connected*):

$$\text{ID6}\quad \text{con}\langle R \rangle a \equiv \forall xy \ulcorner x \varepsilon a \supset . y \varepsilon a \supset xRy \vee yRx \urcorner$$

It can be seen that the definition of R-connection reverses the inner implication of that of R-closure. If each of a group of people is such as

[8] We make use in what follows of the following notation and terminology from the logic of relations in *Principia Mathematica* (WHITEHEAD and RUSSELL 1927); in each case the place where the concept is defined is given in parentheses. $R \dot{\cup} S$, the logical sum of relations R and S (*23.03); $\breve{R}$, the converse of relation R (*31.02); $R{\restriction}a$, the restriction (limitation) of relation R to the class a on both the left and the right (*36.01); R_*, the ancestral of relation R (*90).

to owe to or be owed by everyone else in the group, the group is connected under the relations '. . . owes to or is identical with——' and '. . . is owed to or is identical with ——'. Some link of credit and debt joins any pair in the group. If a is R-connected, then R is reflexive on a. But R-relationships may cross the boundary of a in either direction.

A stronger condition is obtained by reversing the inner implication of either ID3 or ID4: in such a case we say a is *biconnected under R* (*R-biconnected*):

ID7 $\text{bicon}\langle R \rangle a \equiv \forall xy \ulcorner x \varepsilon a \supset . y \varepsilon a \supset xRy \urcorner$

Then every member of a bears R (and hence also its converse) to every other member and to itself. If a is R-biconnected, the restriction $R \lceil a$ of R to a is trivially an equivalence relation, a being its only equivalence class.

If a class a is both connected and closed under R we call it a *closure system under R* (*R-closure system*):

ID8 $\text{cs}\langle R \rangle a \equiv \text{con}\langle R \rangle a \wedge \text{cl}\langle R \rangle a$

Closure systems are maximal connected classes. For example, if Jack owes Jill some money and she owes him some as well, and neither owes to or is owed by any third party, then the pair is a closure system under the relations mentioned above involving identity as a disjunct.

If, on the other hand, we combine biconnectedness with closure, we may say the class a is a *biclosure system under R*:

ID9 $\text{bcs}\langle R \rangle a \equiv \text{bicon}\langle R \rangle a \wedge \text{cl}\langle R \rangle a$

A biclosure system is similarly a maximal biconnected class. If R is the relation of sharing both parents, a biclosure system under this relation is a class of all the full sibling offspring of two particular parents. Where R (as here) is symmetric, closure and biclosure systems are the same.

Relations which are themselves unable to generate closure systems may be traded in for more promising ones. If R is any relation, its disjunction with its converse $R \dot{\cup} \breve{R}$ is symmetric, and the ancestral of this $(R \dot{\cup} \breve{R})_*$ is further reflexive and transitive. The field of R is accordingly divided into disjoint subclasses. In any such class, distinct members are all connected to one another through chains of instances of R and its converse, and none is thus connected to anything outside. They therefore form (bi-)closure systems under $(R \dot{\cup} \breve{R})_*$, and we call them *R-families*:[9]

ID10 $\text{fam}\langle R\rangle a \equiv \text{cs}\langle (R \dot{\cup} \breve{R})_* \rangle a$

For example, if *R* is the parent–child relation, then a family under this relation is a class of organisms each of which is related to every other, whether by ascent, descent, or some combination of the two, no matter how remotely. This is the most extended genealogical family there can be, and, in view of evolution, certainly crosses present species boundaries. It may be that every mammal is part of a single family, for instance. If *R* is the relation '. . . is upstream of or level with ——' on points of rivers, then a family under this relation is a class of points making up a single river system, with tributaries, distributaries, backwaters, and all. If *R* is the relation of being coupled to, applied to rolling stock, then *R*-families are the vehicles making up single railway trains.

As the examples show, families are highly natural and obvious groupings, and the notion of a family as here defined provides the logical basis for our treatment of integral wholes. An object *w* is an *R-integrated whole* (is *integrated under R*) if there is a division of *w* which is an *R*-family:

ID11 $\text{wh}\langle R\rangle w \equiv \exists a \ulcorner a \text{ div } w \wedge \text{fam}\langle R\rangle a \urcorner$

In such a case we call *R* a *characteristic relation* for *w*. In the case of connected graphs, one characteristic relation is that between two vertices between which there is an edge. Another is that between any two vertices which can be connected by a chain. As the example makes clear, there can be more than one characteristic relation for a whole. In the case of a path-connected topological space, the characteristic relation which is most obvious is that of path-connection between points. Points form a basic partition of a space, but we could have taken open neighbourhoods, or a base of open neighbourhoods, and defined suitable path-connectedness relations between them. Sometimes, as in this case, there are several divisions of an object with respect to which it is an integrated whole.

Before we go on to consider various kinds of integrity or wholeness, some general remarks on the varieties of integrity are in order.

It should be remarked that, in view of the systematic ambiguity of

[9] This is slightly different from Whitehead and Russell's concept of a family of a term (WHITEHEAD and RUSSELL 1927: *97, see also CARNAP 1958: §36.) The Whitehead–Russell family of a point in a river under the relation 'upstream of' consists of all points upstream or downstream of this, not all points in the same river system as in our case.

the 'part' predicate uncovered in Chapter 4, the definitions ID1, 2, and 11 are schematic in that *w* can be an individual, collection, or mass. Where a characteristic relation obtains between individuals, it is not thereby clear whether the whole thereby constituted is itself an individual or a collection, since the members of a collection form a division of it, the membership relation being a special case of the part-relation for collections. On this point turns much of the misuse that can be and has been made of the undifferentiated notion of 'whole', for example in social theory. For given the well-founded contention that human beings are linked together by numerous kinds of integrating relation, it is inferred that the resulting communities or collectives are some kind of supra-personal individual. The unacceptability of the conclusion leads opponents of it to ignore, deny, or play down such relations and emphasize the individuality of the individual at the expense of its sociality. The wrong premiss of collectivists is thereby attacked: what is rather to be given up is the (usually tacit) assumption that a grammatical singular signifies an individual.

Now it is far from clear that there is an objectively sharp division between individuals and collections. This does not mean we agree after all to degrees of individuality; but we do have to accept that there are degrees of warrant for accepting the existence of an individual composed of certain parts. We can of course find clear paradigms on both sides, but there are loosely structured individuals and tightly organized collections for which the difference in integrative tightness is at best a matter of degree, and the propensity to see here a collective, there an individual, appears to be in some cases pragmatically or conventionally determined. Matters are complicated by the fact that many relations may play an integrating or disintegrating role with respect to one and the same whole; consider the manifold relationships between the parts of an organism and its environment, for instance. There are cases in biology where the facts do not force a decision either way. Multicellular organisms evolved out of unicellular ones, not directly, but via colonies of unicellular organisms: such colonies still exist, particularly in the sea. In the nature of the case, symbiotic interdependence, specialization of function, and central regulative mechanisms must have developed gradually, so no objectively sharp dividing line exists between colonies of single-celled organisms and single multicelled organisms. Even today some dispute exists as to whether sponges are single organisms or colonies.[10]

[10] Cf. BURTON 1967.

Given the ambiguity of the term 'part', we get no guidance on the matter from linguistic usage (which was to be hoped!). The ambiguity may indeed be turned to the good, since it allows one to *ride out* the problem of whether something is an individual or a collection under harmony of usage, pending investigation of the integrative relations themselves. The mere question of whether an object is an individual or not is itself far less important than the investigation of what exactly holds it together. A case in point is provided by the suggestion of Ghiselin and Hull[11] that biological species are not classes of individuals but rather individuals, of which organisms are parts rather than members. We can take the purely terminological heat out of this debate by *accepting* that a species is an integrated whole of which its members are parts, *leaving aside* whether this whole is an individual or not, and devoting attention instead to the detailed biological investigation of the nature of the unity of the species, its various units, mechanisms of selection and propagation, and so on.

One thing the examples should also have made clear is that there may be natural *hierarchies* of natural wholes. This does not go against our analysis of integrity; it confirms it, since hierarchy amounts to objects which are parts of a natural whole in some respect themselves being wholes in another. The familiar hierarchies of biology are the best example. The guarded acceptance of collections of collections in Chapter 4 now pays its dividend, for under a dogmatic assumption that there cannot be collections of collections, it must be held that at most the top level of a hierarchy be a collection, and everything under it be individual, which would force metaphysical conclusions possibly going against the scientific facts lower down the hierarchy, or else compel non-recognition of higher levels in a hierarchy of collections.[12]

The kind of integration of integrated wholes may vary. A centralized whole is one where the integrating relation consists in all the parts having a common relation to some part, whereas a network of relations may be without such a centre. Between these extremes there is room for a variety of intermediate cases.

Where relations are susceptible of differences of degree, as for instance friendship, or strength of gravitational attraction, the

[11] GHISELIN 1974, HULL 1976, 1978. I am indebted to Marjorie Grene for drawing this debate to my attention.

[12] This leaves the similarly tentative denial of classes of classes in SIMONS 1982*b* untouched, since classes are a special case of collections.

integrity of wholes bound together in such relationships will also come in degrees; we have here an objective warrant for speaking of something's being more integrated than another in a certain respect: one group of people may be more closely knit by friendship than another, for example.

We have stressed that attempts to characterize what distinguishes integral wholes from other things must steer clear of triviality. The definitions of closure, family, etc. given above can be given a trivial interpretation. For if we suppose that a class is a family iff it is a family under *some* relation, or that an object is integrated iff it is integrated under *some* characteristic relation, then the definitions yield nothing useful. For *any* class *a* is a family under the relationship of being comembers of *a*, and *any* object is integrated if we take as division all its parts and as characteristic relation being coparts of it. Hence no *formal* definition of integrity appears to work, since we cannot rule out such relations as coexisting, being different from, being both parts of *s* and so on except by *ad hoc* manoeuvres.

We have a fairly clear instinct as to which relations count as natural and which do not; the point is to try to say what this instinct amounts to. Roughly speaking, natural relations are those which help to *account for* the observed unity of an object; the likeness of colour of the parts of a visual datum and their dissimilarity from parts outside, the constancy of relationships of parts in the motion of a rigid body, the ability to talk one to another as constitutive of a linguistic community. Such relations have a part in explaining why we perceive, think, or otherwise find that certain individual or collective objects 'hang together', and why they are different from arbitrary agglomerations of things from here and there.

We also now have an explanation as to why certain kinds of *undetached* part appear to have lesser claims to objecthood than others. The heart in an animal is certainly connected to the rest of the body in many important ways, and without special care neither the heart nor the animal will survive if these connections are broken. But the heart has a high degree of internal connectedness, and its unity of function, or 'sympathy of parts', as Hume put it, warrants our taking it as an individual in its own right with a relative integrity of its own. The same goes for components in general, and also for sub-systems like the circulatory system, whose functional interconnectedness is higher than that of the heart although it is less obviously a single body. It is because of the importance of its role that the heart is identified as a prominent individual, so that if removed, it reverts to being a mere connected lump of flesh, a heart in the butcher's sense, not the physiologist's.

By contrast, an arbitrary conceptual 'cut' like the northern half of a house has far less claim to our attention, because typically it is not closed under the relation under which its whole is closed. The northern half of a house typically is not discontinuous from other buildings, nor is it typically sufficient to provide shelter on its own. It is the fact that characteristic relations lead outside the part or dangle from it that warrants our taking it to be a fragment, to be something incomplete. But a fragment is not for all that nothing. It is *not* relevant that we can *manufacture* a suitable relation at will. The point is that a fragment has a *potential* for integrity even if it actually does not have it. The southern half of the house may fall down, leaving the northern half standing alone. Or surveyors may draw a new national boundary east to west through the centre of the house, giving each half a new political integrity. So while we should discriminate among undetached parts, and look askance at such as are mere sums, we cannot deny that there are non-integrated undetached parts.

Among those who have tried to analyse what it is to be a whole are Rescher and Oppenheim, who suggest three basically sensible conditions:[13]

(i) The whole must possess some *attribute* in virtue of its status as a whole—some attribute peculiar to it, and characteristic of it as a whole.
(ii) The parts of the whole must stand in some special and characteristic *relation* of dependence with one another; they must satisfy some special condition in virtue of their status as parts of a whole.
(iii) The whole must possess some kind of *structure*, in virtue of which certain specifically structural characteristics pertain to it.

Taken purely formally, these conditions are just as platitudinous as the requirements of being a family, or being integrated under a characteristic relation. *Any* object *s* possesses some attribute exclusively, namely that of being identical with *s*, which neither its proper parts nor its proper containers have. But this is just formal. The first condition is already satisfied by our account, since if *a* is an *R*-family, this being a maximal *R*-connected class means that neither a proper subclass nor a proper superclass of it may be an *R*-family, so this is the

[13] RESCHER and OPPENHEIM 1955: 90.

exclusive attribute we are seeking. Similarly, we have already accounted for condition (ii) in a schematic way, though the hint about dependence needs to be followed up. That will occupy us in §§9.4–5. Then we go to condition (iii) and say something about structure and its relation to the concept of Gestalt. But before that, we pick up the connection between the schematic concept of integrity and the discussion in Part I by considering what mereological or mereo-topological integrity amounts to in extensional theories, and how it is trivialized in the classical case.

9.3 Mereo-topological Integrity

One kind of natural group is the extension of a monadic predicate. It is related to the family concept as follows. If F is a monadic predicate, define the relation FF by

$$x\ FF\ y \equiv Fx \wedge Fy$$

Then a is a family under FF (ID10) precisely when a is all the Fs:

$$\text{fam}\langle FF \rangle a \equiv \forall x \ulcorner x \varepsilon a \equiv Fx \urcorner$$

A *complete whole* of Fs is a whole under FF:

$$\text{cwh}\langle F \rangle x \equiv \text{wh}\langle FF \rangle x$$

A complete whole of Fs is thus an object for which the Fs form a division, and is therefore nothing other than the *sum* $\sigma x \ulcorner Fx \urcorner$ of Fs.

We can now see in what respect classical extensional mereology trivializes the notion of a mereological whole. For every individual is the complete whole of its parts, and every non-empty monadic predicate determines a complete whole, namely its sum.

For a conception of whole in which several objects can form a whole only if there exists something of which they are all parts (i.e. if they are bounded above), it is natural to look at the ancestral of overlapping ($\circ_*$) as a possible characteristic relation. In a classical context this again yields only trivial results, since everything overlaps U, and can hence form a whole with anything else. This applies *mutatis mutandis* to Clarke's mereo-topology (§2.10.2), where the ancestral of the relation $\between$ replaces that of $\circ$, since everything is connected, in the sense of $\between$, with U.

This helps to explain why a number of non-classical extensional mereologies which restrict the existence of sums and least upper bounds do not accept U (Whitehead, Bostock, Needham, Van

Benthem). For instance, Bostock's axiom PA2 (§2.9.2), which says that if the Fs are bounded above, then they have a sum, allows us to see sums as non-trivial complete wholes of Fs, since if Fs do not have an upper bound they need not have a sum. It is precisely adding U to Bostock's system P that renders it classical. The grades of involvement with wholes in non-classical mereology were charted by axioms SA7–22 of §1.4. In particular, if we consider overlapping as a characteristic relation, then axiom SA20 and its binary counterpart SA9 are especially natural. On the other hand, as the interval mereologies of Needham and Van Benthem show, it is more natural to assume that, if individuals are bounded above, they have a least upper bound (SA10, SA21) than to assume they have a sum (SA11, SA22, which is Bostock's PA2), at least if one is thinking of allowing only continuous individuals (which Whitehead, Needham, and Van Benthem are, but Bostock is not).

We turn now to the concept of continuity. It is important to see that this is not a purely mereological concept. The half-open intervals $(0, 1]$ and $(1, 2]$ form a continuous whole, but mereologically speaking they are as disjoint as $(0, 1]$ and $(2, 3]$. Hence to do the idea of connectedness or continuity justice, we shall need to use the mereo-topological concepts of §2.10.

If R is any relation and w any individual, define the associated relation R^w (read 'R in w') as

ID12 $\quad xR^w y \equiv x < w \wedge y < w \wedge xRy$

Note that if we put U for w the result is just R again. Now recall the concept of externally touching or adjoining: x adjoins y (we write $x \asymp y$) if and only if they are connected but do not overlap (CLD5 of §2.10.2). We now define parts x and y of w as being *connectible within* w iff there is a chain of parts of w starting with x and ending with y, each touching its neighbours externally. We express this using the ancestral and the 'in w' operator of ID12:

ID13 $\quad x \operatorname{cnbl}(w) y \equiv x(\asymp^w)_* y$

We can now define an individual y as being *continuous* iff each part of w either overlaps or is connectible within w with every other part:

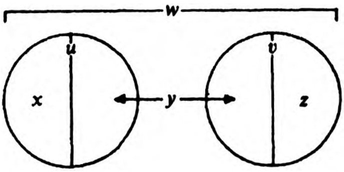
ID14 $\quad \operatorname{cts}(w) \equiv \forall xy \ulcorner x < w \wedge y < w \supset x \circ y \vee x \operatorname{cnbl}(w) y \urcorner$

Consider the class $\operatorname{pt}(w)$ of parts of w. Then it is easy to show that w is continuous iff $\operatorname{pt}(w)$ is a closure system under $\operatorname{cnbl}(w)$ and w is its associated whole:

IT1 $\text{cts}(w) \equiv \text{wh}\langle \text{cnbl}(w) \rangle w$

This is a satisfying result, because it shows that our schematic concept of integrity is applicable in a natural way within mereo-topology itself. The non-triviality arises because we chose cbl(w) as characteristic relation: the mere relation of being coparts of an object w by contrast does not distinguish any objects as being more integrated than others.

It is important to see that the existence of disconnected sums does not disturb this definition. For instance, if w is the sum of the two disconnected discs u and v (see below), the fact that u and v exhaust w



means that they are not only disjoint but also not connectible within w. This is so even though there are separated parts x and z which are connectible within w by means of the discontinuous part y. Here the fact that we are able to define external touching is crucial: if, instead of the definition of continuity given above, we attempted a purely mereological one, using the formula

$$\forall xy \ulcorner x < w \wedge y < w \supset x(\circ^{w})_{*} y \urcorner$$

then, since $u \circ w \circ v$, we are unable to discriminate between continuous and discontinuous individuals as long as the latter are admitted.

It is interesting to compare the definition of continuity just given with Tiles's definition of self-connectedness (§2.10.1):

TID8 $\text{cn}(w) \equiv \forall xy \ulcorner w = x + y \supset x \between y \urcorner$

This is presumably modelled on the topological notion of connectedness according to which a topological space is connected iff it is not a disjoint union of topological spaces. In a suitably strong mereology, in which, if $x \ll y$, the difference $y - x$ exists, our definition and that of Tiles coincide. Here is an outline proof.

Suppose E!w and cn(w). Let $x < w$ and $y < w$. If $x \circ y$ we are home, so suppose $x \wr y$. Then $x \ll w$, so E!$(w - x)$ under our assumption, and y is part of $(w - x)$. x is disjoint from $(w - x)$ and since $w = x + (w - x)$, by TID8 $x \between (w - x)$, and so $x >\!\!< (w - x)$. If $y = (w - x)$ then

$x \bowtie y$ and we are again home, so suppose $y \ll (w-x)$. Then $\mathrm{E!}(w-x-y)$ and $(w-x-y)$ is disjoint from both x and y. But since $(w-x) = (w-x-y)+y$ and $x \bowtie (w-x)$, we have either $x \bowtie y$ or $x \bowtie (w-x-y)$. Similarly, since $(w-y) = (w-x-y)+x$, we have either $y \bowtie x$ or $y \bowtie (w-x-y)$. In all cases, we therefore have $x(\bowtie^{w})_{*}\, y$, since x and y either overlap, or touch externally, or are connectible via $(w-x-y)$. Hence if $\mathrm{cn}(w)$, then $\mathrm{cts}(w)$. Conversely, suppose $\mathrm{E!}w$ and $\mathrm{cts}(w)$ and that $w = x+y$. If $x \circ y$, then $x \between y$. So suppose $x \wr y$. By definition of cts, we have that $x(\bowtie^{w})_{*}\, y$. Since x and y do not overlap, there is $z < w$ which touches x externally. Since $w = x+y$ and $z \wr x$, $z < y$. But since x touches z, and $x \wr y$, $x \bowtie^{w} y$, so in every case x and y either overlap or touch externally, and are therefore connected. Hence if $\mathrm{cts}(w)$, then $\mathrm{cn}(w)$, so

IT2 $\quad \mathrm{cts}(w) \equiv \mathrm{cn}(w)$

Notice that combining Tiles's definition with Whitehead's second definition of joining, WD5′—individuals are joined iff they have a binary sum—yields the result that in Whitehead's system every individual is self-connected, which conforms with his intention.

The fact that we are able to find a natural application for the integrity schema within mereo-topology does not constitute a retrospective endorsement for extensional mereology: the arguments against extensionality remain unaffected by the considerations of this section. Hence the systems examined in §2.9, which restrict the existence of sums and least upper bounds, but remain extensional, are still unacceptably strong as general theories of part and whole, though this is not to deny their local usefulness.

Continuity is only one characteristic relation among many, and while important, it is far from being the only important characteristic relation. Where the holding of a characteristic relation requires continuity, so that the only associated integral wholes are continuous (as are human beings, for instance), this does rule out discontinuous sums of individuals of the relevant kind. We may sum the chemicals in two disconnected human beings, but not the human beings themselves. But for some characteristic relations, continuity may be of little importance, so discontinuous wholes cannot be ruled out a priori. The political ties linking Alaska with the rest of the United States override its contiguity with Canada and make it part of a geographically disconnected country. It is, then, to some other important forms of integrity that we now turn.

9.4 Ontological Integrity

It is natural to consider forms of ontological dependence as possible characteristic relations for wholes. The first person to attempt an account on these lines in complete generality was Husserl, who, however, studiously avoided using the loaded term 'substance'. What Husserl has to say about what he calls 'wholes in the pregnant sense' (he recognized even sums as wholes in the widest sense) is well worth quoting:[14]

> By a whole we understand a range of contents which are all covered by a single foundation without the help of further contents. The contents of such a range we call its parts. Talk of the singleness of the foundation implies that every content is foundationally connected, whether directly or indirectly, with every content.

It requires little thought to see that this is a definition of an object which is integrated under the relation of foundation. But which relation of foundation? We have several to choose from. We have the relation which Fine and Smith designate by 'wf', we have 7 and $\not 7$, and on top of this we have the associated generic dependence concepts. Null's definition of 'individual' (ND2, § 8.6.2) amounts precisely to saying that an individual is a wf-integrated whole. The Fine–Smith notion of the foundational closure f(x) of an object x does not yet give us a wf-integrated whole, since there may be something outside f(x) which is dependent on something which x also depends on. So not all independent objects are wf-integrated wholes. Some are as it were too small. Others may be too big. If we take two disjoint independent objects, or even two disjoint wf-integrated wholes, their sum (assuming they have one) is independent, but no foundation relations cross between the summands, so the whole is not integrated.

When we look at families under the stronger foundation relations $\not 7$ and 7, we do not necessarily obtain individuals which sum the associated families, because occurrents may depend on continuants and vice versa. Since some objects are dependent on more than one individual, as a collision is dependent on the two objects which collide, tracing the foundation relations back and forth to give a family may result in bringing in individuals of the same category which cannot plausibly be said to form a natural whole. For this reason, it is better to remain with families rather than look for wholes integrated under

[14] HUSSERL 1984: A268 f./$B_1$275 f., 1970*a*: 475.

foundation relations. It is notable that Husserl begins his discussion of foundation by explicitly setting aside the relations an object has to others outside it,[15] which is cheating, since only then is it plausible to say that all objects which are foundationally connected form a whole of which they are parts, and the difficult cases are not confronted.[16]

The class of objects with which something is $\not{7}$-connected is of course different from that with which it is 7-connected (the one includes itself and its essential parts, the other not). Even supposing we could form the sums of both families and add the object itself to the 7-sum, while the sum $s + \sigma x \ulcorner s \, 7 \, x \urcorner$ is certainly part ($\subset$) of $\sigma x \ulcorner x \not{7} s \urcorner$, the converse need not hold, since there might be an s' on which s is weakly dependent, and this s' just *happens* not to be part of s, but could have been part of s. This case at least cannot be ruled out without further assumptions, whose credentials are unclear.

By including in a family all the things on which an object is rigidly dependent (in the weaker sense, for the sake of argument), we still do not collect together a self-sufficient family of objects. For an object may be *generically* dependent. Suppose we have an object which *must* stand in a certain relation R to some particular other object, i.e.

$$\Box(\mathrm{E!}s \supset sRs') \wedge \sim \Box \mathrm{E!}s' \wedge s \neq s'$$

then provided R satisfies a falsehood principle, s is $\not{7}$-dependent on s'. But often an object requires *an* object to which it stands in R without requiring a particular one, i.e. we have the weaker

$$\Box(\mathrm{E!}s \supset \exists x \ulcorner sRx \wedge s \neq x \wedge \sim \Box \mathrm{E!}x \urcorner)$$

It is in this sense that men need oxygen molecules, since they need some to respire and perform suitable bodily functions, but there is no single oxygen molecule which any man specifically needs. (As can be seen, the relation may be quite complex.) So to collect together the family of objects which *actually* sustain it, we have to include not just those on which it rigidly depends, but also those which *support* it, in doing the job of standing in the appropriate relation to it (for example, being an oxygen molecule which actually does get respired). Such

[15] 'A part is everything that an object "has" in a real sense, in the sense of something actually making it up, the object, that is, in itself, in abstraction from all contexts into which it is woven' (HUSSERL 1984: A225/B₁228). Findlay's translation (HUSSERL 1970a: 437) is misleading. Husserl then goes on to divide parts in this broad sense into dependent and independent, i.e. moments and pieces.

[16] Here the liberality of the underlying mereology in forming cross-categorial upper bounds tells against the Fine–Smith approach.

families usually include both continuants and occurrents, so again cannot form a natural sum. But there is also not much that is natural about them, especially when we take time into account. It is worth remarking again that any attempt to give a *formal* account of a relation like support is doomed to failure. For suppose we take the *supporters* of an object s to be that class of objects such that for *some* R, the above generic requirement holds, and these objects are actually such that s has R to them. Then the supporters of any object which cannot be the only contingently existing object (which includes most things) will be *all other* contingently existing objects, which is useless. When we construe support *so* weakly, it seems the only independent and integrated whole is the class of all contingently existing objects. If we understand dependence as weakly as possible, i.e. as wf, interpreted as above as

$$s \text{ wf } s' \equiv \Box(\text{E!}s \supset \text{E!}s')$$

then since trivially

$$\Box(\text{E!}s \supset \exists x^{\ulcorner} x = s \vee x \neq s^{\urcorner})$$

the supporters of any object in any world in which it exists are all the objects there are in that world.

These results indicate a general rule: the weaker the concept of dependence, the more comprehensive the associated independent families. For what there is is partitioned into equivalence classes by relations of foundational connectedness, and these classes are the larger, the weaker the characteristic relation. So weak dependence relations favour holistic or monistic concepts of independent object, and stronger ones favour pluralistic concepts. It is no accident that monists stress the interconnectedness and interdependence of things (the stress on internal relations among Anglo-Hegelians, for example) while atomists stress their independence and unconnectedness. Leibniz emphasizes the causal unconnectedness of monads, Wittgenstein the logical independence of states of affairs. In one case we end up with the Absolute, the One True Substance, in the other the world is a mere sum (*Gesamtheit*) of small independent objects, externally (contingently) related to one another. Hume's attack on necessary connections is part of his psychological atomism, and succeeds in showing that the necessity joining cause and effect is weaker than had generally been held. Since there are many different concepts of dependence, there is room for several concepts of independent object, so holistic and atomistic approaches can in theory live side by side.

When applying the concept of a family to ontological dependence in Husserl's fashion, the resulting independent wholes are less prominent in our experience than we might have expected. The one case where we get something easily recognizable is the strong relation of accident to substratum, provided we confine attention only to *monadic* (one-legged) accidents. The resulting families and wholes comprise precisely substances and their non-relational accidents at a given time. Perhaps it is not so surprising that Husserl's analysis best fits the kinds of example he most obviously had in mind.

One advantage of the foundational family analysis of complex wholes is that it offers a general solution to the problem of what holds complex unities together. We are familiar with this problem, especially as applied by various logicians to the unity of judgements or propositions. Traditionally this job was meant to be done by the copula, the binding element of a proposition, but the copula is an element of a sentence in which it occurs no less than any other word, so what binds it to the others? Frege's solution was to suppose that one or other part of a proposition was inherently incapable of separate existence, which is precisely ontological dependence of some kind, while Wittgenstein thought it was not a *part* of an elementary proposition which held it together but simply the hanging together of its parts (names). Husserl's analysis allows many different forms of hanging together, but the cement which binds a whole together is ultimately always a purely formal dependence relation: where objects cannot exist without one another, it is nonsense to look for chains to link them together.[17] It is tempting to take the availability of a foundational analysis of Husserl's sort as the occasion to do away with substances, substrata, and other bearers, and get by with bundles of mutually founding particulars. But while Husserl's approach *allows* such solutions, it does not enjoin them.

9.5 Functional Integrity

Most of the promising examples of integral wholes which we gave are not cases where the parts are linked by ontological dependence, but by other, quite ordinary material relations. Another significant kind of

[17] HUSSERL 1984: A272/B_1 279, 1970*a*: 477.

integral whole is obtained by considering a rather different kind of dependence among the parts, these forming what Rescher and Oppenheim call a *dependence system.*[18] To explain what this is, we need some terminology.

The properties an object may have fall into natural groups or spaces of contraries. For bodies, for example, we have the precise (fully determinate) mass, volume, shape, colour, temperature, velocity, and so on. Provided we speak only of fully exact properties, in each of these spaces no object can simultaneously have more than one property—it cannot have two masses, temperatures, etc. (assuming for the moment we are considering uniformly coloured objects with a uniform temperature, etc.) Such spaces of properties are often known by their own names, like 'mass', 'volume', and so on, and we speak of the mass *of* such-and-such a body, and so on. These terms figure in sentences like 'What's the book's colour?' and 'The pendulum's mass is 25 gm'. Following Johnson,[19] we call mass, volume and the like *determinables* and the precise properties making up such a space *determinates.* So for the determinable *mass* there are indefinitely many determinates, including 1 gm, 2 gm, 3.78 gm, and so on, and similarly for other determinables. Determinables are sometimes called 'attributes' and their determinates 'values' of these attributes. As the terminology suggests, we may think of determinables as *functions* from objects to determinates, so understanding a sentence like 'The mass of the pendulum is 25 gm' as an identity of the form '$M(p) = 25$ gm'.[20] We shall also employ the terms 'argument' and 'value' from function theory, but we avoid calling determinables functions for reasons which will become apparent.

Determinables need not just be scalar magnitudes, as the above examples might suggest. We must include vector magnitudes like *velocity* (30,000 km/h in the direction of Alpha Centauri) and *force* (617.82 N towards the centre of the earth, the force now holding me down in this chair). There are determinables which are not magnitudes, like *colour* and *nationality.* There are also multi-placed determinables, like *distance* (always of one thing to another) and *angle*

[18] RESCHER and OPPENHEIM 1955: 95. [19] JOHNSON 1921–4: 173 ff.

[20] The values are here taken to be abstract objects. That we may apply the usual logic of identity to such objects is an assumption that is presupposed in the following (thanks to Edgar Morscher for pointing out the need to make this assumption explicit). We may not be satisfied that this is the final analysis, but it would take us too far afield to see how we might do without abstract objects here.

(of one thing to another with respect to a third). By plugging gaps in multi-placed determinables we get determinables of fewer places, e.g. *distance from Rome.*

The values some determinables take for certain objects may be partly or wholly determined by the values of other determinables. In such cases we speak of *functional dependence* of one determinable on another. If we have a class of determinables ϕ and a determinable f, we say that f is *totally functionally dependent* on ϕ iff for all arguments A and B such that each member of ϕ has the same value for A as for B, f has the same value for A as for B. In certain cases, where f is one of ϕ or where f takes the same value all the time (f is constant), talk of *dependence* or *determination* is not especially apt, but we assume we are not considering such awkward exceptions. Consider an example. According to Newton's theory of gravitation, the gravitational force exerted by one body on another is determined by their two masses, their distance apart, and their direction *vis-à-vis* one another (remember force is a vector), *and nothing else.* In other words, the force exerted by body x on body y (a binary determinable) is totally functionally dependent on: the mass of x; the mass of y; the distance of x from y; and the direction of x from y (NB everything *at a time*). That this holds for all bodies at all times is precisely Newton's Law of Gravitation. Its usual expression in mathematical form (for the scalar magnitude only)

$$F = G\frac{m_1 . m_2}{r^2}$$

actually hides quite a lot of logical structure, involving quantification over bodies and times, and presupposes also a system of measurement, but we leave these details aside. When we say the force is dependent on the distance, we are speaking not of total but of *partial* functional dependence, which is to be analysed by saying that *other things being equal* (i.e. the masses and the direction; other factors are irrelevant), the force can only vary if the distance varies. The analysis of functional dependence along these lines was first undertaken by Grelling,[21] and while there are difficulties with his analysis, there is at present no more satisfactory alternative.[22] For our purposes it suffices if the general idea of functional dependence among determinables is clear.

[21] GRELLING 1987.
[22] On some of the problems, cf. SIMONS 1985c, 1987a.

Consider now a system of bodies in classical Newtonian mechanics. These are subject to various determinables of one and more places, and for the purposes of mechanics irrelevant ones like colour and price are neglected, while for simplicity idealizing assumptions about others are often made, for instance that the bodies are perfectly elastic, rigid, of negligible volume, or spherical in shape. In any Newtonian system the determinables *mass*, *position*, *velocity*, *force*, and *acceleration* remain under consideration. The gravitational force between bodies is functionally dependent on other factors in the way just mentioned; and assuming we are neglecting other attractive and repulsive forces like electromagnetic ones, just this and the forces exerted by collision need to be considered. The laws of mechanics then tell us about other functional relationships obtaining among the various determinables. Consider then the relation 'exerts a gravitational force upon'. The classical properties of gravity ensure that a closure system of objects under this relation is also a family; any such family may be called a *closed gravitational system*. However, what makes such a system into more than just a naturally integrated whole is the relation of complex *interdependence* among the determinables of its members. The relative positions of the bodies determine the forces they exert on one another, which determine their accelerations, which determine how their relative velocities change, which determine how their positions change, which determine how their attractive forces change, and so on. The impossibility of turning this set of simultaneous variations among values of mutually dependent determinables into a neat series of causes and effects is what inspired Russell to reject the classical notion of causality and propose replacing it with that of functional dependence.[23]

We may thus schematically define a dependence system, following Rescher and Oppenheim, as a collection of objects which form a family under a relation, to which a class of determinables apply, such that each member of the family has some determinable from the class which is functionally dependent upon some or all of the determinables of some or all of the remaining members.[24] Grelling and Oppenheim, who were the first to offer a definition of such systems, first called them 'determinational systems' (*Wirkungssysteme*), later preferring Koffka's term 'functional whole'.[25] Classical physics in particular

[23] Cf. RUSSELL 1913.

[24] RESCHER and OPPENHEIM 1955: 98. We have a slightly different terminology.

[25] GRELLING and OPPENHEIM 1938 (1987*a*, 1987*b*).

makes extensive use of such closed systems; in addition to gravitational systems we have closed electrostatic systems (isolated conductors), closed electric circuits, rigid bodies, stress systems, heat exchange systems, hydrodynamic systems, and more. Where these cannot be found in nature, they are simulated or approximated, or disturbing effects are minimized, equalized, or simply ignored for theoretical purposes.

In the physical and other sciences, the general idea is the same: the state of an object, as given by the values of its determinables, whether all of them (total state) or a collection of relevant ones, depends functionally on the values of other, polyadic determinables by which it is linked to other objects. The scientist is interested in the behaviour of such networks of objects, and their variety indeed is such that no single science can encompass them. The recognition of the general importance of relational networks and their ubiquity, in the formal, natural, and human sciences, while it has historical antecedents,[26] is predominantly a modern phenomenon. In certain extreme forms of structuralism the claims of this approach are overrated, but if the ideas of general systems theory are to be of use anywhere, it is perhaps in the study of formal properties of such systems.[27]

Contingency enters into functional wholes in two ways. The first is that, while the state of an object may be dependent on its relationships to others, it will usually be of no consequence to it *which* of these are hanging on the other end of the dependence connections. All that will matter is the values of their state determinables and those of the other polyadic determinables linking them to the object. The second kind of contingency concerns what are usually called boundary conditions. For instance, in a simple gravitational system consisting of just two bodies, what happens to them—whether they collide, fly past one another, or one orbits the other—depends on how things stand 'to start with'. In dealing with limited problems, the statement of boundary conditions amounts to little more than a signal as to where the problem begins and ends. If a statics problem begins: 'A ladder rests against a wall . . .', it is simply inappropriate to ask how it got there. But on a cosmic scale the apparent inescapability of boundary conditions points to contingence or accident as an objective feature of

[26] Cf. the works by J. H. Lambert in LAMBERT 1974. Kant also included reciprocity among his categories.

[27] On some of the unclarities surrounding the notion of system, cf. SIMONS 1987*a*. It seems that systems theory is awaiting its Grelling.

the world, and not the mere product of ignorance or deliberate limitation.[28] The two kinds of contingency interact in that how it is that, for example, just *this* body comes to be here exerting such-and-such a force on that body may in the end be a matter of chance.

The nature, strength, and complexity of the connectedness of functional wholes varies according to the kind considered. For rigid bodies, as Aristotle noted, their characteristic feature is their unified motion, resulting as it does from the rigid interconnection of their small parts, ensuring that incident forces are up to a limit internally transmitted so as to resist shearing or deformation and result in translation or rotation. The connectedness of bodies of liquid is more one of cohesion, since they do not resist shearing, while that of a body of gas is either compelled by external circumstances (enclosure in a vessel, for example) or due to gravity.

As we have seen, the relations binding organisms together are much more complex. Within a range of conditions in which it can survive, an organism maintains itself and its kind not by brute inert durability but by a series of mechanisms by which it adapts to changing external and internal circumstances. These self-regulating or homeostatic mechanisms are of great subtlety, and many of them function within internal sub-systems of high internal connectedness, whereby state fluctuations are compensated for so that crucial determinables remain around some optimal set of values. A simple example is given by the physiological mechanisms maintaining body temperature in mammals. The most striking adaptation mechanism is perception. Such survival-oriented mechanisms are not confined to individual organisms, but—as in the case of reproduction—extend to the whole species, and even across species. In certain cases of symbiosis, the dependence between different species and their members may even be ontological in nature.

The longevity of a species or an organism may be in part a matter of luck, but statistically is more likely to be traced to a relative ontological independence of causal contingencies within the range of admissible ambient conditions. In man this relative dependence is—or perhaps was—enhanced by the use of technology, from fire and clothing on the one hand to immunization and space capsules on the other, all of which serve to extend the range of environments in which

[28] This overlooks micro-indeterminacy. For Ehrenfels's rather dramatic dualistic account of the struggle between order and chance, see his 1916 (1948). The idea goes back to Plato, of course.

a man may survive by neutralizing the effects of otherwise inimical surroundings, frequently by acting as a kind of shield. The importance of neutralization is emphasized in his writing on causality by Ingarden,[29] who describes what he calls *relatively closed* (or relatively *isolated*) systems. These are, for a while, in certain respects and up to a certain limit wholly or largely causally isolated from the rest of the world. Contributing to such isolation are various kinds of *insulator*—thermal or electrical insulators are just two kinds—which screen the system from outside effects. Houses, armour, crash barriers, smoke-screens, and stand-offishness are all artificial insulators, with natural analogues like shells, scales, and warning threats. Such natural and artificial insulators may be highly selective in their action, as for instance the immune system or membranes. Insulators frequently mark a clear boundary between an integral whole and the rest of the world, like the skin of an animal or the walls of a medieval city. Such boundaries, whether natural or artificial, are objective, being marked by causal and material discontinuities. It is perhaps significant that perception functions primarily by focusing on such discontinuities.[30]

We have touched in passing on the idea that the very *existence* of an organism may depend on the *value* of some determinables pertaining to it and to its surroundings remaining within certain limits—for example, a man cannot live at one degree above absolute zero or at a million degrees centigrade. This is a case where ontological and functional dependence come into contact: the continued existence of a man depends on many factors remaining so that the ambient conditions do not endanger his existence to the point of making it impossible. Is it then necessary to subsume ontological and functional dependence under a single rubric after all, or is this some kind of Hegelian switch from quantity to quality?

In fact the examples give the clue to the answer. Any *constituted* object such as an organism continues to exist by virtue of the continuance of certain conditions in its material substrate, specifiable as ranges of admissible values among crucial determinables. Since these determinables concern in the first instance the substrate, there is nothing absurd in the *substrate*'s changing so that some of the values wander outside the range within which the constituted object can continue to exist. When the bronze gets too hot to remain solid, it

[29] On insulators and relatively isolated systems cf. INGARDEN 1974: §91a. The latter concept also comes to discussion in RUSSELL 1913.

[30] Against the exaggerated conceptualism here rejected, cf. WIGGINS 1980: ch. 5.

melts, and the statue that was in it ceases to exist—but not by virtue of the *statue* taking on a temperature at which it is molten, for there is no statue at such temperatures, only its now altered substrate, the bronze. While the constituted object exists, there is no harm in saying that *it* takes on the same values for these determinables as its substrate—both bronze and statue have the same temperature. This was indeed recommended as an important insight into the nature of superposition in Doepke's account in Chapter 6. But once the substrate wanders outside the limits, such a doubling becomes wrong, since the constituted object is no longer there to have determinate properties. This applies *whenever* an object's existence is dependent on more than just that of its component parts. Whenever a complex must have a certain structure or kind of structure, changes in the substrate which destroy this likewise destroy the complex, as, for example, merely altering the distances between a number of people may disperse a crowd. For continuants the requirements take the form of *persistence conditions*, which tell us under what circumstances the continuant indeed continues—to exist.

What does not form a dependence system in a certain respect might be called a *sum* in this respect. This use of 'sum' is a merely privative one, and is not directly connected with the mereological use. Thus for each kind of dependence system there is a corresponding kind of sum. So Köhler's famous three stones lying in different continents form a kinematic sum because they can be moved independently, and they form a gravitational sum not because they exert no gravitational force upon one another, but because they fail to form a gravitational family. The many different kinds of summativity which arise in connection with physical objects were the subject of a detailed investigation by Rausch.[31]

9.6 Temporal Integrity and Persistence

Only for something with temporal parts—an occurrent—does the question of its temporal integrity arise. We may ask of a state, process, or event what unites its different phases to make of them one object. What makes a string of concrete tones into a token of a certain musical

[31] RAUSCH 1937. Cf. the short summary of the main ideas in SMITH and MULLIGAN 1982: 74 ff. Rausch's analyses show that even relativized concepts of sum need not for all that be vague.

theme? This involves, as in the general case, special relations among the parts—in this case the tones—of the token theme, their relative positions in succession, relative lengths and spaces between them, relative pitches. Some of these determinables may vary, but there are limits to this variation. A musical theme may be played faster or slower, may be speeded up or slowed down, but we cannot play a tone per year and still expect to have a theme. Clearly the relative proximity in time of parts of a temporal whole—even if it is not temporally continuous—may be crucial in holding it together. As for spatial wholes, so for temporal ones the variation of such determinables produces different degrees of temporal integrity, with no sharp cut-off below which there are only collections and no individuals.

The example of a melody shows that what counts as a temporal individual is to some extent a matter of human stipulation, practice, or convention. However, melodies are temporal artefacts, and this freedom does not extend to naturally articulated temporal objects. Consider a natural event like an explosion—whether artificially *caused* or not. There is little room for decision as to what counts as part of it, and none whatever as to whether it is a single event. Its successive phases with their successively wider spans are tightly unified by causal relations. Not only different phases of the explosion, but also its different spatial parts are connected in being causally traceable back to the initial spark or germ of the explosion, which had very limited radius. Here causal relations are characteristic for the integrity of the whole. A triggering event need not itself be without proper parts: when a mass of uranium 235 becomes critical, this consists in some 2×10^{24} atoms splitting in a chain reaction over a period of a millionth of a second.

It seems plausible that very many natural temporal individuals owe their unity to causal relations among their parts, although where we draw the bounds of an event, given that causation is transitive, is not always perfectly clear. Other relations also perform a unifying role, however. The individual events befalling a certain person are strung together and called his or her *life*. While there are certainly causal connections among such events, causal relations lead into and out of the person's life, and it is not clear that all events in a life are indirectly causally connected. The unifying relation is simply that of *genidentity*, involving the same continuant, and is applicable not just to persons, but to all continuants. A description of this life is then a biography or history of the object concerned. Again, what is counted as a single individual continuant may be more or less determined by natural connections, and determining the identity of a continuant rests in part on determining when it begins and ceases to exist, under what conditions. This issue was important in showing the iterability of the Ship of Theseus Problem. A continuant which exists intermittently is precisely one whose history (in the sense of its life, not its biography) is temporally disconnected.

Throughout the centuries the identity of a continuant over time has frequently been held to be different from and somehow more suspect than its identity at a time. This comes to clear expression in Hume,[32] who attempts to find a trans-temporal *Ersatz* for identity under the belief that there cannot be real trans-temporal identity; but we can find echoes of it in many places, such as Chisholm's *entia successiva* or the predilection for event ontologies. While there is certainly a difference to be accounted for, this is not a difficulty, but simply another mark of the difference between continuants and occurrents.

The kind and degree of integrity which an individual continuant possesses *at* a time is a matter of the interrelations of the parts of the object which exist at that time. Should the putative parts fail to come up to the mark, we indeed have no individual, but only an interrelated plurality of continuants. For occurrents, the question of integrity *over* time has exactly the same form: one concerning the connections between the putative parts of the event in question, which parts may of course take place at different times. But for continuants it is not the object itself which hangs together over time, but rather its life. Continuants as such can have no temporal integrity, nor can they lack it. The question whether a given continuant exists (E!) stands or falls with the question whether there are occurrents which find themselves together to form a history (i.e. a life) for it. As a connecting relation, genidentity itself is not competent to *secure* the existence of a continuant via the integration of an occurrent, since genidentity itself *presupposes* the existence of the continuant. Genidentity could at best underline the unification of occurrents to a history (i.e. a life) whose integrity is secured by other relations. It is plausible to suppose that in most cases it is the existence of a continuant which binds together via genidentity the disparate events of a history; the continuant enjoys ontological priority over the life. Since this continued existence then looks like a bare fact, unsupported by any objective integrity, we can

[32] *A Treatise of Human Nature*, bk. I, pt. iv, §2.

structure—and the continuation of sustaining processes. In a highly complex case like that of an organism, all of these play a part, and the individual is neither something purely inert like a medium, nor a pure disturbance like a wave, but combines features of both. The particular conditions under which a continuant of a kind continues to exist are its persistence conditions. These may even allow that the continuant go out of existence and come back into existence again. In that case, the life of the continuant is not temporally connected, but nothing in the concept of a continuant requires that a life always be connected, so long as we can find a basis for identifying across the gap. The concept of a mass of matter, like the concepts of continuant sum considered in Chapter 5, is that of a continuant whose persistence conditions approach the minimal—structure, spatial distribution, properties do not matter. All that matters is that none of the stuff is destroyed completely.

9.7 Structure and Gestalt

Any complex will, as Rescher and Oppenheim rightly say in their third condition, have a *structure*. It is not immediately clear what this means, but the *Oxford English Dictionary* points the way to finding out with the following gloss for one very general meaning:

> The mutual relation of the constituent parts or elements of a whole as determining its peculiar nature or character

Used in this way, 'structure' means something abstract: the *manner* in which something is constructed or the *way* its parts are related. The concrete sense in which 'structure' refers to the object which *has* structure in the first sense:

> An organized body or combination of mutually connected and dependent parts and elements

we shall not employ, but rather speak of the structur*ed* object or complex.

From this preliminary gloss we can extract four conditions for something to be a structured whole or complex:

(1) It must consist of several parts.
(2) These parts must stand to one another in certain relations (principally of dependence).

(3) These relations must connect the parts of the complex to one another.
(4) The total relation of all the parts is characteristic for the kind of complex in question.

These conditions are formal: nothing is said about the kinds of parts or relations. The first condition (1) rules out (as was to be hoped) absolute simples such as points, atoms or monads. These are all *structureless*. Conditions (2) and (3) are automatically taken care of if we consider *R*-integral wholes: all the parts (of some division) are connected to one another by chains of instances of *R* and its converse. The idea of condition (3) is to rule out unconnected sums. But when are objects *unconnected*? Not, surely, when they are unrelated, since any objects are related, if only by difference or dissimilarity. Why then do we discount such relations as connections? Here, it seems, we must draw a distinction, similar to that drawn above in §9.3, between purely *formal* relations like identity, difference, similarity, dissimilarity, and *material* relations like being to the left of, touching, causing, and being twice as large as.[37] Objects would be connected between which some positive material relation holds, and unconnected where not. This is little more precise than what we started with, but seems to show what is needed.

So we cannot be exactly sure as to what counts as unstructured, even where it has more than one part. As we have emphasized, sums (among individuals) and classes (among pluralities) appear as such to be structureless. But in the case of classes, even they may differ in the *number* of members they have; this is characteristic for their kind. It may be overstretching the word to call the number of elements in a class its *structure*, but we shall find that it pays to include such marginal cases along with other, richer ones, when considering abstract structures themselves.

What we have not covered hitherto is the idea, found in (4), of the *total* or *overall* relation of all the parts, as distinct from the multitude

[37] A slightly different distinction is drawn by Meinong between ideal and real relations. Cf. MEINONG 1899 (1971: 394 ff.). FINDLAY 1963: 143 suggests that ideal relations fail to bind their terms into a unity, whereas real relations produce a whole quite as real as its parts. For Meinong there is a characteristic kind of complex for each kind of relation. The most attenuated kind of complex is precisely a class, held together—if that is the right expression—by the mere relation of coexisting. Such classes arising solely out of 'and'-relations, are recognized by Bolzano, Reinach, and Russell, *inter alia*. Cf. SMITH and MULLIGAN 1982: 103, n. 127.

of binary and other relations between and among the parts. In fact, we need even to consider the various kinds and properties of the individual parts as part of this overall relation or pattern. In algebra or model theory, it is usual to define a 'relational structure' as a *sequence* $\langle D, R_1, R_2, \ldots \rangle$ where D is the domain (set of parts) and R_1, R_2, etc. the specific relations among these elements. The structure is here as it were a sort of *super-relation* on the domain and relations. The kind of the structure is determined by conditions on the size of domain, number, adicity, and properties of the relations, usually set out in algebra as postulate sets (for example, for a group, ring, vector space, or affine space). It is clear, however, that the use of a mere list (plus postulates) is above all a notational expedient and, while it gets all the needed details down on paper, does not represent a deeper analysis of the idea of a structure. For one thing, one and the same kind of structure can be represented in model-theoretic or algebraic terms in distinct ways, a good example being the many equivalent ways of defining a Boolean algebra. The structure in this case is so rich that a number of different selections of its properties enable us to infer the rest. Another set of considerations not presented perspicuously by the algebraic approach are those about how one structure may be contained in another, or definable in another, how a structure may be 'loose' in allowing numerous differently structured realizations. Simply lopping off some of the relations or some of the conditions on them is too insensitive.

The alternative approach which we follow stems from the attempts of Grelling, Oppenheim, and Rescher to analyse the concept of a Gestalt, which is scarcely to be distinguished from that of a structure. This is the original sense of 'Gestalt' going back to the work of Ehrenfels, and not to be confused with the concept of a functional whole, for which the term also came to be used. It is worth noting here briefly that while the work and terminology of Ehrenfels was historically the most influential, going back to his original 1890 essay,[38] the same concept was elaborated and exploited at the time by two other former students of Brentano, Meinong and Husserl, who used the terms 'founded content' and 'figural moment' respectively for what Ehrenfels called a 'Gestalt-quality'.[39]

[38] EHRENFELS 1890.

[39] MEINONG 1891, which is a commentary on Ehrenfels's essay, and HUSSERL 1970*b*: ch. 11, which is independent of Ehrenfels. Meinong later used the term 'founded object'.

The works of Grelling, Oppenheim, and Rescher constitute an attempt—perhaps the only one—to give an exact analysis of Ehrenfels's concept in logical terms in such a way as to both fit the usual examples and make *truthful* sense of Ehrenfels's two criteria:

E1 A complex is something other than the sum of its parts (Supersummativity)

E2 A Gestalt remains invariant under transposition from one complex to another (Transposability)

In giving an account of what a Gestalt is, while we follow their account in most things, some of the terminology and some of the details are slightly different.[40]

We start again with determinables, as described previously. Where the arguments of a determinable form a topological space, they are known as *positions* and the determinable an *S-determinable* ('S' short for 'state'). Examples of such positions are points and instants. In fact, the requirement that the arguments be points of a topological space can be trivially satisfied, since the trivial or *discrete* topology is the one in which every subset of the domain is an open set, and a sub-space is simply a subset. It is the topology induced by the trivial or discrete *metric* on a set, for which the distance between any two distinct points is equal to 1. In normal applications, it is clear that what is meant is a non-trivial topology, though the trivial case can be illuminating in considering mathematical Gestalten.

We take a domain of positions *D* and a class *G* of S-determinables all of which are defined (make sense) on such positions, such that each S-determinable is assigned a single value for each position (if it is monadic) or each *n*-tuple of positions (if it is *n*-adic), and call the resulting assignment a *configuration*.[41] Again, Rescher and Oppenheim specify that the range of determinates which are possible values for each determinable be subject to a topological ordering (for example, the three-dimensional ordering of all possible colours). In concrete cases it is usually clear what this amounts to, though again the condition can be vacuously satisfied.

[40] For a fuller account of the differences and our reasons for making them cf. SIMONS 1985*c*, 1987*a*.

[41] GRELLING and OPPENHEIM 1938 use the term 'complex', but we have used this term for the concrete whole, whereas a configuration may be a quite arbitrary section of a complex, e.g. an array of points and for each point a light-intensity (no mention of colour). RESCHER and OPPENHEIM 1955 use the term 'configuration' in a somewhat different sense from us.

Let C_1 and C_2 be two configurations. Then they are in *perfect correspondence* iff the following conditions obtain:

PC1 The domains of positions D_1 of C_1 and D_2 of C_2 are homeomorphic under the topologies employed (let h: $D_1 \rightarrow D_2$ be the homeomorphism).

PC2 Each involves the same class G of determinables.

PC3 The values of the determinables are equal for corresponding positions, i.e.

$$\forall g \varepsilon G \ulcorner \forall x \varepsilon D_1 \ulcorner gx = g(h(x)) \urcorner \urcorner$$

For example, two visually identical reproductions of an old master in two copies of an art book are, in respect of visual determinables such as colour, lightness, relative distance, and angle of points on the surface of the pages, in perfect correspondence. Note that by including polyadic determinables like distance and angle we ensure not only topological homeomorphism but also geometric congruence—were these to be left out of the set of S-determinables in the configuration, the pictures might be distorted with respect to each other in various topologically permissible but artistically inadmissible ways.

Focusing attention on a single S-determinable, we may call the assignment of particular values of it to each position in the domain of configuration a *distribution*. Two distributions which are exactly alike we call *equal*. Two configurations then stand in perfect correspondence to one another when all their S-determinables have equal distributions. However, distributions may also be *similar* in various ways without being equal, and where a number of distributions are similar, they have common attributes. These may themselves be considered determinates of determinables applicable to the distributions, which we accordingly call *D-determinables*. Two distributions are then equal when *all* their D-determinables take the same values. For example, if our two reproductions of an old master are such that one is half the size of the other and is in black and white, then the S-determinables *distance* and *hue* are different between the two. But other determinables concerning the distribution remain constant, such as the *proportions* between corresponding stretches and *darkness* of corresponding points. The smaller picture is not equal to the larger, but is still very similar to it. A quite different similarity is obtained, for example, by keeping the size and colour the same but reversing the picture as in a mirror.

Two configurations *correspond* when PC1 and PC2 are retained but PC3 is replaced by:

C3 For some, but not necessarily all determinables $g \varepsilon G$, the distributions for C_1 and C_2 are similar in some, but not necessarily all respects, i.e. are alike in values of some D-determinables.

Perfect correspondence is then a special case of correspondence. The *kind* of correspondence is determined by the topological nature of D_1 and D_2, the members of G, and the particular D-determinables of the distributions of the selected members of G which remain constant.

Each specific correspondence defines an equivalence relation on configurations, and that which all configurations in an equivalence class under this relation have in common is their Gestalt. If we call the operation or mapping taking one configuration into another, corresponding configuration a *transposition*, then a Gestalt is that which remains invariant through all transpositions.[42]

Ehrenfels's first and still the clearest example of a Gestalt is a *melody*. In this case the configurations are concrete tone sequences. These may be played at different tempi, on different instruments, with different dynamics, in different keys. All such variations are admissible transpositions, although only the last is a transposition in the narrow musical sense. But throughout the transpositions, some D-determinables keep the same values over the sequence of tones, namely the relative lengths and successive intervals (pitch proportions) of the tones. In practice there are limits to how quickly or slowly, how low or high the melody may be played, but we may suppose these to be wide enough to allow sufficient variation within them. A melody, however, is just one kind of musical Gestalt. There are many more. Some are clearly related to melody, for instance the inversions and retrograde sequences of canonic and twelve-tone music (in the latter the Gestalt is the tone row), or a motif heard now in the major, now in the minor mode (not exactly a melody), or the constant ground bass in a *chaconne*. Others, such as characteristic rhythms or keys, have little to do with melody.

From the examples given, it can readily be seen how complexes fulfil the two Ehrenfels criteria. The second is fulfilled by definition, while the first is fulfilled so long as the Gestalt in question is richer in

[42] The transpositions form a group, and the Gestalt is the group invariant. On the equivalence of equivalence relation and group formulations, cf. GRELLING 1969.

structure than simply that of having so many parts. Since the limits on transpositions can be freely altered, relaxed, tightened up, there is all the generality we require in determining the concept of Gestalt. For instance, tightening up the conditions on admissible performances of a work of music, as in total serialism, takes musical works (themselves large Gestalten) in a quite different direction from the loosening allowed in aleatoric music, or the freedom allowed in improvisations and cadenzas.

Since a Gestalt is an attribute, albeit a highly complex one, of a configuration, and a configuration is simply a selection or 'section' out of a complex, Gestalten are attributes of complexes. It is also an attribute of the complex as a whole (as Rescher and Oppenheim required in their first condition) and only rarely also an attribute of proper parts of the complex. An exception, where the same Gestalt may be found both in a complex and in proper parts of it, is the interference pattern found in a hologram. From both perspectives, it appears that Ehrenfels's own term 'Gestalt-quality' is quite appropriate, though we have followed later tradition in using the shorter term. There is the occasional danger with 'Gestalt', however, of using it for the complex instead of its characteristic form, a danger which is real, because the word did acquire this ambiguity, as Grelling *et al.* pointed out.

That a given kind of complex must have a certain Gestalt implies neither that it cannot change its parts nor that it cannot change its Gestalt—if it is a continuant complex. For a continuant may take any one of a number of more specific Gestalten and yet, because these are all specifications of a looser Gestalt, still have the latter. There are some geometrical shapes that a man cannot take on and still continue to exist, such as that of a perfect cube, but a man's actual shape changes whenever he moves, so all these more specific shapes must be within the range allowed for the topological form of a man. For just this reason, a continuant may also within limits gain or lose parts and still continue to exist. While temporal complexes of a given kind cannot *change* their parts or Gestalt, we can still for them, as for continuants, find *variation* among different instances of the one kind, of the sort typified by different interpretations of one and the same piece of music.

Concluding Remarks

> A est B idem est quod A esse coincidens cuidam B. seu A = BY . . . si A sit B et B sit C, A erit C . . . cum . . . A = BY et B = CZ. Ergo A = CYZ. seu A continet C.
>
> Leibniz, *Generales Inquisitiones de Analysi Notionum et Veritatem*, §§16, 19

> omnes partes integrales habent ordinem quendam ad invicem. sed quaedam habent ordinem in situ: sive consequenter se habeant, sicut partes exercitus; sive se tangant, sicut partes acervi; sive etiam colligentur, sicut partes domus; sive etiam continuentur, sicut partes lineae. quaedam vero habent insuper ordinem virtutis: sicut partes animalis, quarum prima virtute est cor, et aliae quodam ordine virtutis dependent ab invicem. tertio modo ordinantur ordine temporis: sicut partes temporis et motus.
>
> Thomas Aquinas, *Summa Theologiae* III, q. 90, 3, ad. 3

As we found it, mereology was dominated by a single theory: classical extensional mereology (CEM), present in two logical guises—the Calculus of Individuals and Mereology—each in a number of variants. CEM is algebraically neat: only a complete Boolean algebra is neater. It is also strong: how strong can be seen from § 1.4. CEM is

(1) tenseless
(2) non-modal
(3) upholds extensionality of parts; and
(4) upholds the conditioned existence of general sums.

Of these characteristics (1) and (2) are privative, (3) and (4) are positive. Among approaches at variance with CEM, most retain (1)–(3) and drop (4) in favour of some weaker conditional existence principle (see § 2.9).

In the face of apparent temporal and modal variation, two major strategies have been followed. The first ignores modality and attempts to retain (3) by recourse to an ontology of four-dimensional objects. This fails because modality still distinguishes objects which (3) would identify. A parallel move to five-dimensional objects, with modality as the fifth dimension, has not been seriously contemplated, which in view of the conceptual difficulties facing the four-dimensional strategy is perhaps as well. The second strategy, Chisholm's, takes both time and modality seriously, but preserves (3) by putting forward an ontology, opposed to common sense, of modally and temporally invariable objects. The problems this approach has are to find good positive arguments in its favour and to account for appearances. In my view, the price paid for retaining (3) is too high whichever strategy is followed.

Nevertheless, if (3) and (4) are not universally acceptable, there are areas where (3) alone or both together may be correctly applied (Chapter 4). Seeing this involves recognizing two categories of particular against which there exists a deep prejudice in philosophy: pluralities and masses. Taking these into account shows that the concepts of mereology have not one but a number of analogous interpretations; what these have in common are the formal properties of these concepts as captured in their algebra.

The question is then what mereology looks like when *none* of (1)–(4) is followed. We followed up consecutively the effects of adding time (Part II) and modality (Part III). But here let us isolate the result of giving up (3) and (4) in a non-temporal, non-modal context. What is the minimum we can require of a relation if it is to be one of proper part to whole? I suggest we need the following four principles:

Falsehood	$x \ll y \supset E!x \wedge E!y$
Asymmetry	$x \ll y \supset \sim (y \ll x)$
Transitivity	$x \ll y \wedge y \ll z \supset x \ll z$
Supplementation	$x \ll y \supset \exists z \ulcorner z \ll y \wedge \sim z \approx x \wedge \sim x \ll z \wedge \sim z \ll x \wedge \sim \exists w \ulcorner w \ll z \wedge w \ll x \urcorner\urcorner$

We have left the quantifiers off: if we universally quantify, the first principle is derivable from the usual quantifier laws in free logic, but if we take the variables instead to be free (i.e. as parameters), Falsehood is required. Notice which logical concepts are presupposed: identity and existence. The import of the principle Supplementation is clearer if we define

Df. $<$ $\quad x < y \equiv x \ll y \vee (E!x \wedge x \approx y)$

Df. $\circ$ $\quad x \circ y \equiv \exists z \ulcorner z < x \wedge z < y \urcorner$

Supplementation then emerges in the familiar guise

$$x \ll y \supset \exists z \ulcorner z \ll y \wedge \sim (z \circ x) \urcorner$$

This is then the acceptable Weak Supplementation Principle of §1.4, adjusted to allow for free logic as a basis.

This, I suggest, is the formal skeleton of the meaning of 'part'. For the temporal version, modify all modifiable predicates by 'at *t*' and slip in '$\forall t$' after other quantifiers with wide scope, i.e. add 'always'. For modal and modal/temporal versions, replace universal closure by necessary universal closure (§7.1). Now we see the point of the principle of Falsehood in the basic version.

If this is all there essentially is to the part-relation, why can stronger principles sometimes apply? The answer lies not in the part-relation itself but in the nature of the objects to which it applies. Among certain *regions* of objects we have extensionality and essentiality of parts; such objects fulfil these principles, but the principles are not constitutive of the part-relation, which is *formal*, i.e. applies in all regions. So we must distinguish *global* mereology, for which the four principles above provide the formal properties, and various *local* mereologies, where these alone do not suffice to capture the mereological properties of the objects in question. The fault of CEM is essentially that of making global what is only local.

The net effect of rejecting CEM in full generality is to make mereology more complicated, but also more interesting. That most modern ontology passes mereology by is due to the inadequacy of CEM as a conceptual instrument capable of use in the variety of issues found in ontology, coupled with a historically misinformed supposition that mereology is something for nominalists only. If I am right about the formal nature of mereology, it should be *neutral* on the issue of nominalism/realism. If mereology can be applied universally (and that has not been shown here, because we have not discussed abstract objects), then it should regain a central position in ontology; along with existence and identity, it should take us to the heart of many ontological issues. The topics covered in Parts II and III are meant to show this: Part II for existence in and through time, for identity, matter, and form, and Part III for essence, dependence, substance, unity, integrity, and form. It is notable how many of the issues in Part III are under-represented in the contemporary literature, although they loom large in traditional ontology, where it was felt to pay to be discriminating about different kinds of parts, as the quotation from Aquinas at the beginning of this section shows.

The contemporary field ontologist is better equipped than his predecessors because he is familiar with formal systems, a device we owe to Leibniz. The acquisition of this tool does not render the old resources—experience, wit, authority, the lore of language—obsolete, but it shifts the ontologist's role. He now has a theoretically endless supply of formal templates to hold up to the untamed phenomena, and his job now consists in fair part in constructing such formal systems and testing them for their applicability. It is tempting to be led by the attraction of internal properties of the formalism either into taking the world to be tamer than it is, or into a relativistic, pragmatic attitude to ontology which can be seen at its most significant in Quine. Such attraction, for which again Leibniz is responsible, lies behind CEM's two errors of omission and two of commission. For different regions, we need different templates, and it is mainly the templates which must be bent to fit, not the world. In the case of mereology, this fails to descend to utter relativism because the theory has a formal skeleton and a range of analogous fleshings out which provides unity in the diversity.

As to the *content* of the ontological theses I have upheld as emerging from a rejection of CEM, I am aware of a chastening old-fashionedness in having emphasized, among other things

(1) the variety of meanings of 'part' and cognate concepts
(2) their analogous connections
(3) the centrality of continuants in ontology
(4) the paradigms of which are natural units, especially organisms
(5) the distinction of matter and form (structure)
(6) the importance of composition and constitution
(7) the distinction of essential from accidental and normal parts
(8) the distinction between dependent and independent particulars
(9) the idea of integrity, and its degrees

all of which points, within limits and suitably qualified, back to Aristotle.

Bibliography

Notes:

(1) While this list of works referred to covers mereology fairly comprehensively, it does not aim at completeness. For an annotated bibliography of extensional and post-Brentanian mereology and dependence theory see SMITH 1982: 481–552 and SMITH 1985: addenda. A similar bibliography of Gestalt theory is in SMITH 1987.

(2) Classic works, such as those of Aristotle, Descartes, Spinoza, Locke, Leibniz, and Hume are not listed, but referred to by title in the notes, and references are given in a way which enables most editions to be used.

ALLETON, D. J., CARNEY, E., and HOLDCROFT, D. (eds.) 1979, *Function and Context in Linguistic Analysis*. Cambridge, Cambridge University Press.

ALTMAN, A., BRADIE, M., and MILLER, F. D. 1979, On Doing without Events. *Philosophical Studies* 36, 301–7.

ANSCOMBE, G. E. M. 1979, Under a Description. *Noûs* 13, 219–33.

ARMSTRONG, D. M. 1978, *Universals and Scientific Realism*. 2 vols., Cambridge: Cambridge University Press.

BÄCK, A. T. 1987, *On Reduplication: Logical Theories of Qualification*. Munich: Philosophia.

BÄUERLE, R., SCHWARZE, C. and STECHOW, A. v. (eds.) 1983, *Meaning, Use, and Interpretation of Language*. Berlin: de Gruyter.

BEALER, G. 1982, *Quality and Concept*. Oxford: Clarendon Press.

BENCIVENGA, E. 1976, Set Theory and Free Logic. *Journal of Philosophical Logic* 5, 1–15.

—— 1980, Free Semantics for Definite Descriptions. *Logique et Analyse* 23, 393–405.

BLACK, M. 1964, *A Companion to Wittgenstein's 'Tractatus'*. Ithaca: Cornell University Press.

—— 1971, The Elusiveness of Sets. *Review of Metaphysics* 24, 614–36. Reprinted in BLACK 1975: 85–108.

—— 1975, *Caveats and Critiques*. Ithaca: Cornell University Press.

BOCHEŃSKI, I. M., CHURCH, A., and GOODMAN, N. 1956, *The Problem of Universals*. Notre Dame: University of Notre Dame Press.

BOLZANO, B. 1827, *Athanasia, oder Gründe für die Unsterblichkeit der Seele*. Sulzbach: Seidel. 2nd edn. 1838. Reprinted as vol. 2 of *Dr. B. Bolzano's gesammelte Schriften*, Vienna: Braumüller 1882. Facsimile reprint of the 2nd edn., Frankfurt/Main: Minerva, 1970.

—— 1834, *Lehrbuch der Religionswissenschaft*, vol. 1. Sulzbach: Seidel. Reprinted as vol. 3 of *Dr. B. Bolzano's gesammelte Schriften*, Vienna: Braumüller 1882.

—— 1975, *Paradoxien des Unendlichen*. Text edited by Fr. Přihonský. Edited, with an introduction and notes, by B. van Rootselaar. Hamburg: Meiner. (1st edn. 1851).

BOLZANO, B. 1981, *Wissenschaftslehre*. 4 vols. 2nd reprint of the 2nd edition, Leipzig, 1929. Aalen: Scientia (1st edn. 1837).

BOSTOCK, D. 1979, *Logic and Arithmetic*, vol. 2: *Rational and Irrational Numbers*. Oxford: Clarendon Press.

BRAND, M. 1976, Particulars, Events, and Actions. In BRAND and WALTON 1976: 133–58.

—— 1981, A Particularist Theory of Events. *Grazer Philosophische Studien* 12–13, 187–202.

—— 1984, *Intending and Acting: Toward a Naturalized Action Theory*. Cambridge, Mass: MIT Press.

BRAND, M. and WALTON, D. (eds.) 1976, *Action Theory*. Dordrecht: Reidel.

BREITKOPF, A. 1978, Axiomatisierung einiger Begriffe aus Nelson Goodmans *The Structure of Appearance*. *Erkenntnis* 12, 229–47.

BRENTANO, F. 1933, *Kategorienlehre*, ed., with introduction and notes, A. Kastil. Leipzig: Meiner. Reprinted Hamburg, 1974. English translation: BRENTANO 1981.

—— 1976, *Philosophische Untersuchungen zu Raum, Zeit und Kontinuum*. From the *Nachlaß*, with notes by A. Kastil, ed. and intr. S. Körner and R. M. Chisholm. Hamburg: Meiner.

—— 1981, *The Theory of Categories*, trans. R. M. Chisholm and N. Guterman. The Hague: Nijhoff.

BROAD, C. D. 1933, *An Examination of McTaggart's Philosophy*, vol. 1. Cambridge: Cambridge University Press. Reprinted New York: Octagon, 1976.

BRODY, B. A. 1980, *Identity and Essence*. Princeton: Princeton University Press.

BUNGE, M. 1966, On Null Individuals. *Journal of Philosophy* 63, 776–8.

—— 1977, *Treatise on Basic Philosophy*, vol. 3: *Ontology I: The Furniture of the World*. Dordrecht: Reidel.

BURGE, T. 1977, A Theory of Aggregates. *Noûs* 11, 97–117.

BURKE, M. B. 1980, Cohabitation, Stuff and Intermittent Existence. *Mind* 89, 391–405.

BURTON, M. 1967, Sponges (Phylum Parazoa). In *Larousse Encyclopedia of Animal Life*. London: Hamlyn, 1967: 33–9.

BUTLER, R. J. (ed.) 1965, *Analytical Philosophy, Second Series*. Oxford: Blackwell.

BUTTS, R. and HINTIKKA, J. (eds.) 1977, *Logic, Foundations of Mathematics and Computability Theory*. Dordrecht: Reidel.

CAMPBELL, K. 1976, *Metaphysics: an Introduction*. Encino: Dickenson.

CANTOR, G. 1932*a*, *Gesammelte Abhandlungen mathematischen und philosophischen Inhalts*, ed. E. Zermelo. Berlin: Springer. Reprinted Hildesheim: Olms, 1966.

—— 1932*b*, Aus dem Briefwechsel zwischen Cantor und Dedekind. In CANTOR 1932*a*: 443–51. English translation of the letter of 28 July 1899, CANTOR 1967.

—— 1967, Letter to Dedekind. In VAN HEIJENOORT 1967: 113–17.

CARNAP, R. 1956, *Meaning and Necessity: A Study in Semantics and Modal Logic*, 2nd edn. Chicago: University of Chicago Press (1st edn. 1947).

CARNAP, R. 1958, *Introduction to Symbolic Logic and its Applications*. New York: Dover.
CARTWRIGHT, H. 1963, Classes, Quantities and Non-Singular Reference. Ph.D. Dissertation, University of Michigan.
CARTWRIGHT, R. 1975, Scattered Objects. In LEHRER 1975: 153–72.
CHISHOLM, R. M. 1973, Parts as Essential to their Wholes. *Review of Metaphysics* 26, 581–603.
—— 1976, *Person and Object: A Metaphysical Study*. London: Allen & Unwin.
—— 1978, Brentano's Conception of Substance and Accident. In CHISHOLM and HALLER 1978: 197–210. Reprinted in CHISHOLM 1982: 3–16.
—— 1979, Objects and Persons: Revisions and Replies. In SOSA 1979: 317–88.
—— 1982, *Brentano and Meinong Studies*. Amsterdam: Rodopi.
CHISHOLM, R. M. and HALLER, R. (eds.) 1978, *Die Philosophie Brentanos*. Amsterdam: Rodopi.
CHURCH, A. 1940, A Formulation of the Simple Theory of Types. *Journal of Symbolic Logic* 5, 56–68.
—— 1956, *Introduction to Mathematical Logic*, 2nd edn. Princeton: Princeton University Press.
—— 1976, Schröder's Anticipation of the Simple Theory of Types. *Erkenntnis* 10, 407–10.
CLARKE, B. L. 1981, A Calculus of Individuals Based on 'Connection'. *Notre Dame Journal of Formal Logic* 22, 204–18.
—— 1985, Individuals and Points. *Notre Dame Journal of Formal Logic* 26, 61–75.
CLAY, R. E. 1961, Contributions to Mereology. Ph.D. Dissertation, University of Notre Dame.
—— 1965, The Relation of Weakly Discrete to Set and Equinumerosity in Mereology. *Notre Dame Journal of Formal Logic* 5, 325–40.
—— 1968, Consistency of Leśniewski's Mereology Relative to Real Number System. *Journal of Symbolic Logic* 33, 251–7.
CRUSE, D. A. 1979, On the Transitivity of the Part–Whole Relation. *Journal of Linguistics* 15, 29–38.
DĄMBSKA, I. 1978, François Brentano et la pensée philosophique en Pologne: Casimir Twardowski et son école. In CHISHOLM and HALLER 1978: 117–30.
DAVIDSON, D. 1980, *Essays on Action and Events*. Oxford: Clarendon Press.
DAVIDSON, D. and HARMAN, G. (eds.) 1972, *Semantics of Natural Languages*. Dordrecht: Reidel.
DE LAGUNA, T. 1922, Point, Line, and Surface, as Sets of Solids. *Journal of Philosophy* 19, 449–61.
DOEPKE, F. C. 1982, Spatially Coinciding Objects. *Ratio* 24, 45–60.
—— 1986, In Defence of Locke's Principle: a reply to Peter M. Simons. *Mind* 95, 238–41.
DORN, G. and WEINGARTNER, P. (eds.) 1985, *Foundations of Logic and Linguistics*. New York: Plenum.

DRETSKE, F. 1967, Can Events Move? *Mind* 76, 479–92.

DUNN, J. M. and BELNAP, N. D. 1968, The Substitutional Interpretation of Quantifiers. *Noûs* 2, 177–85.

EBERLE, R. A. 1967, Some Complete Calculi of Individuals. *Notre Dame Journal of Formal Logic* 8, 267–78.

—— 1968, Yoes on Non-Atomic Systems of Individuals. *Noûs* 2, 399–403.

—— 1969, Non-Atomic Systems of Individuals Revisited *Noûs* 3, 431–4.

—— 1970, *Nominalistic Systems*. Dordrecht: Reidel.

EHRENFELS, C. VON, 1890, Über 'Gestaltqualitäten'. *Vierteljahrsschrift für wissenschaftliche Philosophie* 14, 249–92. Reprinted in WEINHANDL 1967: 11–43, and in EHRENFELS 1987. English translation, EHRENFELS 1987*a*.

—— 1916, *Kosmogonie*. Jena: Diederichs. English translation, EHRENFELS 1948.

—— 1937*a*, Über Gestaltqualitäten. *Philosophia* (Belgrade) 2, 139–41. Reprinted in WEINHANDL 1967: 61–3, and in EHRENFELS 1987*b*. English translation, EHRENFELS 1987*b*.

—— 1937*b*, On Gestalt-Qualities (Last Remarks). *Psychological Review* 44, 521–4. Reprinted in SMITH 1987.

—— 1948, *Cosmogony*. New York: Comet.

——1987*a*, On 'Gestalt-Qualities', trans. B. Smith. In SMITH 1987.

——1987*b*, *Philosophische Schriften*, Bd. 3: *Psychologie, Ethik, Erkenntnistheorie*, ed. R. Fabian. Munich: Philosophia.

EMBREE, L. (ed.) 1983 *Essays in Memory of Aron Gurwitsch*. Lanham, MD: University Press of America.

EVANS, G. and McDOWELL, J. (eds.) 1976, *Truth and Meaning: Essays in Semantics*. Oxford: Clarendon Press.

FABIAN, R. (ed.) 1985, *Christian von Ehrenfels: Leben und Werk*. Amsterdam: Rodopi.

FINDLAY, J. N. 1963, *Meinong's Theory of Objects and Values*, 2nd edn. Oxford: Clarendon Press.

FINE, K. 1975, Vagueness, Truth and Logic. *Synthese* 30, 265–300.

—— 1977, Postscript. In PRIOR and FINE 1977: 116–61.

—— 1978*a*, Model Theory for Modal Logic, Part I: The *de re/de dicto* Distinction. *Journal of Philosophical Logic* 7, 125–56.

—— 1978*b*, Model Theory for Modal Logic, Part II: The Elimination of *de re* Modality. *Journal of Philosophical Logic* 7, 277–306.

—— 1981*a*, Model Theory for Modal Logic, Part III: Existence and Predication. *Journal of Philosophical Logic* 10, 293–307.

—— 1981*b*, First-Order Modal Theories I: Sets. *Noûs* 15, 177–205.

—— 1982, Acts, Events, and Things. In LEINFELLNER *et al.* 1982: 97–105.

FINE, K. and SMITH, B. 1983, Husserl (Part One): The Pure Theory. Mimeograph, Manchester.

FITCH, F. B. 1973, Natural Deduction Rules for English. *Philosophical Studies* 24, 80–104.

FORBES, G. 1983, Thisness and Vagueness. *Synthese* 54, 235–59.

—— 1985, *The Metaphysics of Modality*. Oxford: Clarendon Press.

FREGE, G. 1884, *Die Grundlagen der Arithmetik: Eine logisch-mathematische Untersuchung über den Begriff der Zahl*. Breslau: Koebner. Reprinted Hildesheim: Olms, 1977. English translation, FREGE 1953.

—— 1895, Kritische Beleuchtung einiger Punkte in E. Schröders *Vorlesungen über die Algebra der Logik*. *Archiv für systematische Philosophie* 1, 433–56. Reprinted in FREGE 1967: 193–210, and in FREGE 1976: 92–112. English translation, FREGE 1960*b*.

—— 1953, *The Foundations of Arithmetic: A Logico-Mathematical Enquiry into the Concept of Number*, trans. by J. L. Austin, 2nd edn. Oxford: Blackwell.

—— 1960*a*, *Translations from the Philosophical Writings of Gottlob Frege*, ed. P. T. Geach and M. Black, 2nd edn. Oxford: Blackwell.

—— 1960*b*, A Critical Elucidation of Some Points in E. Schröder's *Vorlesungen über die Algebra der Logik*. In FREGE 1960*a*: 86–106. Reprinted in FREGE 1984: 210–28.

——1967, *Kleine Schriften*, ed. I. Angelelli. Darmstadt: Wissenschaftliche Buchgesellschaft.

—— 1976, *Logische Untersuchungen*, ed. and intr. G. Patzig, 2nd edn. Göttingen: Vandenhoeck & Ruprecht.

—— 1984, *Collected Papers on Mathematics, Logic, and Philosophy*, ed. B. McGuinness. Oxford: Blackwell.

GABBAY, D. M. and MORAVCSIK, J. M. E. 1980, Verbs, Events, and the Flow of Time. In ROHRER 1980: 59–83.

GANDY, R. and HYLAND, M. (eds.) 1977, *Logic Colloquium 76*. Amsterdam: North-Holland.

GARRETT, B. J. 1985, Noonan, 'Best Candidate' Theories and the Ship of Theseus. *Analysis* 45, 212–15.

GEACH, P. T. 1949, On Rigour in Semantics. *Mind* 58, 518–22. Reprinted in GEACH 1972: 198–201.

—— 1965, Some Problems about Time. *Proceedings of the British Academy* 51, 321–36. Reprinted in GEACH 1972: 302–17. (Page references are to the reprint.)

—— 1969, *God and the Soul*. London: Routledge & Kegan Paul.

—— 1972, *Logic Matters*. Oxford: Blackwell.

GHISELIN, M. T. 1974, A Radical Solution to the Species Problem. *Systematic Zoology* 23, 536–44.

GINSBERG, E. 1929, Zur Husserlschen Lehre von den Ganzen und Teilen. *Archiv für systematische Philosophie* 32, 109–20.

—— 1931, W sprawie pojęc samoistności i niesamoistności. *Księga pamiat-kowa Polskiego Towarzystwa Filozoficznego we Lwowie*. Lwów, 143–68. English translation, GINSBERG 1982.

—— 1982, On the Concepts of Existential Dependence and Independence. In SMITH 1982: 265–88.

GOGUEN, J. 1969, The Logic of Inexact Concepts. *Synthese* 19, 325–73.

GOLDMAN, A. I. 1970, *A Theory of Human Action*. Englewood Cliffs: Prentice-Hall. 2nd edn. 1977.

GOODMAN, N. 1956, A World of Individuals. In BOCHEŃSKI *et al.* 1956.

Reprinted in GOODMAN 1972: 155–72.
—— 1969, *The Languages of Art*. London: Oxford University Press.
—— 1972, *Problems and Projects*. Indianapolis: Bobbs-Merrill.
—— 1977, The *Structure of Appearance*, 3rd edn. Dordrecht: Reidel.
GRELLING, K. 1969, On Definitions by Equivalence Classes and by Group Invariants. *Methodology and Science* 2, 116–22.
——1987, A Logical Theory of Dependence. In SMITH 1987.
GRELLING, K. and OPPENHEIM, P. 1938, Der Gestaltbegriff im Lichte der neuen Logik. *Erkenntnis* 7, 211–25. English translation, GRELLING and OPPENHEIM 1987*a*.
——1987*a*, The Concept of Gestalt in the Light of Modern Logic, trans. P. M. Simons. In SMITH 1987.
——1987*b*, Logical Analysis of 'Gestalt' as 'Functional Whole'. In SMITH 1987.
GRIFFIN, N. 1977, *Relative Identity*. Oxford: Clarendon Press.
GUPTA, A. 1980, *The Logic of Common Nouns: An Essay in Quantified Modal Logic*. New Haven: Yale University Press.
HACKER, P. M. S. 1982, Events and Objects in Space and Time. *Mind* 91, 1–19.
HÄNDLE, F. and JENSEN, S. (eds.) 1974, *Systemtheorie und Systemtechnik*. Munich: Nymphenburger Verlagshandlung.
HALLER, R. and GRASSL, W. (eds.) 1980, *Language, Logic, and Philosophy*. Vienna: Hölder–Pichler–Tempsky.
HARRÉ, H. R. 1970, *The Principles of Scientific Thinking*. London: Macmillan.
HARRIS, N. G. E. 1986, The Millions of Men in My Shoes. *Mind* 95, 105–6.
HELLER, M. 1984, Transtemporal Parts of Four Dimensional Objects. *Philosophical Studies* 46, 323–34.
HENKIN, L. 1963, A Theory of Propositional Types. *Fundamenta Mathematicae* 52, 323–34.
HENRY, D. P. 1972, *Medieval Logic and Metaphysics*. London: Hutchinson.
HINCKFUSS, I. 1976, Necessary Existential Dependence. *Australasian Journal of Philosophy* 54, 123–32.
HINTIKKA, K. J. J. 1969*a*, *Models for Modalities: Selected Essays*. Dordrecht: Reidel.
—— 1969*b*, Existential Presuppositions and their Elimination. In HINTIKKA 1969*a*: 23–44.
HIŻ, H. 1977, Descriptions in Russell's Theory and in Ontology. *Studia Logica* 36, 271–83.
——1984, Frege, Leśniewski and Information Semantics on the Resolution of Antinomies. *Synthese* 60, 51–72.
HORGAN, T. 1978, The Case Against Events. *Philosophical Review* 87, 28–47.
—— 1980, Non-Rigid Event Designators and the Modal Individuation of Events. *Philosophical Studies* 36, 341–51.
—— 1981, Action Theory without Actions. *Mind* 90, 406–14.
HORNSBY, J. 1980, *Actions*. London: Routledge & Kegan Paul.
HULL, D. L. 1976, Are Species Really Individuals? *Systematic Zoology* 25, 174–91.

HULL, D. L. 1978, A Matter of Individuality. *Philosophy of Science* 45, 335–60.

HUNTINGTON, E. V. 1904, Sets of Independent Postulates for the Algebra of Logic. *Transactions of the American Mathematical Society* 5, 288–309.

—— 1933, New Sets of Independent Postulates for the Algebra of Logic, with Special Reference to Whitehead and Russell's *Principia Mathematica*. *Transactions of the American Mathematical Society* 35, 274–304.

HUSSERL, E. 1891, Besprechung von E. Schröder, *Vorlesungen über die Algebra der Logik (Exakte Logik)*, I. Band, Leipzig 1890. *Göttinger gelehrte Anzeigen*, 243–78. Reprinted in HUSSERL 1979: 3–43. English translation, HUSSERL 1978.

—— 1931, *Ideas: General Introduction to Pure Phenomenology*. London: Allen & Unwin.

—— 1970*a*, *Logical Investigations*, trans. J. N. Findlay. London: Routledge & Kegan Paul.

—— 1970*b*, *Philosophie der Arithmetik*, ed. L. Eley. The Hague: Nijhoff.

—— 1978, Review of Ernst Schröder's *Vorlesungen über die Algebra der Logik*, trans. D. Willard. *The Personalist* 59, 115–43.

—— 1979, *Aufsätze und Rezensionen* (1890–1910), ed. B. Rang. The Hague: Nijhoff.

—— 1984, *Logische Untersuchungen. Zweiter Band. Untersuchungen zur Phänomenologie und Theorie der Erkenntnis*, 2 parts, ed. U. Panzer. The Hague: Nijhoff. 1st edn. 1901, 2nd edn. 1913. Citations are by pagination of the first edition (A) and the second edition (B), which are given in the definitive *Husserliana* edition. English translation, HUSSERL 1970*a*.

INGARDEN, R. 1962, *Untersuchungen zur Ontologie der Kunst*. Tübingen: Niemeyer.

—— 1964, *Time and Modes of Being*. Translation of parts of vol. 1 of INGARDEN 1964–5 by H. R. Michejda. Springfield: Thomas.

—— 1964–5, *Der Streit um die Existenz der Welt*. 2 vols. Tübingen: Niemeyer. partial English translation, INGARDEN 1964.

—— 1974, *Über die kausale Struktur der realen Welt*. Tübingen: Niemeyer.

JOHNSON, W. E. 1921–4, *Logic*. Cambridge: Cambridge University Press.

KARMO, T. 1977, Disturbances. *Analysis* 37, 147–8.

KENNY, A. J. P. 1963, *Action, Emotion and Will*. London: Routledge & Kegan Paul.

—— 1980, *Aquinas*. Oxford: Oxford University Press.

KRIPKE, S. 1972, Naming and Necessity. In DAVIDSON and HARMAN 1972: 253–355, addenda, 763–9. Reprinted as KRIPKE 1980.

—— 1980. *Naming and Necessity*. Cambridge, MA: Harvard University Press.

KÜNG, G. 1974, Prologue-Functors. *Journal of Philosophical Logic* 3, 241–54. Reprinted in MORSCHER *et al.* 1977: 75–88.

—— 1977, The Meaning of the Quantifiers in the Logic of Leśniewski. *Studia Logica* 36, 309–22.

KÜNG, G. and CANTY, J. T. 1970, Substitutional Quantification and Leśniewskian Quantifiers. *Theoria* 36, 165–82.

LAMBERT, J. H. 1974, Drei Abhandlungen zum Systembegriff. In HÄNDLE and JENSEN 1974: 87–103.

LAMBERT, K. 1965, On Logic and Existence. *Notre Dame Journal of Formal Logic* 6, 135–41.

—— 1967, Free Logic and the Concept of Existence. *Notre Dame Journal of Formal Logic* 8, 133–44.

—— 1983, *Meinong and the Principle of Independence*. Cambridge: Cambridge University Press.

LAYCOCK, H. 1972, Some Questions of Ontology. *Philosophical Review* 81, 3–42.

—— 1979, Theories of Matter. In PELLETIER 1979: 89–120.

LE BLANC, A. V. 1983, *A Study of the Axiomatic Foundations of Mereology*. M. A. Dissertation, University of Manchester.

—— 1985*a*, Axioms for Mereology. *Notre Dame Journal of Formal Logic* 26, 429–36.

—— 1985*b*, New Axioms for Mereology, *Notre Dame Journal of Formal Logic* 26, 437–43.

LEHRER, K. (ed.) 1975, *Analysis and Metaphysics*. Dordrecht: Reidel.

LEIBNIZ, G. W. 1969, *Philosophical Papers and Letters*, ed. L. E. Loemker. 2nd edn. Dordrecht: Reidel. (1st edn. 1956.)

LEINFELLNER, W., KRAEMER, E., and SCHANK, J. (eds.) 1982, *Language and Ontology*. Vienna: Hölder–Pichler–Tempsky.

LEJEWSKI, C. 1954, A Contribution to Leśniewski's Mereology. *Polskie Towarzystwo Naukowe na Obczyźnie. Rocznik* 5 [*Yearbook 5 of the Polish Society of Arts and Sciences Abroad*], 43–50.

—— 1957, Abstract of RESCHER 1955. *Journal of Symbolic Logic* 22, 213–14.

—— 1958, On Leśniewski's Ontology. *Ratio* 1, 150–76. Reprinted in SRZEDNICKI and RICKEY 1984: 123–48.

—— 1960, A Re-examination of the Russellian Theory of Descriptions. *Philosophy* 35, 14–29.

—— 1960–1, Studies in the Axiomatic Foundations of Boolean Algebra. *Notre Dame Journal of Formal Logic* 1 (1960), 23–47, 91–106; 2 (1961), 79–93.

—— 1963*a*, A Note on a Problem Concerning the Axiomatic Foundations of Mereology. *Notre Dame Journal of Formal Logic* 4, 135–9.

—— 1963*b*, Aristotle's Syllogistic and its Extensions. *Synthese* 15, 125–54.

—— 1967*a*, A Single Axiom for the Mereological Notion of Proper Part. *Notre Dame Journal of Formal Logic* 8, 279–85.

—— 1967*b*, A Theory of Non-Reflexive Identity and its Ontological Ramifications. In WEINGARTNER 1967: 65–102.

—— 1967*c*, Leśniewski, Stanisław. In *Encyclopedia of Philosophy*, ed. P. Edwards. New York: Macmillan, vol. 4, 441–3.

—— 1969, Consistency of Leśniewski's Mereology. *Journal of Symbolic Logic* 34, 321–8. Reprinted in SRZEDNICKI and RICKEY 1984: 231–8.

—— 1973, A Contribution to the Study of Extended Mereologies. *Notre Dame Journal of Formal Logic* 14, 55–67.

—— 1974, A System of Logic for Bicategorial Ontology. *Journal of Philosophical Logic* 3, 265–83. Reprinted in MORSCHER *et al.* 1977: 99–117.

LEJEWSKI, C. 1977, Systems of Leśniewski's Ontology with the Functor of Weak Inclusion as the only Primitive Term. *Studia Logica* 36, 323–50.

—— 1979, Idealization of Ordinary Language for the Purposes of Logic. In ALLETON *et al.* 1979: 94–110.

—— 1982, Ontology: What Next? In LEINFELLNER *et al.* 1982: 173–86.

—— 1985, A System of Logic for Bicategorial Ontology: An Overdue Postscript. *Conceptus* 19, 73–86.

LEONARD, H. S. 1930, Singular Terms. Dissertation, Widener Library, Harvard University.

LEONARD, H. S. and GOODMAN, N. 1940, The Calculus of Individuals and its Uses. *Journal of Symbolic Logic* 5, 45–55.

LEŚNIEWSKI, S. 1916, Podstawy ogólnej teoryi mnogości I. [Foundations of a General Theory of Manifolds.] *Prace Polskiego Koła Naukowe w Moskwie*, Sekcya matematyczno-przyrodnicza, 2. Moscow.

—— 1927–30, O Podstawach Matematyki [On the Foundations of Mathematics], *Przeglad Filozoficzny* 30 (1927), 164–206; 31 (1928), 261–91; 32 (1929), 60–101; 33 (1930), 75–105, 142–70.

—— 1929, Grundzüge eines neuen Systems der Grundlagen der Mathematik. *Fundamenta Mathematicae* 14, 1–81.

LEWIS, C. I. 1944, The Modes of Meaning. *Philosophy and Phenomenological Research* 4 (1943–4), 236–49. Reprinted in LEWIS 1970: 303–16.

—— 1970, *Collected Papers of Clarence Irving Lewis*, ed. J. D. Goheen and J. L. Mothershead. Stanford: Stanford University Press.

LEWIS, D. K. 1968, Counterpart Theory and Quantified Modal Logic. *Journal of Philosophy* 65, 113–26. Reprinted in LOUX 1979: 110–28 and in LEWIS 1983: 26–38.

—— 1970, General Semantics. *Synthese* 22, 18–67. Reprinted in DAVIDSON and HARMAN 1972: 169–218, and in LEWIS 1983: 189–229.

—— 1983, *Philosophical Papers*, vol. 1. Oxford: Oxford University Press.

LINK, G. 1983, The Logical Analysis of Plurals and Mass Terms: A Lattice-Theoretical Approach. In BÄUERLE *et al.* 1983: 302–23.

LOUX, M. J. (ed.) 1979, *The Possible and the Actual.* Ithaca: Cornell University Press.

LOWE, E. J. 1985, Reply to Baldwin on *De Re* Modalities. *Mind* 94, 101–3.

LÖWENHEIM, L. 1940, Einkleidung der Mathematik in Schröderschen Relativkalkül. *Journal of Symbolic Logic* 5, 1–15.

LUSCHEI, E. C. 1962, *The Logical Systems of Leśniewski*. Amsterdam: North-Holland.

LYONS, J. 1968, *Introduction to Theoretical Linguistics*. Cambridge: Cambridge University Press.

McCALL, S. (ed.) 1967, *Polish Logic 1920–1939*. Oxford: Clarendon Press.

McCAWLEY, J. D. 1981, *Everything that linguists have always wanted to know about logic* *but were ashamed to ask*. Oxford: Blackwell.

McGINN, C. 1976, On the Necessity of Origin. *Journal of Philosophy* 73, 127–35.

McTAGGART, J. M. E. 1921–7, *The Nature of Existence*. 2 vols. Cambridge: Cambridge University Press.

MANSER, A. 1983, *Bradley's Logic*. Oxford: Blackwell.

MARTIN, R. M. 1943, A Homogeneous System for Formal Logic. *Journal of Symbolic Logic* 8, 1–23.

—— 1965, Of Time and the Null Individual. *Journal of Philosophy* 62, 723–36. Reprinted in MARTIN 1979: 82–94.

—— 1978, *Events, Reference, and Logical Form*. Washington: Catholic University of America Press.

—— 1979, *Pragmatics, Truth, and Language*. Dordrecht: Reidel.

MEDAWAR, P. and SHELLEY, J. H. (eds.) 1980, *Structure in Science and Art*. Amsterdam: Excerpta Medica.

MEINONG, A. 1891, Zur Psychologie der Komplexionen und Relationen. *Zeitschrift für Psychologie und Physiologie der Sinnesorgane* 2, 245–65. Reprinted in MEINONG 1969: 279–304.

—— 1899, Über Gegenstände höherer Ordnung und deren Verhältnis zur inneren Wahrnehmung. *Zeitschrift für Psychologie und Physiologie der Sinnesorgane* 21, 181–271. Reprinted in MEINONG 1971: 379–480.

—— 1969, *Abhandlungen zur Psychologie*. Graz: Akademische Druck- u. Verlagsanstalt.

—— 1971, *Abhandlungen zur Erkenntnistheorie und Gegenstandstheorie*. Graz: Akademische Druck- u. Verlagsanstalt.

MELLOR, D. H. 1981, *Real Time*. Cambridge: Cambridge University Press.

MENGER, K. 1940, Topology without Points. *Rice Institute Pamphlets* 27, 80–107.

MIÉVILLE, D. 1984, *Un développement des systèmes logiques de Stanisław Leśniewski. Protothétique–Ontologie–Méréologie*. Bern: Lang.

MOHANTY, J. N. (ed.) 1977, *Readings on Edmund Husserl's 'Logical Investigations'*. The Hague: Nijhoff.

MONTAGUE, R. 1979, The Proper Treatment of Mass Terms in English. In PELLETIER 1979: 173–8.

MORAVCSIK, J. M. E. 1965, Strawson and Ontological Priority. In BUTLER 1965: 106–19.

MORSCHER, E., CZERMAK, J., and WEINGARTNER, P. (eds.) 1977, *Problems in Logic and Ontology*. Graz: Akademische Druck- u. Verlagsanstalt.

MORSCHER, E. and STRANZINGER, R. (eds.) 1981, *Ethics: Foundations, Problems and Applications*. Vienna: Hölder–Pichler–Tempsky.

MULLIGAN, K., SIMONS, P. M., and SMITH, B. 1984, Truth-Makers. *Philosophy and Phenomenological Research* 44, 278–321.

MUNITZ, M. K. 1974, *Existence and Logic*. New York: New York University Press.

NEEDHAM, P. 1981, Temporal Intervals and Temporal Order. *Logique et Analyse* 24, 49–64.

NICOD, J. 1970*a*, *Geometry and Induction*, ed. R. F. Harrod. London: Routledge & Kegan Paul.

——1970*b*, Geometry in the Sensible World, trans. J. Bell. In NICOD 1970*a*: 3–155.

NOONAN, H. W. 1980, *Objects and Identity: An Examination of the Relative Identity Thesis and its Consequences*. The Hague: Nijhoff.

NOONAN, H. W. 1985*a*, Wiggins, Artefact Identity and 'Best Candidate' Theories. *Analysis* 45, 4–8.
—— 1985*b*, The Only X and Y Principle. *Analysis* 45, 79–83.
—— 1986, Reply to Simons on Coincidence. *Mind* 95, 100–4.
NULL, G. 1983, A First-Order Axiom System for Non-Universal Part–Whole and Foundation Relations. In EMBREE 1983: 463–84.
PADOA, A. 1901, Essai d'une théorie algébrique des nombres entiers, précédé d'une introduction à une théorie déductive quelconque. *Bibliothèque du Congrès international de philosophie, Paris, 1900*. Paris: Colin. vol. 3, 309–65. Partial English translation, PADOA 1967.
—— 1967, Logical Introduction to any Deductive Theory. In VAN HEIJENOORT 1967: 118–23.
PANTIN, C. F. A. 1968, *The Relations Between the Sciences*. Cambridge: Cambridge University Press.
PARSONS, C. 1977, What is the Iterative Conception of Set? In BUTTS and HINTIKKA 1977: 335–67. Reprinted in PARSONS 1983: 268–97.
—— 1983, *Mathematics in Philosophy*. Ithaca: Cornell University Press.
PARSONS, T. 1979, An Analysis of Mass Terms and Amount Terms. In PELLETIER 1979: 137–66.
—— 1980, *Nonexistent Objects*. New Haven: Yale University Press.
PEACOCKE, C. 1978, Necessity and Truth Theories. *Journal of Philosophical Logic* 7, 473–500.
PELLETIER, F. J. (ed.) 1979, *Mass Terms: Some Philosophical Problems*. Dordrecht: Reidel.
PLANTINGA, A. 1974, *The Nature of Necessity*. Oxford: Clarendon Press.
—— 1975, On Mereological Essentialism. *Review of Metaphysics* 27, 468–76.
POTTS, T. C. 1965, States, Activities and Performances. *Proceedings of the Aristotelian Society*, Suppl. vol. 39, 65–85.
PRIOR, A. N. 1965–6, Time, Existence and Identity. *Proceedings of the Aristotelian Society*, n.s. 66, 183–92. Reprinted in PRIOR 1968: 78–87.
—— 1968, *Papers on Time and Tense*. Oxford: Clarendon Press.
PRIOR, A. N. and FINE, K. 1977, *Worlds, Times and Selves*. London: Duckworth.
PÜHRINGER, C. 1982, Modelltheoretische und algebraische Untersuchungen über die Theorie der $\cup$-^{c}-partiellen Booleschen Algebren. MA Dissertation, University of Salzburg.
QUINE, W. V. O. 1960, *Word and Object*. Cambridge, MA: MIT Press.
—— 1969*a*, *Set Theory and its Logic*, 2nd edn. Cambridge, MA: Harvard University Press.
—— 1969*b*, *Ontological Relativity and other Essays*. New York: Columbia University Press.
——1976*a*, *The Ways of Paradox and Other Essays*, 2nd edn. Cambridge, MA: Harvard University Press.
——1976*b*, The Ways of Paradox. In QUINE 1976*a*: 1–18.
—1977, Review of EVANS and McDOWELL 1976, *Journal of Philosophy* 74, 225–41.
— 1981, What Price Bivalence? *Journal of Philosophy* 78, 90–95.

RASIOWA, H. 1974, *An Algebraic Approach to Non-Standard Logics*. Amsterdam: North-Holland

RAUSCH, E. 1937, Über Summativität und Nichtsummativität. *Psychologische Forschung* 21, 209–89. Reprinted Darmstadt: Wissenschaftliche Buchgesellschaft, 1967.

REID, T. 1985, *Philosophical Works*, ed. W. Hamilton, Vol. 1. Edinburgh. Reprinted Hildesheim: Olms, 1967.

RESCHER, N. 1955, Axioms for the Part Relation. *Philosophical Studies* 6, 8–11.

RESCHER, N. and OPPENHEIM, P. 1955, Logical Analysis of Gestalt Concepts. *British Journal for the Philosophy of Science* 6, 89–106.

ROBERTS, J. H. 1979, Activities and Performances Considered as Objects and Events. *Philosophical Studies* 35, 171–85.

ROHRER, C. (ed.) 1980, *Time, Tense, and Quantifiers*. Tübingen: Niemeyer.

ROUTLEY, R. 1980, *Exploring Meinong's Jungle and Beyond*. Canberra: Australian National University, Department of Philosophy, Departmental Monograph.

RUDNER, R. and SCHEFFLER, I. (eds.) 1972, *Logic and Art: Essays in Honor of Nelson Goodman*. Indianapolis: Bobbs-Merrill.

RUSSELL, B. 1903, *The Principles of Mathematics*. London: Allen & Unwin.

— 1913, On the Notion of Cause. *Proceedings of the Aristotelian Society* 13, 1–26. Reprinted in RUSSELL 1976: 173–99.

— 1914, *Our Knowledge of the External World as a Field for Scientific Method in Philosophy*. New York: Open Court.

—— 1972, *Russell's Logical Atomism*, ed. with introduction by D. F. Pears. London: Fontana.

—— 1976, *A Free Man's Worship and Other Essays*. London: Allen & Unwin. First published under the title *Mysticism and Logic*, 1917.

RUSZA, I. 1981, *Modal Logic with Descriptions*. The Hague: Nijhoff.

SALMON, N. U. 1982, *Reference and Essence*. Oxford: Blackwell.

SCHLICK, M. 1935, Über den Begriff der Ganzheit. *Wissenschaftlicher Jahresbericht der Philosophischen Gesellschaft an der Universität zu Wien—Ortsgruppe Wien der Kant-Gesellschaft für das Vereinsjahr 1933/34 und 1934/35*, 23–37. Reprinted in SCHLICK 1938: 251–66. English translation, SCHLICK 1979*b*.

—— 1938, *Gesammelte Aufsätze 1926–1936*. Vienna: Gerold.

——1979*a*, *Philosophical Papers*, Vol. 2 (1925–36), ed. H. L. Mulder and B. F. B. Van de Velde-Schlick. Dordrecht: Reidel.

——1979*b*, On the Concept of Wholeness. In SCHLICK 1979*a*: 388–99.

SCHOCK, R. 1968, *Logics Without Existence Assumptions*. Stockholm: Almqvist & Wiksell.

SCHRÖDER, E. 1890–1910, *Vorlesungen über die Algebra der Logik (Exakte Logik)*. Leipzig: Teubner. Reprinted New York: Chelsea, 1966.

SCHULDENFREI, R. 1969, Eberle on Nominalism in Non-Atomic Systems. *Noûs* 3, 427–30.

SHARVY, R. 1978, Maybe English Has No Count Nouns: Notes on Chinese Semantics. *Studies in Language* 2, 345–65.

SHARVY, R. 1980, A More General Theory of Definite Descriptions. *Philosophical Review*, 89, 607–24.
—— 1983*a*, Aristotle on Mixtures. *Journal of Philosophy* 80, 439–57.
—— 1983*b*, Mixtures. *Philosophy and Phenomenological Research* 44, 227–39.
SHORTER, J. M. 1977, On Coinciding in Space and Time. *Philosophy* 52, 399–408.
SIKORSKI, R. 1969, *Boolean Algebras*, 3rd edn. New York: Springer.
SIMONS, P. M. 1978, Logic and Common Nouns. *Analysis* 38, 161–7.
—— 1980, Individuals, Groups and Manifolds. In HALLER and GRASSL 1980: 483–6.
—— 1981*a*, Logical and Ontological Independence in the *Tractatus*. In MORSCHER and STRANZINGER 1981: 464–7.
—— 1981*b*, Brand on Event Identity. *Analysis* 41, 195–8.
—— 1981*c*, Unsaturatedness. *Grazer Philosophische Studien* 14, 73–96.
—— 1981*d*, Review of WIGGINS 1980. *Grazer Philosophischer Studien* 14, 176–82.
—— 1982*a*, The Formalisation of Husserl's Theory of Wholes and Parts. In SMITH 1982: 113–59.
—— 1982*b*, Number and Manifolds. In SMITH 1982: 160–98.
—— 1982*c*, Plural Reference and Set Theory. In SMITH 1982: 199–256.
—— 1982*d*, On Understanding Leśniewski. *History and Philosophy of Logic* 3, 165–91.
—— 1982*e*, Handlungsontologien. *Rechtstheorie* 13, 303–23.
—— 1983*a*, Function and Predicate. *Conceptus* 17, 75–90.
—— 1983*b*, Class, Mass and Mereology. *History and Philosophy of Logic* 4, 157–80.
—— 1985*a*, The Old Problem of Complex and Fact. *Teoria* (Pisa) 5, 205–26.
—— 1985*b*, Coincidence of Things of a Kind. *Mind* 94, 70–5.
—— 1985*c*, Mathematik als Wissenschaft der Gestalten. In FABIAN 1985: 112–35.
—— 1985*d*, Leśniewski's Logic and its Relation to Classical and Free Logics. In DORN and WEINGARTNER 1985: 369–402.
—— 1985*e*, A Semantics for Ontology. *Dialectica* 39, 193–216.
—— 1986, Unkindly Coincidences. *Mind* 95, 506–9.
—— 1987*a*, Gestalt and Functional Dependence. In SMITH 1987.
—— 1987*b*, Brentano's Reform of Logic. *Topoi* 6, 23–36.
SINISI, V. F. 1966, Leśniewski's Analysis of Whitehead's Theory of Events. *Notre Dame Journal of Formal Logic* 7, 323–7.
—— 1969, Leśniewski and Frege on Collective Classes. *Notre Dame Journal of Formal Logic* 10, 239–46.
SŁUPECKI, J. 1953, St. Leśniewski's Protothetics. *Studia Logica* 1, 44–112.
—— 1955, St. Leśniewski's Calculus of Names. *Studia Logica* 3, 7–73. Reprinted in SRZEDNICKI and RICKEY 1984: 59–122.
SMART, J. J. C. 1972, Space-Time and Individuals. In RUDNER and SHEFFLER 1972: 3–20.

SMITH, B. (ed.) 1982, *Parts and Moments: Studies in Logic and Formal Ontology*. Munich: Philosophia.

—— 1985, Addenda to Annotated Bibliography of Writings on Part-Whole Relations since Brentano. In *Parts and Wholes*, vol. 3. Stockholm, Forskningsrådsnämdnen (Swedish Council for Planning and Coordination of Research): 74–86.

——(ed.) 1987, *Foundations of Gestalt Theory*. Munich: Philosophia.

SMITH, B. and MULLIGAN, K. 1982, Pieces of a Theory. In SMITH 1982: 15–109.

SOBOCIŃSKI, B. 1954, Studies in Leśniewski's Mereology. *Polskie Towarzystwo Naukowe na Obczyźnie. Rocznik* 5 [*Yearbook 5 of the Polish Society of Arts and Sciences Abroad*], 34–43. Reprinted in SRZEDNICKI and RICKEY 1984: 217–28.

——1955, On Well Constructed Axiom Systems. *Polskie Towarzystwo Naukowe na Obczyźnie. Rocznik* 6[*Yearbook 6 of the Polish Society of Arts and Sciences Abroad*], 1–12.

—— 1960, On the Single Axioms of Protothetic I, II and III. *Notre Dame Journal of Formal Logic* 1, 52–73; 2, 110–26 and 129–48.

—— 1967, Successive Simplifications of the Axiom-System of Leśniewski's Ontology. In McCALL 1967: 188–200.

—— 1971*a*, Atomistic Mereology. *Notre Dame Journal of Formal Logic* 12, 89–103, 203–13.

—— 1971*b*, A Note on an Axiom-System of Atomistic Mereology. *Notre Dame Journal of Formal Logic* 12, 249–51.

SOKOLOWSKI, R. 1967–8, The Logic of Parts and Wholes in Husserl's Investigations. *Philosophy and Phenomenological Research* 28, 537–53. Reprinted in MOHANTY 1977: 94–111.

——1974, *Husserlian Meditations*. Evanston: Northwestern University Press.

SOSA, E. (ed.) 1979, *Essays on the Philosophy of Roderick M. Chisholm*. Amsterdam: Rodopi.

SPARKES, I. 1975, *Dictionary of Collective Nouns and Group Terms*. London: White Lion.

SRZEDNICKI, J. T. J. and RICKEY, V. F. (eds.) 1984, *Leśniewski's Systems: Ontology and Mereology*. The Hague: Nijhoff.

STENIUS, E. 1974, Sets. *Synthese* 27, 161–88.

STONE, M. H. 1936, The Theory of Representations for Boolean Algebras. *Transactions of the American Mathematical Society* 40, 37–111.

STRAWSON, P. F. 1959, *Individuals: An Essay in Descriptive Metaphysics*. London: Methuen.

SUPPES, P. 1957, *Introduction to Logic*. New York: Van Nostrand.

SURMA, S. J. 1977, On the Work and Influence of Stanisław Leśniewski. In GANDY and HYLAND 1977: 191–220.

TARSKI, A. 1935, Zur Grundlegung der Booleschen Algebra, I. *Fundamenta Mathematicae* 24, 177–98. English translation, TARSKI 1956*c*.

—— 1937, Appendix E to WOODGER 1937: 161–72.

——1956*a*, *Logic, Semantics, Metamathematics*, trans. J. H. Woodger. Oxford: Clarendon Press.

TARSKI, A. 1956*b*, Foundations of the Geometry of Solids. In TARSKI 1956*a*: 24–9.

—— 1956*c*, On the Foundations of Boolean Algebra. In TARSKI 1956*a*: 320–41.

THALBERG, I. M. 1977, *Perception, Emotion and Action*. Oxford: Blackwell.

THOMSON, J. J. 1977, *Acts and Other Events*. Ithaca: Cornell University Press.

—— 1983, Parthood and Identity across Time. *Journal of Philosophy* 30, 201–20.

TILES, J. E. 1981, *Things That Happen*. Aberdeen: Aberdeen University Press.

VAN BENTHEM, J. F. A. K. 1983, *The Logic of Time*. Dordrecht: Reidel.

VAN FRAASSEN, B. C. 1970, *An Introduction to the Philosophy of Time and Space*. New York: Random House.

VAN FRAASSEN, B. C. and LAMBERT, K. 1967, On Free Description Theory. *Zeitschrift für Mathematische Logik und Grundlagen der Mathematik* 13, 225–40.

VAN HEIJENOORT, J. (ed.) 1967, *From Frege to Gödel: A Source Book in Mathematical Logic, 1879–1931*. Cambridge, MA: Harvard University Press.

VAN INWAGEN, P. 1981, The Doctrine of Arbitrary Undetached Parts. *Pacific Philosophical Quarterly* 62, 123–37.

VENDLER, Z. 1967, *Linguistics in Philosophy*. Ithaca: Cornell University Press.

WEINGARTNER, P. (ed.) 1967, *Grundfragen der Wissenschaften und ihre Wurzeln in der Metaphysik*. Salzburg: Pustet.

WEINHANDL, F. (ed.) 1967, *Gestalthaftes Sehen: Ergebnisse und Aufgaben der Morphologie. Zum hundertjährigen Geburtstag von Christian von Ehrenfels*. Darmstadt: Wissenschaftliche Buchgesellschaft.

WELSH, P. J. 1978, Primitivity in Mereology, I. *Notre Dame Journal of Formal Logic* 19, 25–62.

WHITEHEAD, A. N. 1919, *An Enquiry Concerning the Principles of Natural Knowledge*. Cambridge: Cambridge University Press.

—— 1920, *The Concept of Nature*. Cambridge: Cambridge University Press.

—— 1929, *Process and Reality*. New York: Macmillan.

WHITEHEAD, A. N. and RUSSELL, B. 1927, *Principia Mathematica*, 2nd edn. Cambridge: Cambridge University Press.

WIGGINS, D. 1967, *Identity and Spatio-Temporal Continuity*. Oxford: Blackwell.

—— 1968, On Being in the Same Place at the Same Time. *Philosophical Review* 77, 90–5.

—— 1974, Essentialism, Continuity, and Identity. *Synthese* 23, 321–59.

—— 1976, The *De Re* 'Must': A Note on the Logical Form of Essentialist Claims. In EVANS and McDOWELL 1976: 285–312.

—— 1979, Mereological Essentialism: Asymmetrical Essential Dependence and the Nature of Continuants. In SOSA, 1979: 297–316.

—— 1980, *Sameness and Substance*. Oxford: Blackwell.

WILSON, R. J. 1972, *Introduction to Graph Theory*. London: Oliver & Boyd. Reprinted London: Longman, 1975.

WITTGENSTEIN, L. 1913, On Logic, and How Not to Do It. (Review of P. Coffey, *The Science of Logic*.) *Cambridge Review*, 6 Mar. 1913.

WITTGENSTEIN, L. 1953, *Philosophical Investigations*. Oxford: Blackwell.

—— 1961, *Tractatus Logico-Philosophicus*, trans. D. F. Pears and B. F. McGuinness. London: Routledge & Kegan Paul.

WOLTERSTORFF, N. 1980, *Works and Worlds of Art*. Oxford: Clarendon Press.

WOODGER, J. H. 1937, *The Axiomatic Method in Biology*. Cambridge: Cambridge University Press.

WRIGHT, C. 1976, Language-Mastery and the Sorites Paradox. In EVANS and McDOWELL 1976: 223–47.

YOES, M. G. 1967, Nominalism and Non-Atomic Systems. *Noûs* 1, 193–200.

ZALTA, E. N. 1983, *Abstract Objects: An Introduction to Axiomatic Metaphysics*. Dordrecht: Reidel.

ZEMACH, E. M. 1970, Four Ontologies. *Journal of Philosophy* 67, 231–47. Reprinted in PELLETIER 1979: 63–80.

Index

Entries in italics refer to definitions.

'a thorough and painstaking examination of the relationship of part to whole, and the philosophical issues attaching thereto...the detail and thoroughness of his critique and the care with which he elaborates his alternative viewpoint...makes this book a very welcome contribution to the literature.' *Mind*

'Peter Simons is to be congratulated for having produced a very lucidly written, well thought-out work on formal part and whole theory, i.e., mereology. He has brought together in a most competent manner the scattered work on this subject and attempted to rectify its philosophical defects, in itself a considerable achievement.'

History and Philosophy of Logic

'Simons's discussion is almost always marked by fair-mindedness, formal rigour and subtle common sense. *Parts* could easily be the standard book on mereology for the next twenty or thirty years.'

Timothy Williamson, *Grazer Philosophische Studien*

'Simons has produced a clear and careful work both in metaphysics and in the history and logic of mereology.' *The Philosophical Review*

'an important and impressive contribution to mereology . . . and to contemporary ontology.' *Nous*

'Simons' work succeeds on all counts' *Philosophy of Science*

Peter Simons is Professor of Philosophy at the University of Leeds.

OXFORD
UNIVERSITY PRESS

www.oup.com

ISBN 0-19-924146-5

9 780199 241460